Marx's Theory of Commodity and Surplus-Value

The work of the economist from Novosibirsk being a formalised exposition of Marx's commodity and surplus-value theory deals with the process of labour and its factors, the forms of manifestation of value and surplus-value and the characteristics of value as a price law. With the help of extensive statistic data the author shows that the price formation of today corroborates the principles of Marx's theory of value and surplus-value. The book also deals with Marx's concept of the utility of labour products.

The book is intended for specialists and everybody interested in economic theories.

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K. K. Valtukh

Marx's Theory of Commodity and Surplus-Value

Formalised Exposition

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Translated from the Russian by *Boris Kutyrev*
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К. К. Вальтух

ОЧЕРК ТЕОРИИ ТОВАРА И ПРИБАВОЧНОЙ СТОИМОСТИ
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INTRODUCTION

The aim of this book is to present the main points of Marxist-Leninist views on the political economy of capitalism using modern mathematical models. This presentation additionally substantiates the theory scientifically and gives it new opportunities for further development.

When making increasingly extensive use of modeling as a method of cognizance the followers of Marx and Lenin rely, on the one hand, on modern developments in mathematics and technical cybernetics, and the experience of using them in the other sciences and, on the other hand, on the fundamental methodological achievements in this respect accumulated by economic theory itself. The classics of Marxism-Leninism have left us a very rich legacy of ideas in this field. There is no room here to analyse fully the use of mathematics in the economic works of Marx and Lenin. Let us merely point to certain fundamental aspects of their experience in the formalisation of economic theory.

There is almost no chapter in Marx's *Capital* where he has not employed formalised methods, i.e., models, for describing the subject. First, there are the specific non-mathematical formulae for economic phenomena and processes (formulae to express the various forms of value; the formula for the social exchange of matter under commodity production $C - M - C$; the general formula for capital $M - C - M'$; the formulae for the circulation of the various kinds of capital). Second, there are directly mathematical models—numerical and algebraic. With their aid, Marx in fact analysed all the major phenomena and processes of the capitalist economy studied in *Capital*. For instance, there are the formulae for commodity value ($w = c + v + m$), capital ($K = c + v$), the rate of surplus-value, the value composition of capital, the velocity of the circulation of capital, the annual rate of surplus-value, the conditions for simple and extended reproduction, capitalist costs of production, the rate of profit, prices of production, etc. From his analy-

sis of the characteristics of these models Marx derives the main laws and trends in the evolution of the capitalist economy, i.e., the laws of the changing rate of surplus-value and rate of profit, of the realisation of social capital, and the like.

Whole chapters of *Capital* are devoted to an analysis of formal dependences. For example, in Volume I of *Capital* (Chapter IX), three laws relating to the rate and mass of surplus-value are derived from an analysis of the formula for the mass of surplus-value in two specially developed modifications; Chapter XVII is based on the use of magnitudes of a partial derivative-type to examine the dependences of the relationship between the price of labour power and surplus-value on the length of the working day, intensity and productivity of labour (although Marx does not derive these partial derivatives formally).

In a number of cases, Marx made use of numerical examples and charts that are not generally expressed in the shape of formal models and, in this sense, retain the nature of illustrations. They are, however, brought to a level, the very next step beyond which would involve the building of a model. As an example, let us mention the charts used to analyse the formation of the average rate of profit and those of simple and extended reproduction. Marx also drew up a numerical chart of a four-sector economy constituting a prototype of the input-output table with more than two branches of business; in this chart, one branch of business is represented by two industrialists, the description being more general than in the usual modern input-output tables and representing the first step towards building a model of capitalist competition.[1]

Lenin, of course, generalised Marx's charts of the reproduction of social capital for the case of a growth of the organic composition of capital. Let us also point to the numerical chart of an economy consisting of three branches of industry, with six producers, which Lenin used to analyse the problem of the transformation of subsistence economy into commodity economy and then into capitalist economy.[2] This

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[1] See: Karl Marx, "Outlines of the Critique of Political Economy", Karl Marx, Frederick Engels, *Collected Works*, Vol. 28, Progress Publishers, Moscow, 1986, pp. 362-66.

[2] See: V. I. Lenin, "On the So-Called Market Question", *Collected Works*, Vol. 1, Progress Publishers, Moscow, 1972, pp. 79-111.

chart again leads to the verge of the creation of an appropriate model.

There are now, of course, much greater opportunities for systematic use of mathematics than at the time of the classics of Marxism-Leninism. These opportunities first of all result from the development of mathematics itself, especially from the achievements in the field of set theory, probability theory, modern algebra, and the theory of vector spaces, as well as from their application in describing numerous subjects in various sciences. They also result from the mastering of mathematics by economists and the involvement of mathematicians in economic studies. These processes have taken some time to develop. At the same time, it should be emphasised that the legacy of the classics of Marxism-Leninism relating to the methodology in the field of the employment of formal techniques retains its great importance. The main thing here is that, using relatively simple techniques, Marx and Lenin formalised not only the description of the technological component of the economy, but also that of the social relations constituting its form; moreover, the dynamic and probabilistic properties of the economy, generated by law-governed technological progress, are taken into account.

The legacy of the classics of Marxism-Leninism contains, above all, the principles of scientific abstraction, of constructing pure subjects, which is a necessary condition for creating productive models, especially theoretical ones. The attentive reader of *Capital* cannot but notice the thoroughness with which Marx, every time, points to specific features of the real economy from which he abstracts. He demonstrates in particular that, if they were taken into account, they would not change the conclusions of the corresponding step in the study; when further considering problems lying closer to the surface, in the substantial analysis, Marx consequently makes use of the factors he initially disregarded to specify and develops his previous conclusions on this basis, in accordance with the general methodology of ascent from the abstract to the concrete.

To illustrate this let us merely point to the transition from the formula for the rate of surplus-value to the formula for the annual rate of surplus-value—to that of the rate of profit without regard for the velocity of the circulation of capital—to that of the annual rate of profit—to that of the average rate of profit and the price of production—to that of

the rate of profit with regard not only for productive capital but also to capital of circulation —to that of the rate of profit with regard not only to industrial but also to merchant's capital—to the formalised description of the division of average profit into interest and the profit of enterprise—to the formalised description of the transformation of a part of surplus-value into ground rent, with the appropriate specification of the description of the average rate of profit, etc.

It is of profound general methodological importance that Marx was able to present, in a formalised manner, such fundamental points of his socio-economic theory as the concept of the exploitation of the proletariat by the class of capitalists, the general capitalist production development trends (growth of the organic composition of capital and its effect on the general rate of profit), with the ensuing conclusions relating to the class struggle of the proletariat. To create theoretical socio-economic models to express the fundamental causes and inherent laws of the totality of phenomena observed as they develop is the only way to build up a methodologically sound theory. Such models cannot be replaced by empirical ones for formulating a theory. The latter models reflect only the surface of the phenomena, although these do have a certain role to play in science.

The formalised description of the subject at the level of its laws has opened up the natural way to relevantly expressing its probabilistic and dynamic nature. Marx repeatedly indicated directly that the economic laws of capitalism operate through a chaos of chance occurrences as long-term trends, as blindly acting law of mean numbers. The building of theoretical models to reflect the average result of numerous random fluctuations permits deterministic formalisation techniques (both relatively simple and the most developed) to be employed to describe the probabilistic environment.

As the models are built as theoretical ones, expressing the typical properties of the subject, their use means that the national economic nature of economic laws can be reflected. When considering private capital and formally expressing its specific features, Marx disregarded any branch or other local peculiarities; when examining the departments and branches (studying social reproduction, the overall rate of profit, ground rent) he took them as being interconnected with the other branches, and presented their system as a national economic whole. This saves his models from the inev-

itable shortcomings engendered when local subjects and individual parts of the economy are described separately.

The methodological achievements of the classics of Marxism-Leninism in the use of modeling techniques are especially important because they employed them to build the actual *theory* of social production.

In order to distinguish theoretical models from empirical ones, the very concept of theory should be discussed as a certain stage in the cognizance of external objects. Science widely accepts the concept of theory in the form imparted to it by Albert Einstein. Theory is characterised, first, by the attribute of external justification, second, by internal perfection, the former, i.e., accordance with the properties of the facts that science already knows, being a general requirement that science must fulfil at all the stages of acquiring knowledge. Theory as such is specifically characterised by the fact that it reduces the whole set of well-known regularities to a substantially smaller number of their causes (inner laws); theory is the more perfect the smaller the number of assumptions with which the empirically given properties of an object can be explained. This concept of theory, being a formal one, provides a profound answer to the essential definition of theoretical knowledge within the system of stages of cognizance: this is knowledge of the essence of phenomena, which explains their surface and allows specific features of phenomena not yet observed, including the results of human practice, to be forecast.

Since theory explains and forecasts the set of the object's empirically discovered properties from its relatively small number of unobservable, internal properties, the work on building a theory logically consists only in, first, formulating some system of assumptions; second, analysing the properties of this system, deriving as an advanced system of conclusions as possible from the assumptions (for actual verification and practical use). That is why the mathematical form of the building, development, and exposition of a theory is, in principle, suited to its concept. The level of mathematical formalisation of a theory depends, of course, on the development level of mathematics itself, on the degree to which all the logical achievements of human thinking have been mastered. Mathematics seems, in its most general, abstract sections, to develop, in fact, as specific logical techniques created by science as suitable forms for building a theory. (This is but one aspect of the development of mathema-

tics which is, at the same time, abstraction of the quantitative side of various external phenomena and creates techniques for carrying out applied calculations.)

In striving to create sufficiently elaborated theories, economic science cannot but make use of the entire set of opportunities offered by modern mathematics. At the same time, special research justifies the assertion that economic theory cannot, today, be built up only in the form of a system of mathematically formulated axioms and theorems. Moreover, modern mathematics fails to provide the techniques required precisely for the most general and profound results of economic theory.

The most advanced sector of economic theory is the theory of the capitalist mode of production which, at the same time, contains the fundamental methodological ideas of economic theory as a whole. This theory answers the general concept to the highest degree: all the major properties of its subject are derived from one (certainly internally complex) assumption about the commodity as the cell of the bourgeois economy, i.e., they are represented as the development of the internal contradiction between the use-value and value of the commodity. Lenin saw this as a model of any theory in general.

A formalised presentation of the political economy of capitalism using mathematics broadly is, in our opinion, of special scientific importance. By giving a more sophisticated form to this theory, it opens up broad scope for its further development on the basis of the potent, multi-faceted, sophisticated, logical techniques of mathematics and computers. The path to the application of theory (in particular, for forecasting purposes) is substantially shortened when it is represented by models. Theory is directly involved in the flow of mathematically described research, which is of great importance for its comprehension by both economists and scientists engaged in other fields. This is also of importance for its creative mastering by university students in both economic and other studies. Lastly, it offers further opportunities for Marxists to succeed in the ideological struggle against bourgeois economic doctrines; in particular, it deprives bourgeois economists of the opportunity to play on the difference in the mathematical level of the two opposing theories.

Recently, the flow of mathematical economic literature has been swollen rapidly in the West by works devoted to

the fundamentals of Marxist-Leninist economic theory, i.e., the concepts of value and surplus-value; a number of books[1] and many articles have been published. Generally we are dealing with a quite distinctly established special sector of theoretical economic literature. This can be seen as a reaction to the increased interest in Marxism demonstrated in scientific and student circles owing to the obvious bancruptcy of all attempts by bourgeois economists to propose a theory of prices and cycle that corresponds to reality.

This literature requires thorough critical analysis by Marxists. The difference in the authors' outlooks and the disputes among them must not be ignored. Some of them, for example P. Samuelson, J. Steedman, are obviously anti-Marxists, whereas others try to understand Marx's economic theory and treat it as science (for instance, W. Baumol, G. Abraham-Frois and E. Berreby, Y. Fujimori). Various intermediate positions are taken by M. Morishima, G. Maarek, etc.

In this literature, most attention has been attracted by the works by P. Samuelson, M. Morishima, J. Steedman, and other authors who employ modern mathematical techniques in an attempt to refute Marx's economic theory, and the fundamentals of his theory of society as a whole. Outwardly this is usually offered in the form of attempts to expose Marx's theory strictly, to fix its assumptions and conclusions, and to demonstrate that they do not correspond to one another. In fact, the "exposition" either directly contradicts Marx's theory (and reality together with it) or reduces this theory to a particular, almost non-realistic case. In some or other way, the above authors try to remove Marx's economic theory concerning value and surplus-value from economics.

All these attempts to expose or refute Marx's economic theory demonstrate, in fact, that the latter is too much for them. The use of mathematical models merely lays this fact bare. Mathematics is a strict science and Marxian economic

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[1] Michio Morishima, *Marx's Economics. A Dual Theory of Value and Growth*, London, 1973; Khoshimura Shinzaburo, *Theory of Capital Reproduction and Accumulation*, London, 1975; Jan Steedman, *Marx after Sraffa*, London, 1977; Gilbert Abraham-Frois and Edmond Berreby, *Theory of Value, Prices and Accumulation. A Mathematical Integration of Marx, von Neumann and Sraffa*, Cambridge, 1979; Gérard Maarek, *An Introduction to Karl Marx's "Das Kapital". A Study in Formalization*, Oxford, 1979; Y. Fujimori, *Modern Analysis of Value Theory*, Berlin—Heidelberg—New York, 1982; etc.

theory is the same. That is why they are in deep internal mutual correspondence. If mathematical techniques are really used scientifically, it is impossible to refute the theory; on the contrary, its might can be repeatedly demonstrated, and it can be developed further. If, nevertheless, a certain author strives to refute Marxism using mathematical methods, he has to choose one of two ways: (1) instead of mathematical models identical to the subject of study and providing sufficient grounds for studying relevant problems, to propose inadequate models that ignore precisely those characteristics of the subject without an understanding of which the theory cannot be constructed; (2) build more or less adequate models, but interpret them at variance with their own properties.

Yet in either case, mathematical presentation hampers the bourgeois authors: their logical errors show through much more clearly and are uncovered much quickly than if they had chosen a less formalised exposition. In fact, extensive use of mathematics can only serve to convince any objective, unprejudiced researcher of the correctness of Marxism and the invalid nature of criticisms of it. Once again Lenin's words have been substantiated: "The development of science is providing more and more material that proves that Marx was right".[1]

All attempts, without exception, to refute the fundamental concepts of the theory of value and surplus-value mathematically are reduced to the above two variants of scientifically incorrect points. I have published papers refuting the main lines of mathematical "critiques" of Marx: attempts to demonstrate the existence of some contradiction between the theory of value and that of prices of production,[2] attempts to demonstrate that social value can be a negative or indefinite magnitude, etc.[3]

In this work I shall not return to the dispute with Marx's "critics". The main thing that can be offered to the unprejudiced reader is a positive exposition of Marx's theory using modern mathematical techniques. At the same time, the book will demonstrate that the laws of Marxian political

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[1] V. I. Lenin, "The Collapse of the Second International", *Collected Works*, Vol. 21, 1974, p. 222.

[2] See: *Social Sciences*, USSR Academy of Sciences, Vol. XI, No. 4, 1980, pp. 179-97, and Vol. XIV, No. 2, 1983, pp. 211-16.

[3] See: *Social Sciences*, USSR Academy of Sciences, Vol. XIV, No. 1, 1983.

èconomy are supported by mass statistics with great precision, the actual verification being, in any true science, the key criterion for evaluating theoretical conceptions.

The theory of value and surplus-value not only explains the properties of capitalism. This inevitably suggests the conclusion that capitalism must be replaced by a new, communist system, where every member of society is free from exploitation and other forms of oppression, where all the conditions are created for complete satisfaction of material and intellectual requirements, free comprehensive development of the personality of everyone. It is these conclusions that are the reason that the bourgeois apologists strive to undermine the fundamentals of Marx's economic theory. We shall focus on the demonstration of validity precisely of these fundamentals of the theory.

* * *

I am sincerely thankful to I. A. Itskovich for long-term cooperation and valuable aid in theoretical mathematical economic research, and I will always keep in my memory this wonderful man and scientist. A tremendous amount of work on making the calculations published here and on preparing the manuscript was carried out by L. G. Krivoshapova, A. V. Shcherbinskaya and L. P. Veretennikova, and the author avails himself of this opportunity to express his gratitude to them.

Chapter 1

CONDITIONS OF THE RISE AND EXISTENCE OF PRIVATE COMMODITY PRODUCTION

1.1. Technology and the Labour-Process

Every scientific theory is embodied in a system of concepts supplied with a special glossary. Only together with these concepts and glossary can the theory be perceived. In its concepts and terminology, Marxian economic theory differs substantially from the bourgeois theoretical conceptions that preceded it and exist alongside it. These concepts and terms have been thoroughly developed. A system of concepts describing some general characteristics of any process of production and exchange serves as an introduction to commodity theory.

In accordance with the nature of the object, no stagnant, extrahistorical categories exist in the theory. We shall formulate the general concepts in such a way as to make them correspond to the conditions of the fairly developed economy with which the researcher into commodity production deals. In their undeveloped, sometimes embryonic form, however, the phenomena expressed by these general concepts are characteristic of all the previous stages in the evolution of production, beginning with the primitive ones.

Technology. The initial observed general fact constituting the specific subject of the economic research is that Man continually, regularly creates from the objects occurring in Nature products that the spontaneous action of the forces of Nature either does not generate at all or does so only relatively rarely. The perception of this fact provides the initial idea of *production.*

The initial products purposefully manufactured by ape at the start of its protracted evolution into human were a more or less close imitation of certain natural objects (for example, sharp stones). The intentional manufacturing of them (in contrast to simple collecting) became entrenched in ape

behaviour since experience showed this to take less time. In other words, the frequency of the natural occurrence of the stones of required shape over a certain span of time, for example during daylight, proved to be consistently smaller than the number that could be produced artificially over the same period of time. Here the fundamental difference between human production and Nature showed itself in embryonic form: the steady, in this sense *law-governed*, obtaining of results that are of *low probability*, or even completely *improbable* in Nature itself (without human intervention). In the case of sharp stones we are dealing precisely with a product of Nature that is of low probability (occurring rarely, though existing). At the same time, another property of any technology is felt here: man is acting in it as a force of Nature, using objects provided by Nature, and fulfilling the laws of Nature itself. Man does not create any new laws of Nature, all of which are applicable to human production. Human beings master the laws of Nature only by observing them. They merely make deliberate use in technologies of the laws of Nature existing outside their will. For this reason, *technologies are natural (physical, chemical, biological) processes in the form imparted to them by Man.* The content of this initial definition will be further elaborated.

Negentropy as a property of technologies. The reproduction of products simulating certain natural products (occurring, or at least having occurred in Nature) continues to be one of the segments of human production activities. For instance, people grow a forest (but in a place, at a time, and with such a composition that the probability of just such a forest occurring naturally is virtually zero). Yet on the whole, the development of production has gone the way of creating increasingly diverse kinds of products that never occur in Nature. Regularly created are things that do not occur in Nature at all.

People have created an artificial sphere for their habitation (a special flora and fauna, settlements, household furniture and utensils, enterprises of material and nonmaterial production, and the like). This sphere being involved in the natural environment of the Earth and space spatially is connected with them over time by the flows of matter and energy, yet is, at the same time, separated from them as being the creation of the human intellect and human hand, as well as by developing in a specific way. From the standpoint of information theory, the creating and developing of

this artificial sphere is a process of ordering, of increasing variety, i.e., one in which the *negentropic trend* is clearly felt.

Modern science believes that in the known Universe (at any rate on the Earth and in near-Earth space), the predominant processes are of the transformation of less probable states into more probable ones, i.e., there is an increase in entropy and decrease in the level of ordering and variety. Yet production is the transition from relatively probable states of matter (for Nature as such) to very diverse, hardly probable and virtually improbable states (for Nature as such), i.e., the processes of increasing negentropy and the level of ordering and variety. In this sense, technological processes are opposite to Natural ones.[1]

An important, and complicated, question is whether the increase in negentropy in the artificial sphere of human habitation merely as a result of the increasing entropy in the environment is an inevitable characteristic of technologies. When deposits of iron ore are turned into hundreds of thousands and millions of diverse metal goods, negentropy increases; but, in this case, natural landscapes are destroyed, Nature is polluted by industrial waste and undergoes thermal pollution, corrosion and other processes that eventually lead to the dissipation of metal in the environment. Thus, entropy increases in relation to the level of the initial metal deposit.

There are a lot of technologies of this type. The secondary, tertiary, etc. utilisation of raw materials is, however, intensifying, together with the more wide use of solar and wind energy, the power of rivers, sea waves, and tides; for a long time agrotechnical methods have been used in agriculture that not only do not destroy natural soil fertility, but even increase it. It is not quite clear whether these processes of technological development are able to stop, or only slow down, that additional increase in entropy introduced into Nature by anthropogenic activities.[2] This does not mean, of course, any artificial halt to the natural processes of increasing entropy.

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[1] Here we are abstracting from negentropic trends in Nature itself (these being, in particular, characteristic of the evolution of living organisms).

[2] Neither is it clear how the total of the negentropical and entropical properties of technologies may be summed.

To point out the fact that the results of production are improbable (of low probability) as a consequence of the spontaneous action of the forces of Nature is completely insufficient for understanding the form imparted by technologies to objects taken from Nature. This is rather a negative than a positive characteristic: the products differ from those that may occur directly in Nature. A positive characteristic of their specific features is given using the concept of utility.

The utility of the results of production. The quality of the objects provided directly by Nature and the results of production to meet human requirements is called their utility. Requirements are defined as those material, intellectual, and social living conditions the achievement of which is necessitated by the process of society's development and towards the achievement of which society, its particular groups and individuals actively strive. Strictly speaking, certain requirements have as their object living conditions that are not the direct results of production or Nature (for example, the requirement for labour, for a specific job, for contact with other people, and the like). Yet no requirements exist that may be met without using any products at all.

The utility of a thing makes it a use-value. Use-value is thus the totality of the diverse characteristics of products that make them useful, i.e., potentially able to meet certain human requirements.

We do not intend to discuss here the problem of the rise and evolution of requirements themselves (to some extent this is analysed below in the sections dealing with technical progress and private consumption). For the time being it is sufficient to state the existence of various requirements and their change in the process of mankind's development. The initial, basic classification divides needs into productive and personal ones.[1] Both are always partly met by Natural products, which thus acquire a specific characteristic from man's point of view. The existence only of requirements satisfied by natural objects is, however, a property of living matter at an evolutionary stage prior to the appearance of Man. The very fact of the existence of technologies, beginning with the most primitive ones, shows that requirements have

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[1] The more strict term for the second group of requirements is: *nonproductive*. It covers not only the personal, but also specific requirements for means of waging war. In this book, however, we shall not be studying problems of arms production.

arisen or are, at least, sporadically arising, that cannot be met by natural products alone.

Expediency of technologies. To impart to objects taken from Nature a form in which they meet (or acquire the ability to do so) certain human needs is the *goal* of any recurrent technological process. The concept of technology as formulated above now can be specified thus: technologies produce not only products improbable (hardly probable) as a result of the spontaneous action of the forces of Nature, but, precisely, ones that prove useful to Man. In accordance with the classification of requirements, these products have to be divided, with regard to their actual final use, into *means of production* and *articles of consumption*. Products exist that may be used in either way, for example sugar, salt, milk, meat, and the like. The natural form of products is, however, as a rule such that some of them can be used as means of production only (for instance, various machines, metal in ingot form, etc.), whereas others—as articles of consumption only (for example, clothes, footwear, many types of utensil, etc.). For the sake of pedantic strictness alone, we may specify that by this we mean the use of products according to their intended purpose.

The goal is the constituting principle of any technology in contrast to natural processes. The process itself, the combination and interaction of all its components, are subordinated to it. In this sense, one can speak of the *accordance* of technology to the product as its goal. If the products created by two technologies are different, we are dealing with different technologies. Yet no *one-to-one* correspondence exists between products and technologies: technologies differing in the elements used and in the way these interact can produce virtually the same output (as far as the aggregate of useful qualities and physical forms are concerned).

It is apt here to define the concept of virtually *identical* products. Obviously, certain differences between two units of products taken as identical are, in fact, inevitable: not only between two rams, but even between two metres of fabric from a single roll. These differences may even be significant from the consumer's point of view. Nevertheless, the products are assumed, in theory, to be the same if their differences arise technologically by chance, i.e., if, at a given point in time, there exist no technologies that regularly impart these different properties to the products in question. Moreover, theory ignores the fact that the different

units of such products may not be equally efficient in consumption.

Identical products may be produced both by one and the same unit technology (in successive portions) or by different unit technologies.

Accordingly, products are assumed to be *different* if (a) they differ in their potential useful qualities (physical form); (b) these differences are regularly reproduced by technologies.

Some technologies, usually being the majority, produce strictly one kind of output, but there exist technological processes that inevitably generate a set of different products, all of these being the goal of such processes. A vivid example is the breeding of cattle for slaughter, the goal of which is to obtain a lot of useful products: meat of various sorts, hide, horns, hooves, etc. These products cannot be produced separately, but only as *associated* ones (i. e. co-products).

Such technologies should not be confused with multiproduct economies. One economy can evolve several different technologies. Nothing is changed by the fact that these technologies are somewhat similar with regard to the means of production and labour used. One and the same machine-tool plant can produce completely different lathes. If the manufacturing of the different products is distinguished over space and time, it means that they are the results of different technologies.

The situation when really technologically associated propucts are produced (in contrast with multiproduct economies) are found relatively rarely. Below everywhere, unless otherwise specified, we disregard the existence of such technologies. But the offered theory will be shown to cover this particular case as well.

Real technologies very often generate, besides specified products, also ones that were not deliberate. Both useful and useless results can be found among them, and even ones harmful for humans also.[1] We have already mentioned the negative effect of technologies on the natural environment. Technologies are not, however, brought about for the sake of these side effects but for that of obtaining a certain product. Note that it is only by this attribute of the correspondence with the preset goal that any distinction at all may

[1] Note at once that technologies the direct goal of which is to create harmful products are also usually called useful ones, i.e., ones satisfying a certain requirement existing in society.

be drawn between the primary products and collateral products of technologies.

In considering the utility of products, it is not out of place to return to the concept of technology as the negentropic process of the transition from probable states of matter to ones of low probability, and completely improbable ones, from the point of view of the spontaneous action of the forces of Nature. Not only an outward similarity, but also a substantial fundamental inner correspondence with it can be seen in the concept of the *order of use-value* proposed by Marx.

"Any use-value premised as an element for the formation of a new use-value is, in relation to it, use-value of a lower order, since it constitutes an elementary precondition for it; the order of a use-value is, meanwhile, higher, the more labour processes the elements out of which it is formed anew are subjected, and the more mediated, as it were, is its existence".[1]

Technologies acquire their goal-oriented, specific nature from being processes of human labour. Their purposefulness implies, first, goal-setting; second, the drawing up of a plan of action, in this case of technological operations, leading to the achievement of the goal; third, vigorous actions to carry out this plan. All these are innate qualities of subjects, human beings, endowed with consciousness. *An objective property of technologies* (as well as of the whole economy growing up on their basis), *a property that determines their specific features in relation to spontaneous natural processes is the fact that human beings create them and act by means of them.* Production can be correctly (objectively) understood only as a specific activity of a human being endowed with consciousness. The technological activities of humans are called the labour process.

"Labour is, in the first place, a process in which both man and Nature participate, and in which man of his own accord starts, regulates, and controls the material re-actions between himself and Nature. He opposes himself to Nature as one of her own forces, setting in motion arms and legs, head and hands, the natural forces of his body, in order to appropriate Nature's productions in a form adapted to his own wants. By thus acting on the external world and changing

[1] Karl Marx, "Zur Kritik der politischen Ökonomie (Manuskript 1861-1863)", in: Karl Marx, Friedrich Engels, *Gesamtausgabe* (MEGA), II, Band 3, Teil 1, Dietz Verlag, Berlin, 1976, S. 51.

it, he at the same time changes his own nature. He develops his slumbering powers and compels them to act in obedience to his sway. We are not now dealing with those primitive instinctive forms of labour that remind us of the mere animal. An immeasurable interval of time separates the state of things in which a man brings his labour-power to market for sale as a commodity, from that state in which human labour was still in its first instinctive stage. We presuppose labour in a form that stamps it as exclusively human."[1]

Goal-setting. This is conditioned by people's nonproductive requirements. Yet real production creates products to meet these requirements only as final results of entire *technological systems*, including a lot of *unit* technologies. Each of these technologies is provided with its own direct goal, and only at the end of long chains of these is the goal to produce goods for nonproductive consumption. All foregoing technologies are aimed at manufacturing various means of production.

Goal-setting is the initial step in any production, of the labour process. "A spider conducts operations that resemble those of a weaver, and a bee puts to shame many an architect in the construction of her cells. But what distinguishes the worst architect from the best of bees is this, that the architect raises his structure in imagination before he erects it in reality. At the end of every labour-process, we get a result that already existed in the imagination of the labourer at its commencement."[2]

Unit technology. Only by a system of goals can the concepts of unit technology and of the system of technologies be defined. In a certain sense, the goal is always a finished product, but the concept of the latter is not quite definite. Since a product that is intended for productive use, has not completely left the production process, it is not a fully finished, but, rather, some intermediate one. This applies to all man-made means of production. If, however, such an intermediate product has such a form that further processing may be stopped for a time, it may become the specific goal of an individual technology. Whether it will become such, or is regarded merely as a part, as a semi-finished product of some unit technology, it is the matter of the

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[1] Karl Marx, *Capital*, Vol. I, Progress Publishers, Moscow, 1974, pp. 173-74.

[2] *Ibid*, p. 174.

organisation of technologies, of their organisational division. Hence unit technology is a system of technological operations (processes) that yields a product of a certain independent form and is organisationally separated as a technology the immediate goal of which is just such a product. A process previously considered as a component of some unit technology can then be separated out as a special technology, and vice versa.

The concept of unit technology should not be confused with that of an economic unit. The latter may operate several unit technologies.

A system of technologies. The technological system underlying the economy as a whole appears as a complex of interrelated unit technologies, ultimately producing products for nonproductive use. They necessarily include technologies for the primary production of raw materials, for producing instruments of labour, and for manufacturing final output; there are also technologies of the successive stages of the processing of raw materials.

No labour processes, nor, therefore, technologies exist that could be performed without employing the natural environment, its products and laws. There do exist, however, special processes of the *primary* acquisition of natural materials by human beings, the primary extraction of these substances from their purely natural relations. This means the extraction of minerals, the cultivation of plants and breeding of animals, forestry, fishery, and the like. A major specific feature of them economically is that purely natural peculiarities (different characteristics of mineral deposits, active natural fertility of the soil, its potential fertility depending on different methods of cultivation and the use of fertilisers, the natural relief of a fertile locality, as well as the location of all of these natural objects on the Earth and in the space around it) fairly tangibly tell on the level of labour input, connected with the carrying out of the technological process. On the subsequent stages of production the level of inputs is not so greatly affected by natural conditions. The major ones of these are manufacturing, transport (these two, together with the initial stages, make up *material* production), services and intellectual production (making up *nonmaterial* production). These stages are distinguished primarily by their goals and, for this reason, by some general characteristics of technological processes.

The planned character of technologies. The conscious aspect

originating from the subject of production determines not only the goal of technologies. Technologies are a complex of physical (including mechanical, in particular simple spatial movement), chemical, biological, and intellectual processes, the natural outcome of which is the receipt of specific results. This combination of necessary processes consciously determined in advance by humans, first appears as a plan, the technological process always being the subsequent fulfilment of this plan. That is why technologies constitute the results of human knowledge and creative work.

"He [man] not only effects a change of form in the material on which he works, but he also realises a purpose of his own that gives the law to his modus operandi, and to which he must subordinate his will."[1]

There exists a process of the mutual correspondence of technologies and their specific results. Not only should a technology correspond to its goal, but also the goal should be technologically adapted. It must imply processes that do not run counter to the laws of Nature. The failure of the attempts to create a *perpetuum mobile* were eventually understood as a result of the contradiction between this goal and the natural law of the conservation of energy. Situations occur where the goal can, in principle, be attained, but the way to do so for some time remains unknown or is not sufficiently developed. This can be exemplified by regulated thermonuclear synthesis to obtain energy.

The realisation of technologies as a labour process. Real technologies have, as a rule, always presupposed the direct involvement of human beings in the actual performance of the process, their vital deliberate activity. It is this activity on the part of humans in practically performing and regulating technological processes that is the labour process proper as defined above. Marx says in this respect: "This subordination [of the will of the producer to technological procedures] is no mere momentary act. Besides the exertion of the bodily organs, the process demands that, during the whole operation, the workman's will be steadily in consonance with his purpose. This means close attention. The less he is attracted by the nature of the work, and the mode in which it is carried on, and the less, therefore, he enjoys it as something which gives play to his bodily and mental powers, the more close his attention is forced to be."[2]

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[1] Karl Marx, *Capital*, Vol. I, p. 174.

[2] *Ibid.*

So far there have been virtually no even unit technologies that could do without continuous human participation. Economic theory has always taken as its real subject production based just on such technologies. Below we shall in all cases assume that each unit technology needs a certain labour input in order to operate.

As a rule even unit technologies presuppose the participation of many people, who necessarily, therefore, enter into more or less complex relations of labour cooperation. (Here we are faced with one of the primary manifestations of the fundamental characteristic of production that it is possible only as a unity of the technological process and social relations between people.) Accordingly, besides the concept of the individual worker economic theory uses that of the *collective worker* whose labour realises technologies, whereas the labour of an individual is not enough for this purpose.

Labour power and the means of production. Any technological process, in whatever sphere it is performed, implies the use by humans of things external to them. Accordingly, two kinds of necessary preconditions for production are distinguished: (1) its material, objective conditions—means of production, (2) its subjective condition—labour power.[1]

Labour power is defined as the ability of humans to work, i.e., the totality of their physical and intellectual qualities, knowledge, and skills sufficient to perform certain technological processes.[2] This general definition omits, for the time being, any indication of the range of technologies the individual is able to use, or the answer to the question of the need for professional training in order to acquire a real ability to work. It should be pointed out that this ability, if not regularly utilised, is gradually lost, whereas real labour (if performed under conditions excluding excessive intensity and, let alone, mutilations) not just maintains but actually develops, and improves the worker's skill.

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[1] "...In the process of labour, the use-values included in it break down into two aspects, strictly limited in concept, and opposites ... on the one hand, the material means of production, the *objective* conditions of production, and on the other, active labour power, purposefully revealed labour power, the *subjective* condition of production" (K. Marx, F. Engels, *Collected Works*, Vol. 49, Politizdat, 1974, pp. 36-37, in Russian).

[2] "By labour-power or capacity for labour is to be understood the aggregate of those mental and physical capabilities existing in a human being, which he exercises whenever he produces a use-value of any description" (Karl Marx, *Capital*, Vol. I, p. 164).

Means of production are usually divided into objects of labour and instruments of labour. The former are the objects to which the person imparts a special form in the labour process adapting them to his needs; they also include numerous auxiliary materials that do not form the body or physical substance of the product created. It is largely arbitrary to speak of the direct objects for processing in such sectors of material production as power engineering and transportation as well as in the spheres of services and culture. Instruments of labour are the things by means of which a worker either directly influences the object of labour by changing its shape or its location, or creates the necessary conditions for this by insulating the object of labour and the technological process as a whole from the undesirable effects of elemental forces. The first group of instruments of labour, i.e., tools of labour, are sometimes called active, the second group which includes industrial premises—passive means of labour. Yet there is no strict demarcation line between them.

Neither is there any gap between instruments and objects of labour. Many so-called auxiliary materials, such as dyestuffs, actively affect the major object to be processed and adhere to it, etc.

There are great distinctions between means of production and labour power participating in production. Human labour alone is the factor that imparts to the action of the laws of Nature in technologies a specific purposeful character. Unless employed by man means of production are not realised as such, and constitute nothing but ordinary natural objects, although with a specific shape, but in inaction liable to the destructive action of elemental forces. "... Products intended, as use-values, to enter the new process of labour, as it were, being either instruments of labour or unfinished products (i.e., products that, in order to become actual use-values, require further processing)—these products, being, as it were, either means of labour or material of labour for the further labour process, realise themselves as such only when they come into contact with living labour, which removes their dead material nature, uses them and turns them from use-values existing only as possibilities into actual, operating use-values, expending and using them as material factors of its own live movement. A machine that is not used in the labour process is useless, is dead iron and wood. Moreover, it is subject to the destructive effect of natu-

ral forces and the overall exchange of substances: iron rusts, wood rots. Yarn that is not used for weaving or knitting, etc., is only spoiled cotton, cotton that is also useless for any other useful application, the possibility of which it possessed in its nature as cotton, as raw material for yarn."[1] Every production use is certainly inevitably associated with means of production losing their initial useful shape; but in this case, as already said, a useful form of a higher order arises.[2]

The alternative of being simply objects affected by general natural processes or of playing a special role as means of production is one that is decided not simply with regard to the characteristics of certain things, but also to the actions of human beings in relation to these things. "*In the process of labour* these things have no purpose except to serve as *vital means of labour, as use-values of labour*—in relation to living labour itself as its material and means, in relation to the product of labour as its means of production, in relation to the fact that these means of production themselves are already products—products that are means of production for a new product."[3] Marx fixes this specific relation of things as means of production to active human labour in the terms "objects of *labour*" and "instruments of *labour*".

The two-fold nature of social labour. Any of human labour to perform and regulate a technological process is, above all, the aggregate of purposeful, specific actions designed to transform means of production into a specific product, actions in which the skills and abilities to perform this given technological process are used. As such, labour represents *concrete* labour. In the labour process, which is performed under normal conditions and does not result in industrial injuries and the like, the skills and abilities of the worker

[1] Karl Marx, Friedrich Engels, *Gesamtausgabe* (MEGA), II, Band 3, Teil 1, S. 55.

[2] Let us also emphasise Marx's idea that, as the products of nature develop into increasingly artificial forms, they lose the range of further possible use and become, as far as their natural shape is concerned, more specialised, adapted to satisfying an increasingly narrow circle of human needs. In a certain sense, as can be seen, the entropical natural and negentropical production processes sometimes draw together: the final highly specialised form of a product can be just as much of a hindrance to its material substrate being used for production purposes as the dissipation of this substrate within the natural environment.

[3] Karl Marx, Frederick Engels, *Collected Works*, Vol. 49, p. 52 (in Russian).

are consolidated and improved, the latter acquiring more production experience so that his ability to work becomes increasingly developed.

At the same time, any labour process is associated with an *expenditure* of human labour power. It appears directly as the work of muscles, nerves, brain, the spending of the energy accumulated in the human body, i.e., expenditure in the physiological sense of the word. It can already be seen that all the versatile, concrete kinds of labour, totally different in the nature of the production operations performed and the skills themselves, have nevertheless something common, because all of them require the above physiological expenditure.

This fact plays a specific role, since any production is possible only as social production and is based on *cooperation* of labour, both within unit technologies and on a broader scale. In reality, humanity runs production being organised in societies and, within such societies, labour power is the common social resource of production which is, in a manner, allocated among the various unit technologies. The specific skills of the individuals making up this resource definitely impose limitations on its possible redistribution, but society, if necessary, retrains workers, and trains new workers in accordance with the changed demand for specific kinds of labour. In any circumstances, concrete labour is thus a specific manifestation of a certain common *social labour power*. Labour expenditure by individuals is accordingly the spending of this social labour power, of the *social working time fund*. In this quality, labour is *abstract* labour.

The abstract character of labour develops historically as the social cooperation of labour progresses, i.e., as the mutual isolation of societies is eliminated and they turn into national, international, etc. communities up to the appearance of a common world economy. Abstract labour is manifested in a particular form under conditions where commodity-money relations develop; namely, it forms the substance of the *value* of commodities.

Technological input and output norms. Every unit technology is specifically characterised by certain extreme quantites of output it can produce and of production inputs. First, the *maximum* output is determined technologically. If a technology is based on manual tools, this maximum is defined as the quantity of output corresponding to the *productiv-*

ity of the workers employed by the technology, given the normal level of skills and intensity of labour in the society. If the technology is based on machine tools, the maximum output is determined according to the *productive capacity* of the implements of labour and other means of production, which is external in relation to the worker.[1]

The properties of unit technology also determine the necessary expenditure of various means of production and living labour per unit output (input norms). In part, these standards are already given by the laws of Nature themselves: one ton of cotton yarn cannot be obtained from less than one ton of cotton. Moreover, the entropy characteristic of Nature necessarily causes some waste. Even if the amount of it depends on the specific features of the technology used and is diminished owing to the special improvement of technologies, it cannot usually be reduced altogether. The material input standards are also determined by the capacities of the tools employed and their other properties, the durability of the materials from which they are made, etc. The labour input standards depend on the level of mutual adjustment of the means of production and the worker, the working area being thus determined, etc.

Input standards are determined technologically not only for situations where the output corresponds to its technologically possible maximum, but also where it is lower, with the input standards usually depending on the extent of approaching the maximum. Initially, as the output approaches the maximum, as a rule they decrease but, ultimately, it turns out that in order to employ the productive capacity to the full, the material and labour input standards must be increased. The problem of the extent to which it would, in this case, be rational to make full use of the potential ca-

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[1] Certainly, not only the quantity but also the quality of output depends on the character of technologies. It has already been pointed out that if a given technology can yield output of a certain quality not attainable for another technology, the products and technologies are different. Very often, society needs products of a similar purpose but of different quality; for example, there is a demand for cement of various rather than of only the highest grades. This justifies the existence of appropriate technologies. At the same time, the normal quality is fixed for every kind of product, and this is important in economic theory. "...The value of every commodity is determined by the labour-time requisite to turn it out so as to be of normal quality" (Karl Marx, *Capital*, Vol. I, p. 169). In speaking of the maximum output in the text, we mean just output of a given kind and of normal quality.

pacity of technologies is solved economically, rather than technologically, on the basis of an estimate of total labour expenditure per unit output. This problem is not to be analysed in the present book.

In all the above cases the maximum output and minimum input, by kind of input, were determined technologically. This means that it is certainly possible to produce smaller quantities of output, right down to zero, as well as with greater input (with no upper limit). Technology itself does not prescribe the volumes of output and input; it merely determines their extreme magnitudes.[1] Every long-existent social form of production has, however, a *social* mechanism to accustom workers to technological discipline, i.e., to using technologies at the level of technologically possible standards. In this lies the quantitative definiteness of the concept of expediency and the planned character of technologies.[2]

The three epochs in the development of the implements of labour. At the same time, the processes of the technological development of society the ultimate objective of which is to increase labour productivity presuppose as the means for achieving this goal the transfer of more and more technological operations and even whole functions from humans to tools.

Any technological process implies the existence of (1) certain objects the shape of which is changed therein, which in turn means the existence of (2) a source of energy, (3) a transmission device, (4) a tool that is thus put into opera-

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[1] These extreme magnitudes are, of course, not understood here deterministically, as strictly fixed magnitudes but as those of a probabilistic nature, perhaps, as mathematical expectations of the highest possible output and lowest possible input.

[2] "The fact that the new product includes only such quantities of material and instruments of labour as are required for the formation of this product, i.e., that the product includes only necessary working time, which is dictated by these specific quantities, in other words, that neither material nor means of production are wasted here, ... this fact is a condition relating not to it as such, but to the feasibility and productivity of new labour, which uses them in the process of labour as its material and means; it constitutes the definition that should be borne in mind when considering this (new) labour itself. Here, however, it is assumed that, as instruments of labour and material of labour, they are included in the new process only in the quantities actually required, as such, for the accomplishment of new labour, are actual material conditions for the new process of labour". (Karl Marx, "Zur Kritik der politischen Ökonomie", in: Karl Marx, Friedrich Engels, *Gesamtausgabe* (MEGA), Band 3, Teil 1, S. 65; see also *op. cit.*, Ss. 104-105, 110-12).

tion, and finally (5) a regulating organ able to register and remove the deviations from the planned operating conditions. The technological evolution of humanity may be divided into three major epochs, depending on the extent to which the components of the technological process constitute the direct use of organs of the human body. These epochs are defined as the *stages in the development of the implements of labour.*[1]

Manual implements leave for the human body the role of the major (in many labour processes even the sole) source of energy, the transmission device, and the controlling organ. In connection with this, capacity of the unit implement and the number of such implements to be manipulated by one worker, are greatly limited.

A machine consists of a motor mechanism, a transmission device, and a working tool. Although man remains, to one degree or another, the source of energy and the transmission device (especially if mechanisation is not comprehensive), this role is confined to auxiliary production operations. Owing to this, a machine may have several working organs that operate simultaneously; the unit capacity and operating speed of the working organs multiplies manyfold; machines are created that have no analogues among manual tools at all; finally, one worker is able, in a number of cases, to operate several, or even several dozen machines simultaneously. All this means a revolution in the productivity of labour, not a one-time revolution, moreover, but, as more and more sophisticated machines are introduced, one that is manifested in a continuous improvement in the productivity of the whole technological system. In this case, the production process continues to be controlled exclusively or mainly by operators, which again limits the opportunities for technological development given particular types of implements.

Finally, the *automatic and automated systems of machines* tend to remove man altogether from the process of regular control over the running of the technological process, too, leaving him the function of observing the correctness of the automated control (and, in the future, releaving him even of this function). Technological development is, in principle, limited only by the laws of Nature and, at each given stage,

[1] "It is not the articles made, but how they are made, and by what instruments, that enables us to distinguish different economic epochs" (Karl Marx, *Capital*, Vol. I, p. 175).

by the level of cognition of these and the elaboration of ways to apply them productively. The time of the universal application of automated systems in production has not yet arrived.

The level of the automatisation of production excluding man from the direct technological process is the sphere of technical sciences and their practical application rather than of general economic theory. Some kinds of automated technological process have been known to mankind since the time when the simplest forms of plant-growing, cattle-breeding, and industry (such as, fermentation) first appeared. Omitting the history of automatisation, let us merely point out that exceptionally broad opportunities are offered for it by the use of micro-processors. The latter are able to control technological processes physically not accessible to human intervention and, in some cases, to maintain operating conditions with substantially greater precision than man usually can.

Let us also take a closer look at two kinds of economically important division of means of production: those of single use and of multi-time use. Moreover, they can be divided into natural and reproducible ones.

Means of production of one-time use and of multi-time use. Any production is, in a certain sense, discrete: there exists a form of output that represents an integral whole and its individual components are no longer output of the given kind. Like in physics, one can speak here of the concept of a "quantum" of output, but one usually speaks of the unit output. The units of a number of products are strictly fixed by their natural form: a ram, lathe, suit, etc. In other cases, the natural form represents some part of output consisting of several units (a piece of fabric); nevertheless, the primary unit makes itself felt (too small a remnant of fabric might be left that cannot be used for the usual purposes). Roughly production itself can also be viewed as a succession of single acts, those of manufacturing the units of the corresponding products.

Means of production of one-time use are those that, on being used to produce a unit of output, lose their initial form (are consumed) already in this single act, so cannot be used to produce further units of output. These are, as a rule, basic and auxiliary materials (even if, for instance, a fertiliser affects the harvest for a number of years running, i.e., a number of successive crops). In some cases, the length of

service of means of labour also proves shorter than the time taken to produce a unit output: for example, many kinds of instrument used in ship-building are worn out before the ship is completed.

In a certain sense labour power also belongs to productive resources used only once, namely: the working time spent creating one output unit cannot be returned (simply owing to the natural irreversibility of time) and used again to produce any further units.

It is clear from the above that means of production of multi-time use are those that do not lose their specialised useful form in a single act of production and, for this reason, are used in a number of successive production acts (sometimes, for example, in a machine-building works with a multiproduct specialisation, even for successive single acts of making different products). These are, as a rule, instruments of labour, but also in some cases objects of labour.

The time over which any means of production retain their useful form is finite, which is why the aggregate of the acts of production in which means of repeated use are involved is also finite. Economic theory employs the concept of the *time* of the normal *operation* of each kind of means of production (lifetime). Account is also often taken of the fact that this time for the same means of production depends on the technologies involved.

Time of production and time of labour. Any single act of production takes place over time, is performed with a definite speed resulting from the speed of the operation of the natural processes used in it. There thus exists a production time, being a value which is always strictly positive. Since technologies include automatic processes, the production time should be distinguished from the labour time. The latter never exceeds the time necessary for a single act of production; it may be either equal to, or shorter than it.

The maximum speed of natural processes is that of light. Production certainly uses processes with a speed close to this maximum, but the bulk of processes are considerably slower. This simple fact is very important for understanding economic problems: it fills with a specific content the concept of the limitedness of the possible volume of output produced on the basis of any technological system. This limitedness is such that the limit to the possible amount of output has, at least up to now, been consistently lower than the level of full satisfaction of the human requirement for

consumer products. Since the given technological systems cannot produce more output over a certain span of time, this is one of the premises, rooted in the logic of things, that make people invent more and more new, more efficient technological systems. The disparity between the capacity of production and human needs has not, however, been eliminated yet.

Natural (nonreproducible) and reproducible means of production. The primary source of means of production for man is Nature.[1] Not only in its primitive forms but at any stage in its development, production inevitably asks Nature for more and more portions of matter. This applies, above all, to energy because this, having been turned into useful work, regularly assumes a form in which its further utilisation for performing work is impossible. This also applies to substances, because, despite all the special measures aimed at preventing its dissipation (diffusion) within the environment, such dissipation is inevitable. Thus, even the repetition of production on a constant scale and with the same structure requires more and more portions of natural matter. This is particularly true if an expansion of production takes place without a lowering of the material input norms for the unit of output. To prepare the objects provided by Nature for productive use, to separate them from immediate connection with their environment, human labour is needed. Nature, however, exists independently of humanity and the basic natural resources of production are not themselves the result of human labour.

Nothing changes even if, as already said, some products of human production are imitations of natural ones (so-called reproduction of natural resources). An essentially precise line of demarcation may be drawn between resources if nonreproducible ones are distinguished from those reprodu-

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[1] "The soil (and this, economically speaking, includes water) in the virgin state in which it supplies man with necessaries or the means of subsistence ready to hand, exists independently of him, and it is the universal subject of human labour. All those things which labour merely separates from immediate connection with their environment, are subjects of labour spontaneously provided by Nature.

"As the earth is his original larder, so too it is his original tool house.

"...The earth itself is an instrument of labour, but when used as such in agriculture implies a whole series of other instruments and a comparatively high development of labour" (Karl Marx, *Capital*, Vol. I, pp. 174, 175).

cible by people, which they use in production and to meet nonproductive needs. The term *natural* will be used everywhere to mean "nonreproducible", by which we imply the *actual* absence of the reproduction of the resource in question.

The development of production (and of society as a whole on this basis) is determined by that of reproducible means of production, especially tools. Production proper *began* when the ape went over from simply using natural objects and the organs of its body to manufacturing tools. "The use and fabrication of instruments of labour, although existing in the germ among certain species of animals, is specifically characteristic of the human labour-process, and Franklin therefore defines man as a tool-making animal."[1]

The actual use of natural resources as means of production implies the creation by man of special tools for cultivating the land or extracting minerals. That is why, theoretically and practically, technologies cannot exist that use only a natural resource and labour power: certain reproducible means of production are always used as well.

When manufacturing tools and other means of production, man adapts them in advance to the requirements of further productive use, i.e., of those technologies in which they are intended to act as means of production. It is such *specialisation* that determines the effectiveness of these means of production, their productiveness in the process of use. From the economic point of view, there are two basic aspects to this specialisation or (the same thing) the utility of means of production which are essential: their ability in general to act as means of production to produce a qualitatively definite output, and the possible standards of material and labour input in producing the output, which is decisively determined by the technological quality of the means of production.

The specialisation of means of production is carried out precisely as a way to prepare the objective conditions for manufacturing output of a certain purpose and quality, moreover with minimum possible inputs. Accordingly *technical progress* consists either in the creation of means of production for manufacturing new products, being newly introduced into the man-made habitation environment, or in creating

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[1] Karl Marx, *Capital*, Vol. I, p. 175.

means of production adapted to manufacturing traditional products with lower input.

Moreover, means of production are objective factors forming the conditions for human vital activities in the labour process, i.e., working conditions, and factors on which the influence of technologies on the natural environment depends. Their specialisation and technical progress can, correspondingly, be adjusted to certain requirements resulting from such anthropogenic activity.

What has just been said about the specialisation of means of production should not be understood as a simplified statement of the type: the narrower the specialisation of means of production the more effective they are. Efficiency always presupposes a certain specialisation. Indeed, technological progress is often manifested in a narrowing of specialisation of the means of production and, on this basis, the production of special output or a reduction in inputs. Yet everything has a limit. It often turns out that the latter goal can, on the contrary, be achieved with greater success if the specialisation of the means of production is relatively wide: its narrowness may prevent their regular use in mass production. Ecological and labour protection requirements set their own limits to the specialisation of means of production.

Conclusion: a special, constituting role of labour in production; two primary factors of production. The purposeful specialisation of means of production reveals the correlation between man and means of production as factors of production from a new angle. On the whole, this correlation appears as follows: by their labour, people impart to reproducible means of production certain useful qualities that comprehensively determine their potential production effectiveness; thanks to this the objective prerequisites are created for using nonreproducible means of production; by their labour, in employing means of production, people transform their potential utility into actual one. That is why human labour is seen as an active factor of production *constituting* production itself. Labour power is regarded as the main component of the system of productive forces.

Recognition of the special role of labour among all the factors of production distinguishes Marxism fundamentally from bourgeois economic theories. The theory of value is essentially based on this.

At the same time, as is quite clear from the whole exposition, Marx's theory counters any conceptions regarding labour

as the only source of wealth. "The use-values ... are combinations of two elements—matter and labour. If we take away the useful labour expended upon them, a material substratum is always left, which is furnished by Nature without the help of man....

"...Labour is not the only source of material wealth, of use-values produced by labour. As William Petty puts it, labour is its father and the earth its mother."[1]

Generally, the relation between the two primary factors of production is as follows. Production is subject to all the laws of Nature, particularly those of *conservation*. Yet there is no natural law conserving the *form* of matter. On the contrary, if, say, the law of conservation of energy is meant, we have in mind its conservation precisely during a change in form. A certain equivalence of mass and energy is also implied, etc. Production, however, is nothing but the transformation, the alteration of the form of objects given by Nature in order to adapt them to satisfying human needs. In changing their form, man relies upon the laws of Nature, acts like Nature itself acts, i.e., performs only transformation processes that are themselves common in Nature. The fundamental difference, however, is that man, in this case, selects and combines these processes so as to achieve the aims dictated by his own requirements.

"...The history of the development of society proves to be essentially different from that of nature. In nature—in so far as we ignore man's reaction upon nature—there are only blind, unconscious agencies acting upon one another, out of whose interplay the general law comes into operation. Nothing of all that happens—whether in the innumerable apparent accidents observable upon the surface, or in the ultimate results which confirm the regularity inherent in these accidents—happens as a consciously desired aim. In the history of society, on the contrary, the actors are all en-

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[1] Karl Marx, *Capital*, Vol. I, p. 50. Marx scoffed at the idea that "labour is the source of any wealth", introduced into the Gotha Programme of the German Workers' Party (cf. Karl Marx, "Critique of the Gotha Programme", in: Karl Marx and Frederick Engels, *Selected Works* in three volumes, Vol. 3, Progress Publishers, Moscow, 1973, p. 13). Attention would also be drawn to Engels's ironic criticism of this statement by bourgeois political economists in his work "The Part Played by Labour in the Transition from Ape to Man" (Frederick Engels, *Dialectics of Nature*, Progress Publishers, Moscow, 1974, pp. 170-83).

dowed with consciousness, are men acting with deliberation or passion, working towards definite goals; nothing happens without a conscious purpose, without an intended aim. But this distinction, important as it is for historical investigation, particularly of single epochs and events, cannot alter the fact that the course of history is governed by inner general laws.... But where on the surface accident holds sway, there actually it is always governed by inner, hidden laws and it is only a matter of discovering these laws."[1]

In this way, while stressing the specific nature of the development of society and its difference from that of Nature, the Marxian theory at the same time draws a basic distinction between the concept of the conscious principle in society (and, consequently, in production) and the idea of arbitrariness. Production develops in conformity with objective laws, this implying not only the action of the laws of Nature themselves. There exist special social laws differing from natural ones, which manifest themselves in the conscious activities of people and determine the results of these actions. Like the laws of Nature, these laws are objective, i.e., they operate irrespective of whether people know about them or not, and whether they want just such laws to operate or not. By attempting to create a *perpetuum mobile*, people demonstrated that they did not believe in the law of the conservation of energy to exist. Nevertheless the law does operate objectively and realises itself even in the very attempts to create a *perpetuum mobile*. The same is also true of the objective laws of people's social life (including production activities). The task of science is to find such laws in this sphere as well.

This is precisely the goal Marx set himself in *Capital*. As he put it, "it is the ultimate aim of this work, to lay bare the economic law of motion of modern society...". Marx viewed the evolution of the economic formation of society as "a process of natural history...".[2]

The task of this book is to once more show that Marx has performed this discovery, *having found* the really objectively acting laws of the capitalist economy.

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[1] Frederick Engels, "Ludwig Feuerbach and the End of Classical German Philosophy", in: Karl Marx, Frederick Engels, *Selected Works*, Vol. 3, pp. 365-66.

[2] Karl Marx, *Capital*, Vol. I, pp. 20, 21.

1.2. Cooperation and Social Division of Labour

The social form of production. Analysis of technologies and labour processes as such prompts the conclusion that production cannot be reduced to these, that it is not simply the totality of certain physical, chemical, and other processes, but is also a social phenomenon. The organisation of natural processes, in the form of technologies, their combination, the creation and use of means of production, all imply the existence of certain relations between people. Without such relations, technologies would be impossible. "...It is invariably only a definite social corpus, a social subject, that is engaged in a wider or narrower totality of productive spheres."[1] Any production is social production.

Thus we can assert, that *production is the unity of technological processes and relations between people.* "Production is always appropriation of nature by an individual within and with the help of a definite social organisation."[2] Technological processes are the content of production, while the relations between people for this purpose are its social form. Both are processes of people's conscious activities.[3]

Since production itself is nothing but appropriation, to maintain that social relations in production are *relations of appropriation, i.e., property relations*, is tautological. Relations connected with ownership of the means of production and labour power provide the social form for combining these factors of production in society, without which any technological process is impossible. On this basis, further, relations arise of the appropriation of real working time, cooperation of labour, its division into necessary and surplus, the appropriation of the components of working time, the social form of labour discipline ("discipline of the stick" as noneconomic class compulsion to surplus labour, "hunger discipline" as economic class compulsion to surplus labour, and conscious labour discipline are distinguished). Lastly, the

[1] Karl Marx, *A Contribution to the Critique of Political Economy*, Progress Publishers, Moscow, 1977, p. 191.

[2] *Ibid.*, p. 192.

[3] "We have seen that *value* is based on the fact that people relate to one another's labour as to equal and general labour and, in this form, social labour. This is an abstraction, as is all human thinking, and the social relations between people are only possible to the extent that people think and possess the ability to abstract from sensual details and random factors" (Karl Marx, Friedrich Engels, *Gesamtausgabe* (MEGA), II, Band 3, Teil 1, S. 210).

relations of the appropriation of the output arise from all of these relations. This means its distribution and exchange, the relations that complete the process of appropriation and, at the same time, reproduce the initial relations of ownership of the means of production and labour power. When the question is raised of the objective laws of the economy generally (and of the capitalist economy, in particular), it is a matter of the objective laws governing precisely the system of relations of appropriation in its historically given and, at the same time, changing form.

The need for the social form in carrying out production and the historical changes of this form are ultimately rooted in the functioning and development of technologies. This connection has its general expression in the concept of the correspondence of people's relations of production to the character of the productive forces, this being the most general thesis penetrating all materialist Marxist-Leninist theory in general and economic theory, in particular.

The agents of the economy. People, considered as participants in labour process and property relations are persons vigorously acting in the economy, or economic agents. Below we shall extend this concept taking into account the existence of a special sphere of exchange in the economy (commodity and money circulation).

Cooperation. A fact that has demonstrated throughout human history that people can carry out production only jointly is cooperation of labour. Cooperation is the form of labour "when numerous labourers work together side by side, whether in one and the same process, or in different but connected processes ..."[1] The simplest kind of cooperation is the immediate joint performance of one and the same operation by a number of workers, i.e., in the same place and simultaneously. From primitive hunting up to many modern production activities, such joint labour has been a necessary condition for technological operation to be performed at all. The productiveness of technologies, i.e., the achievement of their goals, and, consequently, the productivity of labour too are conditioned mainly by the cooperation of workers in simplest or developed forms.

For further analysis, it is essential to note two aspects that make themselves known even in simple cooperation.

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[1] Karl Marx, *Capital*, Vol. I, p. 308.

First, cooperation is a way to *increase* the productivity of labour, even if productive labour is itself possible as individual for a certain technological operation. The efficiency of joint harvesting, which cuts the time involved and consequently the losses, is but a vivid example.

Second, cooperation averages the quality of the labour power engaged in the technological process, making it virtually independent of the worker's individual specific characteristics. "Any average magnitude, however, is merely the average of a number of separate magnitudes all of one kind, but differing as to quantity. In every industry, each individual labourer, be he Peter or Paul, differs from the average labourer. These individual differences, or "errors" as they are called in mathematics, compensate one another and vanish, whenever a certain minimum number of workmen are employed together."[1]

Division of labour. Specialisation of the workers in the production of different use-values (i.e., the performance of different technological processes in this sense) or in the performance of different technological operations is called the division of labour among them. Speaking of specialisation, we have in mind the fact that the worker is regularly, for a long time, engaged in producing a certain narrow range of use-values, or in performing a narrow set of operations—narrow as compared with the total list of them; that the provision of the whole list of products and operations is, therefore, a function of a number of different workers; specialisation of workers is expressed in their acquisition of corresponding knowledge and skills for performing certain production operations, the knowledge and skills they do not possess in relation to the other kinds of production activities, or they possess to a lesser degree than the relevant workers. In its developed form, specialisation of the worker is secured as his trade.

A distinction is drawn between natural and social division of labour, though they are somewhat integrated and overlap. The natural division of labour takes as its basis the differences between people in terms of sex and age, their specific bodily characteristics. This used to dominate in the early stages of social production, when the implements of labour were poorly differentiated, and is felt to this day

[1] Karl Marx, *Capital*, Vol. I, pp. 305-306.

through the existence of, say, predominantly male and predominantly female occupations and even industries.

The social division of labour is based on specialisation of the means of production, particularly the implements of labour. Its basis is, thus, not differences between people but those between the implements of labour resulted in the fundamental concrete differences in the nature of the production operations performed with their help and, further, in essentially different technologies. For instance, the technology of steel-smelting has nothing in common with that of weavering. The knowledge and skills of a steel worker have nothing in common with those of a weaver. It is quite rightly stated that: "Nothing but trouble will come from a shoemaker baking cakes and a baker stitching boots".

Specialisation of implements of labour and the divergence of technological processes constitute a necessary but not sufficient condition for stable specialisation of producers. Such specialisation becomes a technical necessity when (a) technological processes chiefly presuppose the direct involvement of the worker in performing specific operations to transform objects of labour; (b) the knowledge and skills used here are mainly empirical in character. Technological opportunities for overcoming occupational narrowness appear as soon as the comprehensive mechanisation and automation of production are brought about. These opportunities can be realised only if corresponding social conditions are created subordinating people's production activities to the interests of their all-round development as individuals, the conditions of social ownership of the means of production.

In contrast, private property in one form or another consolidates the occupational division of labour among factory workers. In anticipation, let us here point out that a simple commodity producer is attached to his specialised enterprise as its owner. Narrow specialisation of wage worker is, for the capitalist, one of the conditions for turning the worker into an appendage of a machine, for actually subordinating labour to capital on this basis; moreover any technology including highly mechanised and automated ones, is, in practice, put into operation more efficiently by a worker who has been associated with it in his labour process for a period of many years; as for the negative social effects resulting from this the capitalist does not care about them.

All the forms of division of labour rely on cooperation of labour and represent a further development of cooperation.

In particular, within the bounds of a unit technology, the workers' specialisation is usually a factor that in itself requires their joint engagement in the production process. The elements of simple cooperation as such are also retained, because the collective worker, serving the unit technology, consists of a group of individual workers, each with the same trade and the same production functions to fulfil.

Developed social division of labour takes cooperation beyond the bounds of a unit technology and the individual enterprise.

The exchange of matter in the economy. The social division of labour makes the exchange of activities among humans regular. This exchange takes two forms: that in the form of products and that in the form of activities as such (services). Exchange of activities arises as a further development of the exchange between Man and Nature, which constitutes the essence of the technological process in general.

Any unit technology implies the existence of the above objective and subjective conditions, which we shall call its *input*, and the obtaining of specific products, to be called *output*. In themselves, the physical, chemical, etc. processes of turning resources into output are completely unimportant to economic science. If you wish, it is completely sufficient for this science to see technological transformation as a black box. Yet input and output ought to be examined in the framework of economic theory. Technology then appears as a specific mode of exchange of products of Nature and production, as well as of the working time of labourers, representing the "input" of the technology for its specific products. The completion of the act of production means that the resources used in it have been consumed (in relation to some resources, as already mentioned, this is true only after repeating of single acts) and no longer exist in their previous form, i.e., as resources for the given technological process. In their place, there is the product.[1]

Let us depict this technological exchange of resources for products as follows (Fig. 1).

The technological exchange of matter is an economic phenomenon, first of all because it forms the basis of the entire

[1] In some cases, the product itself can again enter the same technology as one of its inputs (for example, coal can be used as fuel to generate power for the further extraction of a new amount of coal). Such cases are, however, rare and, the main thing, the output of a unit technology never includes the whole list of inputs.

system of *production proportions*. By itself, it is a certain proportion between the product and the resources spent to create it.

If technology is based on an internal division of labour among the workers, the technological exchange includes a direct exchange of activities between workers.

As soon as the resources of a given technology have been consumed, for the performance of a new act of production they

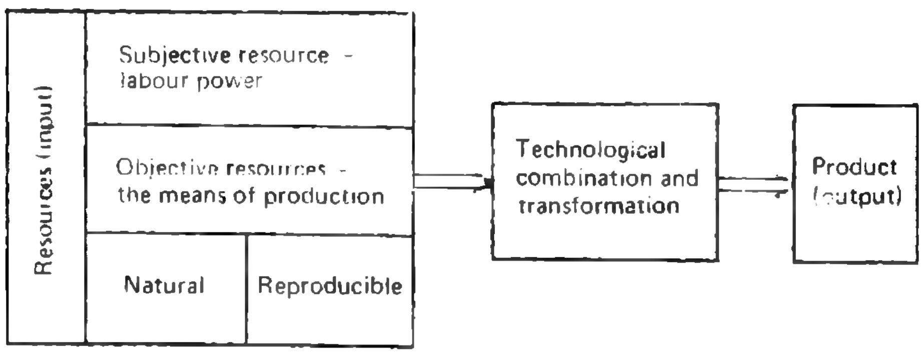

Fig. 1. Technological exchange of matter

have to be reproduced in the natural form of the corresponding means of production and labour power. Further portions of nonreproducible resources are, in this case, drawn directly from Nature. Since, however, natural resources are subject to ownership, this process implies certain relations among the people permitting the producers access to the natural resources. All actual technologies use reproducible resources and labour power. In any case, this means that use must be made of the products of other technologies: they should enter the given technology directly as means of production and enter the consumption of workers as means of reproduction of their labour power. When there is a social division of labour, this means an *exchange* of the products of technologies *among producers*.[1] The resources entering each technology are the products of workers operating in other technologies. "It is a prerequisite for a fairly devel-

[1] Under the natural division of labour, too, such exchange is observed, although in an undeveloped form.

oped division of labour that every one's needs have become very many-sided and his product very one-sided."[1]

It is precisely the system of flows of products and activities from some participants in the system of the division of labour to others that is called exchange. The system of technologies, as defined above, implies regular exchange, without which it inevitably runs down as soon as the resources of each unit technology are turned into its output, this transformation itself becoming meaningless. The specific product of any unit technology is, given the social division of labour, intended not only to be used by the participants in the given labour process but also (and even in the main) to be used by other people. Its real utility is that not for the producer himself, but for the other participants in the system of the division of labour (in the initial stages of the development of this system, part of the output is intended to be consumed by the producer; in the advanced stages, the whole output of an individual producer is intended not for his own consumption, but for that of the other members of society). The social division of labour turns the use-value of output into *social use-value.*

The technological exchange of matter thus is inevitably social in nature.[2] The technological system is reproduced thanks to the exchange of matter within itself, and is finite, closed provided on the whole it uses only products of Nature and products of its own technologies. Not only the unit act of production in each technology separately, but also the reproduction of their system as a whole are possible only within and by means of a certain social form.

Exchange in general and equivalent exchange have to be distinguished. Everything said above about the exchange characterises the need for it in any technological system. So far we have considered only the fact that each unit technology gives its product for use within the system as a whole and gets its resources from the system as a whole. Only under certain social conditions is exchange subject to the principle of equivalence.

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[1] Karl Marx, "Outlines of the Critique of Political Economy", in: Karl Marx, Frederick Engels, *Collected Works*, Vol. 28, p. 130.

[2] "In so far as exchange is a process, by which commodities are transferred from hands in which they are non-use-values, to hands in which they become use-values, it is a social circulation of matter" (Karl Marx, *Capital*, Vol. I, p. 106).

1.3. The Concept of the Commodity. A Necessary and Sufficient Condition for the Existence of Private Commodity Production

Equivalent exchange (initial definition). Logically, the concept of equivalent exchange serves as a further specification of the concept of the exchange of activities in general. The exchange of activities consists of the flows of acts and products from those who produce them to those who use their useful qualities for meeting personal or productive requirements. The initial (qualitative) definition of equivalent exchange is that in any particular case there exist *mutually inverse, counter flows*, moreover, such that *a necessary condition for each particular flow is a counter flow*. The flow from producer A (participant in the system of the social division

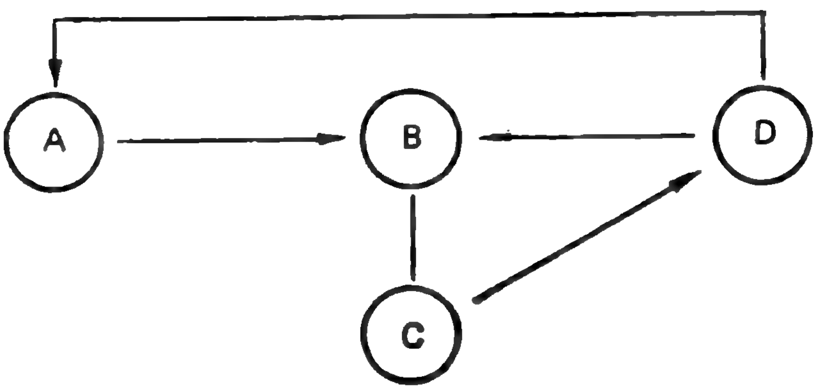

Fig. 2. Circuit of the possible exchange under conditions with no necessity of the equivalence

of labour) to producer B is possible only if there is a flow from B to A. The general concept of exchange and the concept of equivalent exchange can be represented by Figs. 2 and 3 (the arrows depict flows of goods and the circles depict producers).

In either case, each producer, as a participant in the system of exchange of matter, receives something from the system (resources for his production and personal consumption) and contributes something to it (product, actions). In the former case the flows from A to B, from B to C, from C to D, from D to B and A may (as depicted by Fig. 2) lack the counter flows from B to A, etc.[1] Such exchange

[1] Counter flows are not, of course, excluded from this kind of exchange, too. Their existence is a necessary condition for equivalent

can always be observed under the division of labour within a particular economy (primitive commune, landlord estate, capitalist factory, and the like). The owner regularly organises the flows within such an economy, while the participants of the system of exchange fulfil the will of the owner.

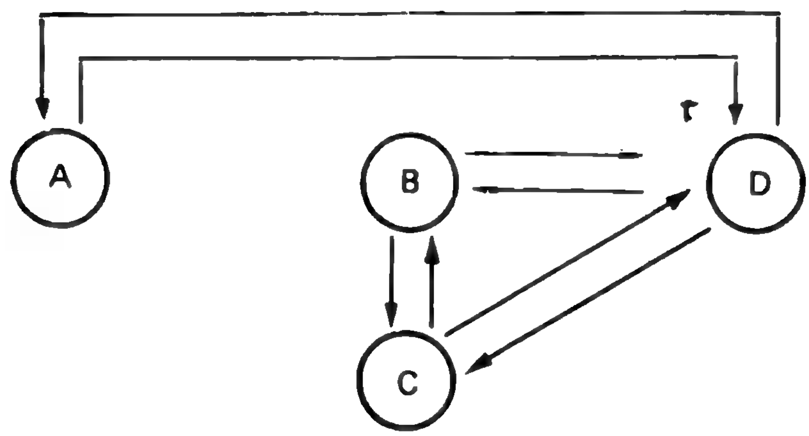

Fig. 3. Circuit of the possible exchange under conditions of the equivalence necessity

The problem of social exchange therefore inevitably requires a special definition of the concept of the unit economy (enterprise) and unit property.

Unit enterprise. As has already been noted, even the concept of unit technology does not yield to a definition if it takes into account only the technological attributes of production. A unit technology is not just a technological, but also an organisational element of production. It is the simplest, basic cell of the cooperation of the workers, isolated organisationally to produce a given output. Its individual component operations do not, in the eyes of those involved, have any independent sense; they are subordinated to the united whole. This is true with respect to the workers performing the given technological operations; each of which is but a part of the collective worker. The above also applies to the conditions of simple cooperation, as well as to those of cooperation with division of labour. Here we face the border line where the technological and social facets of production are combined, where their mutual penetration, or merging, as it were, are given.

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exchange. The latter is, on the contrary, not a necessary condition for the existence of the opposite, mutually-inverse flows of products

A higher form of the differentiation of production in society is the unit enterprise. As its lower limit it has a single unit technology, but it can include several unit technologies. To define enterprise, as a concept, property relations must be analysed.

Property is the distribution of factors and products of production among the members of society, to be disposed of their will.[1] Its major forms are public and private property. The forms of *public property* in history have included group and state property. The latter is public only if the state itself is created by a society without antagonistic classes, i.e., is the organisation of the whole people of a country. The state form implies the existence of state borders within which a corresponding socialisation of property is only valid. Theoretically, property of the whole of the people is possible in the form of nonstate (nonpolitical) economic self-management, carried out by society as a whole; it may involve particular nations or, ultimately, embrace all humanity as a united society.

Unless it follows otherwise directly from the text, everywhere in this work private property is meant. *Private property* is, in its major forms (slavery, feudalism, capitalism), above all the monopoly of one part of society (the classes of slave-owners, feudal lords, capitalists, and landlords respectively) over the basic or all the means of production in the given society. Within this part of society (class of proprietors), the means of production are owned individually or jointly. Ownership of means of production may be supplemented by ownership of workers deprived of personal freedom (full ownership of the labourer—slavery, partial ownership—serfdom). Under capitalism, the worker is a proletarian deprived of ownership of the means of production, but personally free and with his labour power at his own

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[1] "Any distribution whatever of the means of consumption is only a consequence of the distribution of the conditions of production themselves. The latter distribution, however, is a feature of the mode of production itself. The capitalist mode of production, for example, rests on the fact that the material conditions of production are in the hands of non-workers in the form of property in capital and land, while the masses are only owners of the personal condition of production, of labour power" (Karl Marx, "Critique of the Gotha Programme", in: Karl Marx and Frederick Engels, *Selected Works*, Vol. 3, p. 19. See also: Karl Marx, *A Contribution to the Critique of Political Economy*, pp. 202-203; Karl Marx, *Capital*, Vol. III, Progress Publishers, Moscow, 1974, p. 879).

disposal. According to Marxism, state property is also private property if the state itself is the organisation of the minority of the members of society for the purpose of suppressing its working majority. Finally, there have existed forms of private ownership of means of production by independent workers: one or several unit technologies are operated on the basis of family cooperation, so that the family or its head has at their full disposal the corresponding means of production, as well as the family's labour power, and the output produced.

A distinction must be drawn between individual and group private property, the latter occurring when private owners associate to dispose jointly of some part of social production.

In this work we shall call *a unit private economy* (unit enterprise) a complete set of technologies at the full disposal of a single owner or group of owners. This concept thus covers the entire set of technologies subject exclusively to the will of some owner or owners. Any technology within this set and therefore any element of the corresponding means of production, any worker who participates in the given production cooperation and any component of the specific products produced by means of these technologies is a part of a single object of property. The object as a whole covers the multiplicity of partial ones united as belonging to one and the same owner. The most specifically characteristic attribute of a unit economy is that not just one technology, but the whole set of technologies within it (only in the particular extreme case is this set reduced to one unit technology) operates and develops on the basis of a plan, expressing the owner's goal and will.

Let us note, in anticipation, that unit private economies may be *closed* economies, i.e., cover a closed set of unit technologies consisting of all the stages of production from the extraction of raw materials to the manufacturing of the whole range of consumer goods to be used by the owners and workers in a given economy. Unit private economies may be *open*, i.e., where the input or consumer goods of the inner use constitute just a part of the output.

Like any other economic concept, that of the private unit economy develops historically. Two processes are fundamental to modern monopoly capitalism. On the one hand, a complex tangle of private enterprises has arisen: wealthy person may be a co-owner of different enterprises, so that,

in each of them, he will be a member of a special, separate group of owners. Moreover, he may also remain the individual owner of some firm. On the other hand, the owner may have his exclusive property consisting of different firms whose managers fulfil some of the owner's functions. This property is thus internally subdivided not only organisationally, but also economically, in part answering the concept of relations of property, the relations of *subproperty* of a kind or *localisation* of the property. Irrespective of the reasons for such phenomena the concept of the unit economy is losing its definiteness. In such cases, the term *firm* should be applied to parts of social production independently organising their economic activities on the basis of commodity-money relations. But in this work, unless otherwise specified, unit economies are understood in the strict sense defined above.

Equivalent exchange (further definition). The requirement of equivalence does not arise in exchange within a unit firm for the very reason that the participants in the process of exchange are here not the individual owners of the products or services exchanged; they merely fulfil the will of the owner. In some cases, for example, as in the Indian community with advanced social division of labour,[1] they are, at the same time, the joint owners of the products exchanged, but this means that such flows do not proceed from one owner to another. Systematic exchange is a *real necessary and sufficient* condition within a unit enterprise for reproduction of all the components of its internal system of division of labour. That is why the problem of equivalence itself does not arise. In reality, this systematic character reflects one of the forms of the economic realisation of property.

If, however, different owners enter into an exchange, together with the flow of products and services from one agent to another, the former inevitably relinquishes ownership of them, whereas the latter acquires it. This is the key point for understanding the whole theory of equivalent exchange.

Exchange is essentially, as already stated, the flow of products and services from the agents who produce them and for whom they are, at the same time, of no direct usefulness (constitute no use-values) to those who use them and thereby realise their potential utility. In this case the physical substance in which the products are embodied and by means of which the corresponding actions are performed, does not

[1] See: Karl Marx, *Capital*, Vol. I, pp. 49-50, 337-38.

disappear. In consumption, however, the useful form acquired by the physical substance thanks to labour does disappear. Together with this form, ownership also disappears. A person who gives someone else a product for consumption cannot remain the owner of that product: in the course of consumption the product (more precisely—its useful qualities) will be lost. A person who renders another person a service cannot remain the owner of this: in the process of rendering a service its usefulness is used by its recipient and no longer exists as the object of property. Since products or services flow from one owner to another, this inevitably means a change of owner.[1]

Since exchange between different owners is associated with a change of the owner, mutually opposite flows inevitably arise. The loss of ownership of some object should be directly offset by acquisition of ownership of another object. Equivalent exchange in the sense formulated above (see Fig. 3) becomes a *necessary condition* for the very existence of exchange and therefore the system of division of labour the *reproduction* of which it serves.[2] The concept of equivalence of exchange cannot thus be comprehended outside property relations and the problem of the reproduction of the system of the division of labour.

Consider particularly hypothetical situation in which agents A, B, etc. (Fig. 2) are the individual owners of their products, but they act only as shown in this chart: they hand over each other their products to be consumed free of charge. Because property is, in this case, lost, such flows are, in fact, those of gifts. Any individual agent, as an owner, can, of course, present the others with his products, but he has no

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[1] Interrelations of a lease type, i.e., the transfer of the objects of property to another owner for his use for a given period of time are no exception to this statement. In this case, the owner retains his ownership of the given object only in the sense that, as soon as the contract term has elapsed, he regains this object with the *remaining* useful qualities; for the duration of the lease, he has no access to use of the object. That is why here, too, flows are not, in fact, unidirectional; rather they obey the principle of equivalent exchange (because rent is paid).

[2] To understand Fig. 3, it is essential that, in the comments on it, it is nowhere maintained that the mutually opposite flows between A and B, B and C, etc. are the mutually opposite flows of their corresponding products. The flows of usual products are here analogous to those shown in Fig. 2. Fig. 3 actually shows only the inevitability of the emergence of a universal equivalent for sufficiently developed equivalent exchange (see below).

reason to reckon that he will, in turn, receive the necessary consumer goods and means of production as a gift from third persons who are not interested in his gifts (they are not even the recipients of these gifts). Assume that A has handed over his products as a gift, but he must receive the products he himself requires from D, to whom he has not given anything. If, however, A receives nothing from D, A's enterprise will have no input and, thus, no output and A himself will cease to be an owner. The extinction of A's enterprise consequently results in that of the system as a whole. Its survival thus becomes improbable, a virtual impossibility. Reproduction of the system of the division of labour cannot rely upon gifts. It may have as its basis only either conformity to plan (and so is always division of labour within unit property) or equivalent exchange (and so is always the division of labour among different owners).

The principle of equivalence of exchange reads: *I give in order that you give*; *I do in order that you do*; *I give in order that you do, I do in order that you give.* It is necessary and given certain quantitative ratios (see below) sufficient for regular reproduction of the system of the social division of labour among different owners.

All this means that the principle of equivalence forms an objective law, the law of exchange of products and services under the conditions discussed. Its hypothetical alternative is, under these conditions, the exchange of gifts. Yet, as already shown, it is extremely unreliable even for a system with three or four agents, as it internally contradicts the very concept of different owners. Such exchange would mean that they obeyed a certain common social discipline, but this is equal to nonexistence of their exclusive ownership, i.e., is equal to the creation of ownership common to all agents. In such a case we are dealing not with gifts, but with regular flows that belong to the system's agents jointly.

The moral principles of the agents in such an economy are certainly not altruistic but egoistical. One should conclude in this respect: the social moral is here, as in all the other cases, but an inevitable consequence of social being, an *economically necessary* form of human behaviour under given social conditions. A society of altruists can, in conformity with the sense of this concept, be only that of people free from private property, i.e., participants in public ownership.

The receipt of some equivalent property by the agents in ex-

change for the property handed over by them is the essential sense of the concept of equivalent exchange.[1]

It is important to comprehend relations of equivalent exchange as those of property for understanding this phenomenon. This is a fundamental distinction between Marxism and bourgeois theoretical conceptions. Ignoring this key fact leads to the construction of price "theories" that do not correspond to reality.

From the scientific point of view, there is, of course, no need to emphasise that property relations exist only among people, not between man and Nature. For this reason, no relation of *equivalent* exchange arises between man and Nature, although all production constitutes an *exchange of matter* between them. From Nature people derive the materials required for production, and, in the final count, inevitably give it back to Nature, owing to the natural laws of conservation. The changed entropy thus occurred here alters nothing from this standpoint: even if it is assumed that production ultimately increases entropy in Nature, people do not and cannot compensate it to help Nature. We are forced to speak about it merely because bourgeois theoreticians are, in their struggle against Marxian economic theory, prepared at any moment to pretend not to see the elementary differences in the relations between man and Nature and the interrelations among people.

The commodity. Any object of property actually involved in the system of equivalent exchange is a commodity. More precise definitions can be obtained if certain classification attributes of the objects of property themselves are taken into consideration.

Objects of property may be reproducible or nonreproducible goods, so a corresponding distinction must be drawn between reproducible and nonreproducible commodities. A person himself may also be an object of property, as a specific commodity, if he is involved in the system of exchange among different owners. Lastly, capitalism is based on relations in which the employee is personally free (is not part of another person's property), yet, at the same time, deprived of ownership of means of production; his personal freedom thus represents his ownership of a specific commodity, i.e., his labour power.

[1] Everywhere below, unless it follows otherwise from the text, we shall simply employ the term "exchange" to express the concept "equivalent exchange".

The *usual reproducible commodity* can be defined as a useful outcome of human activity (unit product in the form of a good or service) that actually enters relations of exchange according to the principle of equivalence. This definition means essentially that the reproducible commodity is a product of human labour with two characteristics: (1) social use-value and (2) exchange-value, i.e., proportions in which it is exchanged for other products.

The two cases when products enter equivalent exchange are distinguished according to the two stages in the formation of relations of such an exchange: (1) product is produced in the economy in which its use-value can be realised, where, therefore, similar products are regularly consumed but a given item is, instead of it, exchanged for the products of another owner; this product is produced not specially for exchange, but is in fact exchanged for some reasons; (2) product is produced specially for exchange from the very beginning, i.e., it has no use-value for its producer. In the former case there exists commodity exchange, but commodity production has not yet been established. In the latter, production is of a commodity type, and commodity exchange does exist on the basis of it. Commodity production may, in turn, cover only the excess of the output of a unit economy over internal consumption or it may be completely of a commodity type (all products produced as commodities of a unit economy). Historically and logically the rise of commodity production is preceded by that of commodity exchange.

Product can be produced specially for exchange but not actually enter into exchange, if there is no the counter equivalent flow of goods. Such a possibility arises as a natural result of property relations, which turn products into commodities: the mutually opposite flows exist only as a manifestation of the free will of two owners, the will of one of them not being enough for this. If the product does not realise itself in exchange as a commodity, it is not really a commodity. Yet a society of commodity producers can exist only if the overwhelming majority of products, being commodities in the opinion of their producers, actually realise themselves as commodities. For this reason, the main specific characteristics of such a society are economically analysed assuming that all commodities are realised. Only on the basis of a study of these characteristics should the real circumstances that cause deviations from the condition just formulated be analysed.

From the above follows the definition of the necessary and sufficient condition for turning products into commodities, as *relations of the social division of labour among different owners.* The social division of labour itself is necessary, but not sufficient for the existence of commodity exchange. If there exists a social division of labour but only within a unit economy, such an economy is of a subsistence type, the latter denoting the absence of any commodity exchange (but not the absence of the exchange within the given economy). If there thus exists a multiplicity of owners, but there is no division of labour among them, their economies are closed, subsistence ones. At the same time, if there is a division of labour among two or more owners, exchange arises among them, moreover, exchange precisely according to the principles of equivalence.[1]

It follows from the concept of the commodity that less than two kinds of commodity cannot exist in society. The quantitative increase in the kinds of commodity belonging to different owners forms the basis of the qualitative transformation of the subsistence economy into the commodity one.

The evolution of commodity exchange. Commodity production. Commodity exchange historically and logically passes through a number of formation stages. The history of commodity exchange's development confirms the condition for its existence as formulated above. The division of labour among different owners may appear by chance and, accordingly, exchange appears among them by chance. Such a division did occur sporadically among primitive communes giving rise to commodity exchange.[2]

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[1] Note that a natural division of labour (by sex and age) can provide the basis for the division of labour among different owners only haphazardly. Any sufficiently steady division of labour among different owners implies that the agents involved in exchange ignore any limitations of sex and age patterns, but that they are specialised on the basis of differences in the means of production used. Sex and age differences may in this case, play only a secondary role, or even be economically unimportant at all.

[2] For understanding the general issues of equivalent exchange it is important to know that "...exchange between different tribes or peoples—and this, not private exchange, it is its first form—begins only when a surplus is purchased (obtained by trickery) from an uncivilised tribe, a surplus which is not the product of its labour but the natural product of the soil and of the region in which it dwells" (Karl Marx, "Outlines of the Critique of Political Economy", in: Karl Marx, Frederick Engels, *Collected Works*, Vol. 28, pp. 102-103). Below we shall deal with the problem of equivalent exchange of non-reproducible goods but here we'll note that this phenomenon

Often it took the form of an exchange of gifts: the quantitative proportions of chance exchange are, in essence, themselves inevitably occasional.[1] Nevertheless, the fundamental specifics of commodity exchange are seen from the very outset. Marx proposed an economic formula reflecting a unit act of exchange:

$$x \text{ (commodity } A) = y \text{ (commodity } B),$$

or, which is the same:

$$C - C.$$

Those involved assume the exchanged products to be equivalent to each other, i.e., equal in some respect, their use-values being obviously different. The former specific feature is expressed in the formula by the equality sign, the latter by the different names of the commodities.

Not simply different use-values are engaged in exchange. In addition, exchange is worthwhile, appears and exists only due to the fact that both parties, as a rule, benefit from the use-value. "So far as regards use-values, it is clear that both parties may gain some advantage. Both part with goods that, as use-values, are of no service to them, and receive others that they can make use of. And there may also be a further gain. A, who sells wine and buys corn, possibly produces more wine, with given labour-time, than farmer B could, and B on the other hand, more corn than wine-grower A could. A, therefore, may get, for the same exchange-value, more corn, and B more wine, than each would respectively get without any exchange by producing his own corn and wine."[2] This gain expresses the economic effect of the social division of labour.

The gain of both contracting parties in use-value specifically characterises exchange from the start. Under a sufficiently developed social division of labour among different owners, the gain becomes absolute, i.e., infinite in magnitude: the utility of a narrowly specialised producer's own commodity is equal, for him, to 0 ("non-use-value"), but in exchange he gets commodities with a strictly positive utility

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may be understood only on the basis of the analysis of the exchange of reproducible goods.

[1] The chance component also remains inevitable for the proportions of a unit act of exchange at all the stages in its subsequent evolution. Their conformity to law has been gradually coming to the fore (in the form of historical continuity, as can be seen below).

[2] Karl Marx, *Capital*, Vol. I, p. 155.

for him. As regards utility, the proportion of exchange for party A is $U_A/0$, where $0 < U_A < \infty$; for party B the proportion is $U_B/0$, where $0 < U_B < \infty$. This fact is crucial in explaining why the use-values of commodities cannot underlie the proportions of exchange.

A unit act of exchange is always carried out according to the above formula. Given a sufficiently narrow specialisation of the contracting parties, however, the latter do not necessarily produce (moreover, they do not produce, as a rule) those products for which they have a mutual need for productive and personal consumption. The producer of looms (user of metal) does not produce anything that has a positive use-value for the steel works owner. This situation is shown in Fig. 2: a direct exchange of products has no sense here. Nevertheless, equivalent exchange requires mutually counter flows (Fig. 3). The paradox is negated by money as a commodity, which is actually and regularly taken as the universal equivalent by commodity producers.

Even if the contracting parties (Fig. 3) have no mutual interest in each other's commodities, they may still effect the mutually counter flows of goods: each of them gets money (as the universal commodity) in exchange for their commodities. If, for example, money is represented by commodity A, commodity exchange can be shown as follows (Fig. 4).

Producer A immediately becomes the owner of the universal commodity as his own product. He gives it, in exchange for commodity D he requires, to the producer of the latter. The universal commodity from D reaches C in exchange for his commodities; from C to B, etc.[1] When agents B, C, D act as vendors, they give their commodities in exchange for money; as buyers they get the commodities they need in return for money.

Money. We shall not here describe the history of the evolution of exchange which resulted in the appearance of money.[2] Let us merely note that as Fig. 3 shows a universal

[1] Note that, in contrast to the situation shown in Fig. 2, A's commodity does not arrive directly at B: A is not interested in the opposite flow from B. Nevertheless, B receives A's commodity via the system.

[2] See: Karl Marx, *Capital*, Vol. I, pp. 54-75; Karl Marx, "Outlines of the Critique of Political Economy", in: Karl Marx, Frederick Engels, *Collected Works*, Vol. 28, pp. 97-119; Karl Marx, "The Form of Value", in: Karl Marx, Frederick Engels, *Collected Works*, Vol. 49, pp. 139-64 (in Russian); Frederick Engels, "The Origin of the Family, Private Property and the State", in: Karl Marx and Frederick Engels, *Selected Works* in three volumes, Vol. 3, pp. 317-23.

equivalent may become necessary as soon as more than two kinds of commodity, belonging to different owners become involved in exchange: given three different commodities, it is already possible that their producers in pairs will not be interested in each other's commodities on a one-to-one basis and will transfer their commodities along a chain of exchanges. If the requirement of equivalence of exchange is not observed in this case, exchange will not take place and

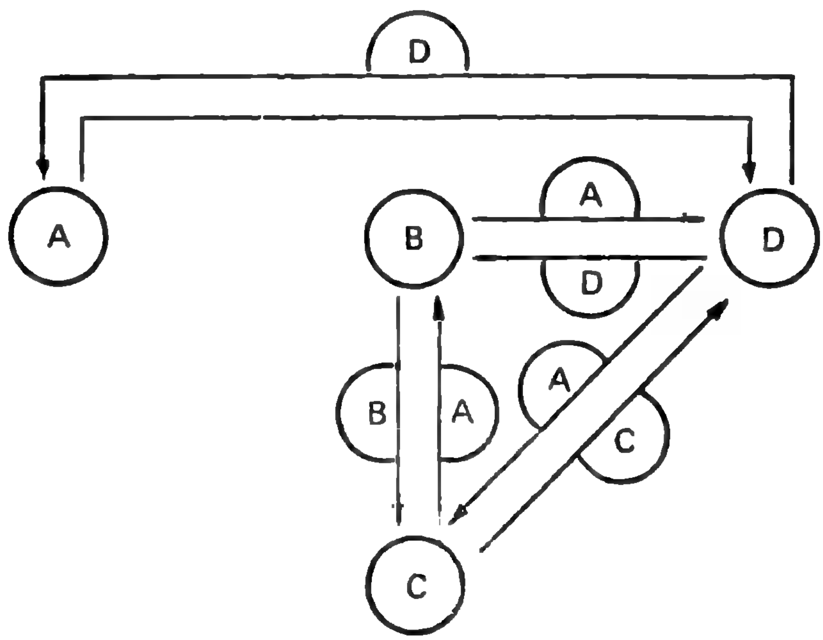

Fig. 4. Circuit of the equivalent exchange with money as medium

the system of the division of labour among such separate producers will not reproduce itself. If such a situation is possible for the case with three different contracting parties, it is virtually inevitable for 5 or 10, to say nothing of a system of millions of kinds of various commodity belonging to thousands of specialised producers. In such a system, the division of labour covers vast areas and implies that the supply of commodites by contracting parties and the acquisition of commodities by them for their productive and personal use are separated over time and space. The formation of money is a necessary condition for reproduction and so, also, for further development of the system with a developed division of labour among different owners. "*Together with money, the possibility of an absolute division of labour is given*, for the independence of labour on its specific product, on the

direct use-value of its product for this labour, is also given."[1]

The history of the formation of money, the role of which was played by various commodities, covers several thousand years. Its end may be taken as the time when gold and silver coins began to be minted by the state.[2] This first happened inthe State of Lydia in Asia Minor, at the end of the 7th century B.C. The formation of money is the historical process of the evolution of commodity exchange, leading to the rise of commodity production. Strictly speaking, *commodity production is production specially for the exchange of products for money*, i.e., for *sale*. It implies that a universal equivalent is operating regularly in the sphere of exchange.

The necessary and sufficient condition for the existence of private commodity production. While commodity exchange arose on the basis of communal property, commodity production was supported by private property. It is this that provided the social form for the quite deep division of labour that has necessitated money in exchange and engendered the manufacture of products specially for exchange for money. As a historical phenomenon, money may disappear if the conditions giving rise to commodity production die out.

Private property provided the form for the initial separation of crafts from agriculture and livestock-breeding, for the rise of a number of separate crafts and, thereby, of various specialised productive activities aimed (at least, in part) not at the producers' own consumption, but at consumption of products by everybody throughout the system. Given developed crafts, the number of contracting parties in exchange, differing in specialisation, is obviously far greater than two. That is why just the division of labour among private owners necessitated money, as well as the formation of commodity production. Given the substantial changes in the forms of private property that have taken place since it appeared and are continuing today, its general feature

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[1] Karl Marx, *Grundrisse der Kritik der politischen Ökonomie*, S. 114.

[2] The social role of money has become attached to gold and silver owing to their natural form, which corresponds best to this role: they have a high value per unit weight and are, at the same time, extracted in considerable amounts; they can be subdivided into small ingots, the quality of ingots of equal weight being virtually equal; they are also physically long-lasting. This makes it possible for commodity exchange to cover various areas, and for the sale and purchase of commodities to be separated over time.

is retained: the segments of the social division of labour exist as private economies, which can only be commodity ones.

Historically and logically, the social division of labour among private owners is a necessary and sufficient condition for the existence of private commodity production.[1]

Everywhere in this work, unless otherwise indicated, we take commodity production to mean private commodity production.

As long as the specialisation of economies was wide enough to enable them to consume some part (as regards agriculture, most) of their own output, only the excess of output was involved in commodity exchange, while the economies remained mainly subsistence ones. Such production was partly commodity production. Commodity production became universal under the conditions dominated by a specific — capitalist—form of private property.

The functions of money. According to the conditions that gave rise to it, in a society of commodity producers money performs certain specific functions that are in some way or another necessary for the continued reproduction and development of the system of the social division of labour among private owners. In the totality of these functions, the social quality of money is realised as a universal commodity by means of which any specific commodity may be easily obtained, meaning the quality of money as the social embodiment of wealth as such. The commodity acting as money does so only within the sphere of exchange.[2]

The *first* function of money is price-formation. Price is the name given to the proportion of exchange of commodity

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[1] "...Social division of labour ... is a necessary condition for the production of commodities, but it does not follow, conversely, that the production of commodities is a necessary condition for the division of labour. ...Only such products can become commodities with regard to each other, as result from different kinds of labour, each kind being carried on independently and for the account of private individuals" (Karl Marx, *Capital*, Vol. I, p. 49).

[2] A commodity representing money can, in accordance with its specific physical characteristics (in contrast to its purely social characteristics, which it possesses within the sphere of commodity exchange), be used for productive or personal consumption. A gold coin can become raw material for a jeweller, but it will then play the role not of money, but simply of an ingot of gold. Gold, silver, etc. are money only in the sphere of commodity exchange, including the accumulation of money to be exchanged later.

for money, i.e., the exchange value of commodity expressed in money terms.[1] Thus,

$$p_j = \frac{q_g}{q_j}, \tag{1.1}$$

where j is the index of commodities ($j = 1, \ldots, n$, where n is the total number of all kinds of commodity; $n \geqslant 2$); q_g and q_j are respectively the quantities of money and commodities of kind j which are acknowledged as equivalent.[2] As all commodities $j = 1, \ldots, n$ have prices, the proportions of their mutual exchange by means of money are thereby given. For any two kinds of commodity j and j', such a proportion (given q_g) follows from the equation:

$$p_j q_j = p_{j'} q_{j'}. \tag{1.2}$$

The quantities of commodities q_j and $q_{j'}$ corresponding to this equation are seen, on the market, to be equivalent.

The determination itself of the price of a commodity does not require the commodity to be exchanged for money directly: it is enough for the commodity to be equated mentally to a certain quantity of money. Shortly, we mean here not real, but ideal money. It is a different matter that the process of real exchange can alter the mentally fixed price, even to the extent that the commodity will not be sold at all. For this reason, an original seller's price (determined mentally by the seller when offering a commodity on the market), an original buyer's price (determined mentally when forming the demand for a commodity), and a real price, i.e., the proportion in the actual exchange of a commodity for money, must be distinguished.

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[1] "The exchange value of the commodity expresses the totality of the quantitative proportions in which all other commodities can be exchanged for it... Exchange value expressed in money, i.e., equated to money, is price" (Karl Marx, "Outlines of the Critique of the Political Economy", in: Karl Marx, Frederick Engels, *Collected Works*, Vol. 28, pp. 119-20).

[2] In accordance with formula (1.1) money itself also has a price if the monetary commodity is taken as one of the n kinds of commodity. Yet this price is rather conventional, being always equal to unity, and merely reflects the simple fact that some quantity, say, of gold of a given karat, is always equal to the same quantity of gold of the same karat. Logically, however, it is impossible to acknowledge the existence of relations of equivalent exchange without this simple fact. "Its (money) price, so-called coin price, is just a denomination of the definite quantities of its own material" (Karl Marx, *Grundrisse der Kritik der politischen Ökonomie*, S. 768).

As a phenomenon of equivalent exchange to express property relations, the price can, by definition, be only a strictly positive magnitude: $p_j > 0$ for all $j = 1, \ldots, n$. Only a strictly positive price answers the requirement of mutually counter flows of the objects of property. A zero price for some commodity would, in essence, mean that the owner of this commodity simply presented it to the consumers: it has been already shown that the economy of such an owner would collapse (if he presented his whole output) thus demonstrating that such behaviour contradicts the laws of commodity production and exchange. A negative price is sheer nonsense from the standpoint of these laws: the owner not just presents the commodity, but also pays extra to the recipient of this gift, i.e., presents him, in addition, with some sum of money.[1]

The above applies primarily to the actual prices paid in acts of sale and purchase. Since the actual prices are such, however, so are those fixed earlier by buyers and sellers in their minds. These may differ from the final price but, given normal behaviour of the agents, they are positive magnitudes from the very beginning.

Note that the assertion of the price as a strictly positive magnitude is absolutely universal, i.e., irrespective of the ratio of the demand for the given commodity to its supply over a certain span of time. This assertion applies, in particular, to the case when supply exceeds demand. In this event, some commodities remain unsold, but this is not the same thing as selling at, say, a zero or negative price: the commodity has not been sold and remains with the seller. The commodities that are sold in such a situation have a strictly positive actual price. An excess of commodities over demand presses the real sales price, brings it down, but does not turn it into zero.

To point out the fact that price is a proportion of the exchange of commodity for money does not in itself characterise the magnitude of this proportion, since it is rather superficial statement of the facts observed, a formal definition of price. Accordingly, formula (1.2) merely states the existence of equivalent quantities of commodities. The task of the *theory* is not simply to state the fact of the existence of prices

[1] A negative commodity price must not be confused with fines for emitting industrial waste polluting the environment. The latter are not commodities.

but to discover the law that governs them, i.e., the basis on which the proportions of the exchange of commodities for money and, by means of money, of the exchange of one commodity for another may be fixed. The formal definition is the formulation rather than the solvation of the problem. In anticipation it is worthwhile pointing out that, once the law of prices is discovered, the prices of reproducible commodities can be determined as a monetary expression of their labour values. Accordingly, the first function of money is that of the *measure of value*. The demonstration of this forms the basis of the Marxist-Leninist theory of capitalism and will be expounded in detail in the next Chapter.

The *second* function of money is that of a medium of circulation. Money performs this function when being directly exchanged for commodities, i.e., is a means of purchase. Now a unit act of commodity exchange $C - C$ is expressed in the specific form:

$$C - M,$$

where M is the universal commodity—money. The exchange of commodities in which each party of the system of the social division of labour introduces his commodities into the system to be used and receives the other's commodities for his productive and personal consumption is described as a tangle of equivalent exchange processes. These processes have the following form for each party:

$$C - M - C,$$

where the original C is the commodity the agent puts into the exchange process, and the final C is the set of commodities the same agent receives as a result. Such exchange of commodities by means of money is called *simple commodity circulation*. This is characteristic of any commodity production.

The place where interrelated acts of commodity exchange for money take place is called the *market*. Each such act is one of the *sale and purchase* of a commodity.

In each real sale contract for commodities, two parties are involved: the *seller* who provides a specific commodity and the *buyer* who pays money for it. The former relinquishes ownership of the commodity and acquires, in exchange, ownership of money and thereby also a real possibility of buying any other commodity. The seller's gain relating to use-value consists in the fact that, instead of a specific commodity of no use-value for him and not possessing the quality of

being directly exchangeable for other commodities, he obtains a special commodity possessing this quality. The use-value of money means that it provides access to any commodity on the market. Yet money as such has no other use-value, the single one being realised in the purchase of commodities. The gain relating to use-value is thus enjoyed not only by the seller who gives his commodity for money, but also the buyer who gives money for the commodity, while turning the potential possibility of purchasing the needed commodities into their practical acquisition. The general rule still holds: both contracting parties benefit as far as use-value is concerned.

The process allowing the seller to benefit from use-value passes, from his point of view, through two stages: the sale of his commodity—the exchange of a non-use-value for the universal commodity; the purchase of the commodities of others—exchange of the universal commodity for those specific commodities that are, for him, real use-values, i.e., meet his personal or production requirements. The process is completed only after both stages have been passed. The fundamental difficulty he has to overcome is just the first stage, when his own specific commodity has to be sold; at this stage, it is demonstrated whether this commodity has any real social use-value, whether somebody needs it for consumption, and, therefore, it is really a commodity. The second stage, when money has to be realised, raises no difficulties by the very definition of the universal equivalent.[1]

In fulfilling its second function, money plays a real but transient role: a commodity is sold in order to purchase others, so money is not held long but passed from hand to hand virtually continuously. For this reason it can, without causing any substantial interference or negative consequences, be replaced by special state symbols, i.e., coins which, in their natural form, have no utility outside circulation, and can serve practically only within this sphere. Strictly speaking, the way to this was already paved by minting gold and silver coins: coins of the same nominal denomination may prove to be unequal in weight be it simply because they were minted at different times and, for this reason, have been

[1] The commodity needed by the money-owner might be missing from the market, in which case the realisation of the money would take extra time. Access to all the available commodities is, however, open to the money-owner, which is not the case for the owner of a specific commodity alone.

worn down to different degrees. For the purposes of direct commodity circulation, such coins may be assumed to be equal. In this case, the role of the coin may be played by paper money and even just notes in the mutual accounts books of sellers and buyers.

Note that the formal description of the concept of price remains unchanged, but this time q_g denotes not the quantity of the monetary commodity as such (e.g., gold) but the quantity of money in accordance with a certain state nominal.

The *third* function of money is that of its accumulation or hoarding. In its embryonic form it is connected with the second function: all economic processes, including those of commodity circulation, take place over time, and as long as the money is with a given owner, it constitutes a certain accumulated wealth, in the given case stocks of coins. The formation of such stocks is imposed on the contracting parties by the finite speed of the processes of circulation. Yet money may be deliberately withdrawn from circulation and, in just such a case, it constitutes money specially accumulated, i.e., treasure. Although treasure is money withdrawn from circulation it is still money: it has been accumulated not as a raw material or consumer good, but as a universal commodity. Moreover, it is in this function that the fundamental specific feature of money as the general social expression of wealth as such comes to the fore.

Strictly speaking, the transient function of commodity circulation money may perform (and historically has performed) not only as a stable universal equivalent, but also as chance universal equivalent serving only some series of acts of commodity exchange. In other series of acts of exchange other universal equivalent may then be used. Money is, of course, the universal equivalent and, as such, performs the function of the medium in commodity exchange. Yet not every universal equivalent is money. Money appeared historically much later than the series of acts of exchange with the aid of some general medium. Once money had appeared, however, other (chance) universal equivalents no longer came into being.

A certain commodity plays the role of money not simply because it has functioned as the medium in a series of contracts but because it plays this role steadily and regularly for a long time, i.e., it represents the complete social commodity wealth as such. Acknowledgement that such a com-

modity really exists can be seen just in the fact that it is deposited as treasure.

If some commodity is used to assess commodities in comparing them and to play the role of intermediary in their exchange, it does not necessarily become money, although money usually does play this role. The fact that a commodity constitutes money as such can be socially confirmed if it begins to be accumulated as treasure (and also to function as a means of payment). Yet the most widespread approach used in bourgeois general economic equilibrium theories and models in opposition to the Marxian theory of capitalism is precisely disregard for the functions of money as treasure and a means of payment, i.e., those without which no money exists.

Transient economic roles of seller and buyer correspond to transient money with the function of medium of circulation; the regular economic role of the collector of treasure, a specific economic person other than the producer (although one person can play either role) corresponds to money as such; treasure collectors form a special social group, as opposed to traditional producers and agents of circulation. Here, for the first time, the increase in the amount of money becomes a specific goal of activities, this making the treasure collector and the capitalist similar. Both historically and logically, the accumulation of wealth is a process leading ultimately to the emergence of capital. Thus as soon as money has appeared, it immediately begins to turn into capital. At the same time, there are fundamental differences between capital and treasure.

The *fourth* function of money is that of means of payment. It comes into being if the mutually counter flows forming equivalent exchange are broken over time, i.e., the seller gives his commodity to the buyer on deferred terms. The time factor, without which no economic phenomenon can be understood, here comes to the fore.

The function of means of payment obviously implies that the role of money is socially attached to a certain commodity: the seller needs to be assured that, having received money for his commodity after a given time, he will actually be able to exchange this money for the other commodities he needs.

The quite stable economic roles of creditor and debtor correspond to the function of means of payment. They persist over time for the parties of some unit credit-sales con-

tract until the debt is repayed; moreover, usually partner *A*, who currently plays the role of creditor in relation to partner *B*, continues to be the same even once *B* has repayed the debt, because he regularly enters into such contracts. Similarly, anyone in the red is often forced to run up even greater debts, not necessarily with a single creditor. New debts are entered into in order to repay old ones, and so forth. If, moreover, commodities are lent cum interest, this is a form of relations characteristic of *capital*.

In contrast to the first and second functions, the third and fourth cannot be without substantial difficulties for economic persons when fulfilled by simple bank-notes the purchasing power of which may change rapidly *over time*, irrespective of any processes in the sphere of production, simply owing to the oversaturation of circulation channels with such notes. It can be hampered if the state fixes a mandatory stable exchange rate for notes in relation to gold and silver (or if the stability of prices is maintained compulsorily).

Yet these conditions may prove unrealisable. Moreover, the second one cannot be realisable in practice for long enough, because this contradicts the characteristics of the structural dynamics generated by commodity-money relations themselves. For this reason the role of money as such cannot be attached to the notes issued into circulation by the state, this objective specific feature of commodity production having been repeatedly confirmed historically by all attempts to overcome it. The history of such attempts is no less instructive than that of the attempts to create a *perpetuum mobile*, though the former has certainly cost humanity infinitely more. "*When paper money receives its denomination from a metal standard* (*of any sort in general*), its convertibility into gold or silver becomes an economic law, irrespective of whether it is a law in the legal sense or not. Arguments centring on convertibility thus become purely theoretical—*how* to provide for this convertibility: by legislative means or not, etc."[1]

A study of the last two functions in which money plays its specific role insistently leads the researcher to the concept of *value*. This is particularly clear when the problem of the devaluation of soft money arises. But the concept of value has not so far been introduced here which is

[1] Karl Marx, *Grundrisse der Kritik der politischen Okonomie*, S. 103,

why the theory of money has not yet been presented in detail.

Finally, the *fifth* function of money is that of world, or universal, money. Money with this function fulfils the same four functions but on a scale of world trade. Here it can be seen particularly clearly that money in its original shape should be distinguished from coins, which now play the role of simple ingots of gold or silver of a given karat and weight, irrespective of their nominal value. We should note that in this work we shall not deal with the special issues of world money.

Since money exists, in conformity with objective laws it inevitably generates *capital.* We have thus come right to the problem that constitutes the actual subject of this work, that of the laws governing the operation and evolution of the capitalist economy.

Chapter 2

VALUE AND SURPLUS-VALUE

The *essence* of capital under capitalism is defined as follows: capital is value yielding surplus-value on the basis of the exploitation of wage labour. The purpose of this chapter is to demonstrate the truth of this assertion. To begin with, we'll give a brief outline of the general characteristic of capitalism as a form of social production.

2.1. Capitalism: the Highest Form of Commodity Production

Capital. The theory distinguishes the concepts of capital and capitalist production. Historically, capital arose in the sphere of commodity-money circulation; for a long time it existed here alone and only afterwards began to dominate production. In its most general scientific description (and, moreover, formally[1]) *capital* is defined as *money yielding extra money*. This definition is expressed by Marx in his general formula of capital:

$$M - C - M',$$

where M and C denote money and commodities respectively, with M' standing for the money originally advanced plus a certain increment ($M' = M + \Delta M$, where ΔM denotes the increment in the quantity of money, i.e., *profit*; $\Delta M > 0$); the dash in this formula, like that in the foregoing formulae by Marx, symbolises equivalent exchange.

Capital is thus a certain amount of money that its owner can actually and does use, not simply to serve reproduction

[1] It should be kept in mind that Marxian theory uses the concept of *formal* (definition, relation, etc.) in the sense that it describes social phenomena in their external, i.e., superficial, form. Here we deal with a generalisation on the superficial level of phenomena,

in some unit economy (segment of the system of the social division of labour), but in a certain specific way, so that this quantity increases.[1] The owner in this case does not hoard money as the collector of treasure does: he does not withdraw it from circulation but uses it repeatedly to buy commodities. The ultimate goal of this purchasing is the sale of commodities (the same or others, appearing as a result of productive use of the commodities bought), this sale being intended to yield profit. Compared with simple commodity circulation, only a rearrangement of the succession of the selling and buying acts takes place: in one place the sale precedes the purchase, in another, vice versa. Yet this formal rearrangement expresses a fundamental difference in goal: in the first case the ultimate goal of the process is obtaining the commodities from the market; in the second case the goal is to increase quantity of money as a result of market operations. A private owner who behaves according to the general formula of capital is a *capitalist.*

The process in which capital arises in its initial (antediluvian, as Marx put it) forms is the same as that in which money arises. *The existence of money is the necessary and sufficient condition for the existence of capital.* In order for money to become capital, it needs only the quantity of it held by one person to exceed the current demand for the commodities to be acquired for the owner's personal and productive use, and do so by an amount sufficient for purchasing commodities for the purpose of resale or lending them to yield interest. Yet an uneven distribution of money is inevitable in a society of private owners, because there is no mechanism to level this distribution. Precisely which participant in commodity production and circulation will come to possess the surplus money and in what way this will take place is, of course, a matter of *chance.* The fact is, however, that *in conformity with objective laws*, such a surplus quantity does accumulate in someone's hands. Strictly speaking, the accumulation of treasure is already based on this. Let us men-

[1] When speaking about an increase in the quantity of money as the goal of capital, they mean money as a universal embodiment of wealth opening the access to real wealth, i.e., commodities. The increase in the amount of money correspondingly means the increase of its purchasing power. The increase in the quantity of money as the result of the decrease of its purchasing power (devaluation) is an opposite phenomenon. The reason for the devaluation is not important be it a decrease in the value of the unit weight of gold or inflation of paper money not exchangeable for gold.

tion that an inevitable characteristic of price-forming is deviations of prices from some normal level, unequal profitability of different technologies, and the like. *In conformity with the laws* is also the fact that having obtained a surplus money as above, the owner thereby gets a tool for further increasing the surplus money, perhaps by lending money on interest. Here we are dealing with *the system of relations with a positive feedback displaying over time*—a fundamental characteristic of commodity-money relations to be encountered incessantly in our analysis. It is this feedback that is, strictly speaking, represented by the general formula of capital: some cause (initial money) generates a process that strengthens the cause itself (increasing the amount of money in the hands of an owner). The rich become richer. That is the verbal sense of the formula, which is constantly confirmed by everyday life.

It must be emphasised that the laws of commodity production operate merely as general trends and not in each given case. There is the probability that any individual capitalist may be ruined, owing to the same laws of the market that inevitably engender capitalists in general. These laws will be discussed below. Here, at this point without proof, let us merely note that the probability of ruin, while being a strictly positive magnitude for each capitalist, is at every given moment for the majority of them much closer to 0 than to 1, i.e., such that most capitalists are at any given moment not going bankrupt; on the contrary, they are becoming richer.[1] Individual capitals are destroyed, but not capital as economic phenomenon. It must be noted that to understand the laws of commodity production one must study the subject of the research as a probabilistic environment. This is an important logical point to consider.

Antediluvian forms of capital are *merchant's* and *usury* capital. It is these that appeared together with money. The activity of the merchant who resells at a higher price the very commodities that he bought (for example, the commodities bought relatively cheaply wholesale and resold retail at a higher price; the commodities bought beneficially due to the price differences between the various local market places or due to seasonal differences, and the like) shows the gener-

[1] The reason, to put it briefly, is that, owing to exploitation of the working class, all capitalist industries and, therefore, most firms in them *can be* profitable and prices which are regulated by the law of value *make them actually* as a rule profitable. (See 3.2.)

al formula of capital. The usurer lends money, and the formula of capital may seem here to be simply $M \ldots M'$. Yet the borrower does so not for hoarding. It is not a way to make a treasure. The real purpose of the deal is to use the loan to buy immediately. For this reason, the usurer's money moves along the same way as described by the general formula (though it passes the stage of buying commodities in the hands of others).

Merchant's and usury capital existed for thousands of years before the rise of capital engaged in the production of commodities. These two kinds of capital were, at the same time, a necessary prerequisite for the rise of capitalist production. Under definite conditions, accumulated money capital began to be used to create enterprises to produce commodities; the existence of wholesale trade has served as a condition for ensuring the sale of commodities under mass capitalist production. In this book we shall not go into the history of the formation of industrial capital,[1] but shall point out the conditions under which production becomes capitalist production; at the same time we also note that only on this basis capital becomes dominant over the circulation of commodities as well.

Capitalism. Capitalism is the highest form of the evolution of commodity production, where virtually all output is produced as commodities and labour power is a commodity too. Capitalism is a total commodity production.

Logically and historically, the transformation of all output into commodities and that of labour power into a commodity are mutually conditioned processes with a common source. While the existence of money is the necessary and sufficient condition for the existence of capital in general, *the turning of labour power into a commodity is the necessary and sufficient condition for the rise and existence of capitalist production*. Capital can penetrate production if and only if there is the possibility of buying labour power.

The history of the transformation of labour power into a commodity, i.e., that of so-called primitive accumulation, will not be considered here. We shall merely suggest a logical analysis of the existing relations, given such a status of workmen.

The transformation of labour power into a commodity reflects an essential, specific feature of the property relations

[1] See: Karl Marx, *Capital*, Vol. I, pp. 667-715; *Capital*, Vol. III, pp. 324-27, 593-612.

on which capitalist production is based. This itself means dual economic freedom of the production worker: his freedom from means of production and circulation, and freedom from any personal dependence. The workers forced into a state of such dual freedom are called *proletarians.* While not being the owner of the means of production and money, the proletarian is also deprived of any immediate access to consumer goods and cannot set up a business of his own. He may be connected with means of production and create product only in an enterprise that does not belong to him, hence only by consent of the owner. Yet the worker himself forms no part of the property belonging to the enterprise owner; he is fully at his own disposal and hence disposes of his ability to work, which is not separable from himself. For this reason, the enterprise's owner cannot, in turn, directly connect means of production with labour power without the consent of the latter's owner, cannot carry out real production without such consent. While not producing any output independently, the proletarian can purchase consumer goods for himself and his family only on the market, i.e., for money. Money, however, may be obtained only in exchange for some commodity. In the situation under consideration, the only possible commodity is his labour power.[1]

The system in which the connection of labour power with means of production has the social form of the sale and purchase of labour power is called the system of *wage labour.*

Since workers are deprived of ownership of the means of production and are personally free, the social prerequisites for the existence of subsistence, or even chiefly subsistence, economies are undermined. Such economies preceding capitalist ones were mainly the agricultural economies of feudal lords or peasants as private owners. The transformation of labour power into a commodity became equal to the destruction of the social basis of both economies: of the former owing to the freeing of the workers from personal dependence, of the latter owing to the freeing of the workers from the

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[1] It is hardly necessary to point out specially that nothing essentially changes if the possibility of borrowing is taken into account. Even if some proletarian is able to obtain credit, he will have not only to earn money to continue to exist, but also to cancel the credit. Moreover, the majority of the working people cannot live on credit: they have to work in order to produce the commodities that are bought for money by those obtaining credit.

means of production. Proletarians have to acquire consumer goods on the market, hence most consumer goods produced in society should be commodities.

The social necessity itself of the dual liberation of the worker was based on the requirements of the development of production: labour power was required by capitalist manufactories. These production units were, in their technological basis, inevitably narrowly specialised enterprises mass-producing a relatively narrow product mix. They developed in the towns, which resulted in migrations from the countryside to the towns. Their mass product could not be consumed within them; it was intended for consumption by society as a whole and was, therefore, commodity one; accordingly, such enterprises could not derive means of production from their own output and had to buy them on the market. With regard to their technological basis, manufactories could exist only as entirely commercial economic units,[1] and the migration of labour power to the towns (which simultaneously saw the development of merchant's and usury capital) created the necessary social prerequisites for their diffusion. As a reverse action, these processes also transform agricultural economies from subsistence into specialised commodity ones: the latter supply the other sectors with raw materials and consumer goods.

All this means that the social existence of labour power as a commodity is, first, the necessary condition for the transformation of all production activities into commodity (namely, capitalist commodity) ones; second, the sufficient condition, because any other production is impossible, given such a status of labour power. The system of wage labour can only be the capitalist system of production. The system of capitalist production can only be that of wage labour. It is impossible to separate them from each other. Rejection of the capitalist mode of production and its replacement by a new, more advanced social mode is possible only if the social form of the existence of labour power characteristic of it is also rejected.

The technological level of production reached before capitalism and further developed by the latter is character-

[1] Such exceptions as the consumption by mines of their own coal as fuel or sporadical production of machine-tools by engineering firms for their own enterprise does not change this basic fact. Such cases are so exceptional that the accounts department records them as purchases by the capitalist of the components of his capital from himself.

ised by the following socially fundamental property: the productivity of labour is such that it *is possible* to produce surplus-product. This is the name given to the part of the total output produced in society that remains (a) after the means of production used to produce this output have been replaced and (b) in addition to the set of consumer goods to be used as means of subsistence for productive workers and their families. This set should be sufficient to maintain the traditional (historically established) living standards of the working people, without which normal reproduction of labour power is impossible.

The necessary condition for the existence of all societies based on private property, including capitalism, is that the workers *could* produce output for their own consumption during less working time than they are able to devote to production. The actual extension of their working day in excess of these limits is no longer technological, but social in nature: it is achieved *by compulsion to surplus labour*, being non-economic under the slave-owning and feudal systems and economic under capitalism. The latter is exercised by capital by means of wages: the amount of money needed to buy a normal basket of consumer goods is obtained by the workers only if they work longer than necessary to produce that basket, i.e., they create a surplus-product. Compulsion to surplus labour in favour of capitalists is based precisely on relations between the two classes as regards ownership of the means of production and labour power: the workers can have the access to means of production only on condition that their labour exceeds the amount necessary for their own subsistence.

Compulsion to surplus labour is *exploitation.*

The capitalist form of property. Even at the risk of repeating ourselves, all this should be summarised by specifically characterising capitalist property relations as a specific form of private property. This form of private property is characterised by two major features.

First, property relations between the two classes, complete separation of the producer from means of production. The class of capitalists disposes of the means of production as exclusively their private property, whereas the class of proletarians, who constitute the majority of the population, is deprived of any means of production, though the capitalists' private property does not cover production workers. The latter are personally free and formally dispose fully of their

labour power. By selling it on the market, they receive money for it. After this sale, capitalists combine in their hands, as their own property, means of production and labour power (for the time for which the latter has been sold by the workers); these become components of capital. Accordingly, capitalists are the owners of the products arising as a result of the productive interaction between means of production and labour power, and the workers give the capitalists money to buy a portion of their own product as means of subsistence. After this, the capitalists remain the owners of the means of production and circulation, as well as of their own means of consumption and other products of nonproductive consumption.

Second, property presents relations within the class of capitalists. This is a distribution of means of production among them making each individual capitalist, or group of capitalists the private owner of one or several unit enterprises forming constituent parts of the system of the social division of labour. The output of these enterprises is produced specifically for sale and, being commodities, reach the final consumers (capitalists or workers) via market.

The reader can see that, in one respect or another, this form of property differs from any other forms of private property (for example, the private property of slave-owners, feudal lords, or simple commodity producers not employing wage labour). Social production undertaken in such a form is called the *capitalist mode of production* or, briefly, *capitalism*.

The three kinds of capital. Under capitalism, capital operates in three social forms: industrial, commercial, and loan capital. *Industrial* capital is the capital engaged in the production of commodities. The main economic formula to describe this concept is:

$$M-C\begin{cases} L \\ MP \end{cases} \ldots P \ldots C'-M'.$$

Here L denotes labour power as a commodity, MP—the means of production as commodities, P—production, C'—the commodities that result. It is clear that the formula for industrial capital is a concrete variant of the universal for-

mula for capital when it is engaged in producing commodities. In the first act reflected by the formula, the capitalist buys means of his production and labour power on the market, these are then used in production, and a new commodity comes into being, to be sold with profit.

The formulae characterising merchant's and usury capital remain valid for *commercial* and *loan* capital.

Advanced capital and capitalist cost-price. The universal formula for capital and, accordingly, the formulae for its three kinds show that any movements of capital begin with money being *advanced* to buy commodities. The term "advanced" is employed here to show that commodities are purchased for the ultimate purpose of regaining the money, and in larger amount. The following sale of commodities allows this goal to be achieved.

The velocity at which the money is regained differs above all from one sector and firm to another. The velocity also differs for various elements of capital within a unit firm.

The major differences will be shown with reference to industrial capital.

Money capital invested in purchasing means of production of one-time use, as well as in purchasing labour power, is completely recovered after the sale of each batch of commodities. Such capital is usually called circulating capital. The velocity at which it is recovered is determined by the time taken by the purchase and transportation of the required means of production and labour power, by the time of production and the sale.[1] In the overwhelming majority of sectors, this part of capital is regained in a few weeks or months after being advanced. It rarely takes a year or more, though this does happen in sectors with a very long production time, such as the construction of such large projects as railways, ports, big industrial plants, etc.

In sectors where the circulating capital advanced is regained in less than a year, it may be used again during the same year to buy required means of production and labour power. Here we see the difference between the circulating capital (a) *advanced* and (b) *applied during the year*: the latter is equal to the former multiplied by the weighted year-

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[1] In determining these time spans, vital role is played by the piling up of normal (including reserve) stocks of raw materials, semi-finished, and final products.

ly average number of circuits of the components of this capital.[1]

Accordingly, the amounts of circulating capital *advanced* and of *expenditure* of it for producing commodities differ. Every new batch of commodities requires labour inputs and means of production to be spent at technological rates, i.e., they should be bought out of capital. For all further batches, however, the same advanced capital may be used, provided it is regained by the capitalist after the sale of the preceding batch of commodities.

Money capital invested in means of production of multiple use forms *fixed* capital.[2] All of it is *applied* in the process of the production of commodities, but only a proportional part of it is included in the *expenditure* of capital on creating each unit commodity: for each element of fixed capital that part is equal to the total divided by the number of unit commodities produced by it.

Expenditures of circulating and fixed capital per unit commodity form the capitalist *cost-price*. After the sale of the commodities, the cost-price returns to the capitalist in money form. Until the time comes to replace a certain element of fixed capital in physical form, the corresponding amount of money received for commodities forms the *depreciation fund*. For some time it may be used by the capitalist for other purposes (to enlarge the scale of production, to make credit, etc.). When the elements of the fixed capital are actually withdrawn, more money capital has to be advanced to replace them.

Part of the capital advanced involved in the direct technological process of creating commodities forms *productive capital*. In addition, some part of any capital is engaged in the sphere of circulation as *capital of circulation*: labour power, premises, equipment and materials serving the circulation process, as well as commodities and money. Loan capital and commercial capital form capital of circulation only.

So far we have classified the elements of capital according to their superficial attributes. Marxian theory reveals the

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[1] They circulate at different velocity owing to the differences in the time taken by transportation, stockpiling and the like.

[2] Different elements of fixed capital serve for different periods of time and are therefore involved in the production of different amounts of commodities. They are all classified in one group mainly because they are engaged in the production of more than one batch of each type of commodity.

indepth division of capital relating to the essence of phenomena, that into *constant* and *variable* capital. The former is capital advanced to buy means of production, the latter to buy labour power. It has been demonstrated that the real origin of profit is *only* its variable part. The following paragraphs of this chapter are devoted to theoretically demonstrating this point which is the essence of the Marxian theory of capitalism, its fundamental difference from all bourgeois conceptions.

Competition of capitals. Each commodity producer tries to gain as much money as possible on the market for his commodities. With regard to capitalist producers, this means attempts to maximise the total sum of profit. The producers are then operating under conditions when there is no attachment between producers and consumers, so any buyer may request a commodity from any seller and any seller is free to sell his commodity to any buyer. This fact is reflected in the concept of the *single market.*

In reality there exist, of course, numerous obstacles to the establishment of absolutely single markets. These are, first of all, state borders, as well as restrictions concerning the transportation of commodities within the state because of their specific physical properties. There may also exist a variety of obsolete forms in which consumers are linked to producers. Yet capitalism does its best to overcome such obstacles. Theory assumes that this task has been accomplished. If currently insurmountable barriers continue to exist for the formation of a single market for some kind of commodity, the latter is considered as a set of kinds in accordance with the set of its markets, irrespective of the affinity of their physical form.

On the single market a *single price* tends to be formed. The price is, of course, formed immediately in the process of each individual act of commodity sale and purchase, but the freedom to buy commodities from anyone at all and to sell commodities to whomever you like leads to all contracts over a given period ultimately being carried out at similar prices. This is an effect of the *competition* between sellers, trying to maximise prices, and buyers, attempting to minimise them. In the process of such competition, every seller is, of course, free to give his commodity at a lower price than that established earlier and every buyer is free to pay a higher one. Under certain conditions, this freedom would lead to an overall drop in the price and, under others, to an

increase (see below, paragraph 3.2, section "Regulation of the proportions of production. The mechanism of the law of value"). The processes of competition as a whole on the market for one kind of commodity are called *intraindustry competition*. The trend towards a single price for equal commodities is a consequence of this.

There also exists *interindustry* competition between commodity producers. This is the change they make in their firms' specialisation to maximise the amount of money to be made on the market. Here the unity of the commodity market as a whole thus manifests itself backed by the unity of all the sectors of commodity production, the unity of the economic system based on a broad division of labour. Interindustry competition gives rise to a tendency towards a single *level of profitability* in all industries. The concept of profitability itself tends to change depending on concrete historical conditions. Under some conditions, it means merely reimbursement of the money outlays of producers in each industry (on the average) on running production (conditions of simple commodity production). Under other conditions, it means that all industries obtain approximately equal profit on the unit capital advanced (pre-monopoly capitalism).

Interindustry competition, together with monopolistic effects, may also engender more complex principles of profitability, which will be considered below, at the end of Chapter 3 and in Chapter 4. The main content of this work will, however, be displayed without regard to the impact made by monopolies on price-formation.

The subsequent analysis will be devoted mainly to industrial capital. Loan and merchant's capital will be taken into consideration only in discussing the forms in which surplus-value is distributed. Yet to do this, the general sense of the concept of *surplus-value* itself needs to be clarified, this being created in the production of commodities as part of their *value*.

2.2. The Initial Conditions Under Which the Theory of Value Is Built

The theory of value may be initially built for a certain ideal subject, i.e., universal commodity production (the capitalist mode of production) taken in its pure form, which the theory obtains by ignoring a number of specific features of the economies of actual capitalist countries. There are

two lines of abstraction: first, the economic components immanent to forms of social production extraneous to and made outdated by capitalism (primitive, slave-owning, feudal, petty commodity production, etc.) are ignored; second, some actual properties of capitalism as such are excluded from consideration, i.e., those that are historically and purely logically inevitable for this socio-economic formation but insignificant with regard to this work, i.e., ones that would not affect the outcome of the research even if taken into account but would complicate the process of obtaining this outcome in the course of the research and exposition. As regards the second group of actual specific characteristics, once the main results have been obtained, they have to be gradually taken into consideration in a certain order which makes it possible, first, to verify the truth of the assumption that taking them into account does not affect the conclusions, second, to develop further and specify these conclusions with reference to real economic affairs. In none of the stages of research are properties of the real-world subject neglected if they are incompatible with the conclusions to be drawn, i.e., no preconceived conclusions are forced upon the subject. On the whole, such an approach to abstracting is the essence of the method of ascending from the abstract to the concrete, widely employed by science in general and by the Marxian political economy of capitalism in particular.

Hence it is further assumed that the economy consists of only capitalist firms. It is useful to start with mentioning the specific characteristics of such firms and of the economy made up of them, these actually existing in capitalist countries but initially being ignored in a study aimed at finding out the law of prices.

(1) Actual capitalist society consists of three classes: capitalists, big landlords, and proletarians. In building the theory of value and the closely related one of reproduction initially only two classes are assumed to exist: capitalists and proletarians. Theoretically, it is assumed that all land is nationalised and the class of big landlords does not exist.

(2) The actual capitalist economy in any country is not absolutely closed: there are exports and imports of commodities, capitals, profits, labour power, etc. In building the theory of value, an absolutely closed capitalist society is treated first: it itself produces all the products it consumes and itself consumes everything produced, does not employ any outside resources (for example, it employs as labour

power only the corresponding part of the population of a given country and engages only the natural resources of the same country). In other words, at this stage all forms of international economic relations are disregarded.

(3) The actual capitalist economy is included in the system of social relations as a whole and is, in particular, affected in a variety of ways by the superstructural elements of this system, especially by politics. These impacts (for example, wars) may substantially disrupt the normal course of economic development. In building the theory of value and reproduction, the economic system is first analysed disregarding the non-economic spheres of the life of society in general, i.e., the system is taken as developing according to its immanent laws. Nevertheless, some impacts of non-economic spheres of the life of society (for instance, the influence of the development of scientific ideas) are taken into consideration in this book, but they are analysed only as resulting from economic development and, in a sense, as involved in it.

Since the above-mentioned spheres of the life of society are not considered, the same applies to their demand for output.

(4) Actual production is, from time to time, confronted by limitations on certain natural resources; to overcome these limitations is an aim of technical progress. In building the theory of value and reproduction, we originally disregard the limitedness of natural resources. Moreover, at this stage the expenditure of natural resources in production is completely ignored and the existence of non-reproducible resources is disregarded. Accordingly, it is, first, assumed that, even if natural resources of different qualities are actually used, this does not mean that natural reserves of resources of a certain quality, including the most efficient, have been exhausted; second, in the initial stages, the theory of value disregards in general the differences in the economic quality of natural resources, i.e., the dependence of labour and reproducible resources input on the quality of natural resources used.

(5) Actual commodity production implies the existence of two spheres: the production of commodities proper and commodity-money circulation. In building the theory of value and reproduction it is initially assumed that, even if the sphere of circulation does exist, the time involved in the acts of commodity sale and purchase is equal to 0, since

no time ex penditure is required, it is also assumed that other expenditures in this sphere, namely material and labour ones, are equal to 0; lastly, capital advanced to make deals in the sphere of circulation is accordingly assumed to be equal to 0 too.

Pure operations of circulation as such, i.e., changes in the forms of value (the commodity turns into money, money back into a commodity; see the formula for commodity circulation given above), and continuation of the processes of production in the sphere of circulation are distinguished here. At the beginning of the study, it is not expenditure in general in the sphere of circulation that is disregarded but expenditure (accordingly, capital advanced) on performing only pure operations in the sphere not related to changing the use-value of commodities.

(6) Actual commodity production covers sectors where the technologically necessary production time is more than a year (building of big ships, plants, etc.). The theory of value and the theory of reproduction (and at all the stages of the political economic research) are built on the assumption that any kind of product can be produced during a year. This assumption does not, at any stage, refute the conclusions because the production time is finite and precision would require only that the unit span of time be represented by a period equal to the maximum production time. Political economy simply takes this span of time as a year. Yet the essence of the problem remains unchanged, irrespective of what it is called.

The theory of value takes into account the fact that, in some industries, more than one period of production may occur during a year.

(7) Actual relations between the class of capitalists and the working class, in conformity with the objective laws of this society, imply that the proletarians receive wages after completing some set of labour acts (say, after a week, two weeks, or a month of labour), as payment for labour processes performed. This means that in fact the workers advance the capitalists. The theory of value is built on the assumption that, on the contrary, the capitalist pays the proletarian his wage for future labour, hence advances the worker. It is in this case assumed that the worker, having received his wage, works it off in the same way as he would have worked if expecting to obtain his wage in the future. Since this condition is accepted, the above assumption

does not affect the conclusions, but simplifies the reasoning and calculation.

(8) An actual capitalist firm usually produces not one but a certain set of kinds of output (if they are analysed in accordance with the concept of the unit kind of product given above). The theory of value initially assumes that the division of labour has led to maximum specialisation of firms in strictly a single kind of output. Accordingly, it may be assumed that each firm uses strictly one unit technology. Also, the existence of by-products is also ignored in the initial stages.

(9) Actual commodity production includes firms that use part of their own output in kind, as means of production or consumer goods for the workmen. The theory of value disregards such cases (occurring rather rarely under capitalism), i.e., assumes that all the firm's output is intended for the market. Any actual commodity production may thus be described by simply excluding the corresponding amounts of output consumed in kind both from the volume of output and from that of input.

(10) The actual development of commodity production embraces the appearance of new kinds of product and the cessation of the manufacture of ones produced earlier, as processes constituting inevitable elements of technical progress engendered by deep-running characteristics of commodity-money relations. But the theory of value, while revealing the inevitability of these processes, deals directly with the problem of the law of the prices of commodities produced at a given moment of time, hence the list of goods is taken as given.

(11) The actual ratio of demand to supply of commodities on the market is never a strict equality at a given place and at a given moment (or this equality may occur merely by chance). The theory of value and the theory of reproduction initially study precisely the situation where demand and supply are equal. The reason is that, as the theory itself finds out, discrepancies between demand and supply not only regularly appear as a result of the action of the law of value, but also, in conformity with the law, cancel each other out, so that the natural outcome as an average for sufficiently long time periods of its action consists in an equilibrium of demand and supply. It is demonstrated in particular that the receipt of this outcome in the theory is not a consequence of the fact that it has been originally taken as a premise.

(12) A difference does exist between the capital advanced and expended during a year, as a result of that in velocity of turnover of the various elements of capital. The theory of value is usually exposed initially on the assumption that this velocity is the same, namely equal to one turnover per year; only subsequently (namely in a mathematical description of the subject) is the above difference taken into account. Below we shall make use of the assumption concerning the velocity of turnover of capital being equal for all industries, firms, and elements of capital (one turnover per year) just as an example.

(13) A difference really does exist between the annual total depreciation of fixed capital and the total amount of its removal per year. The theory of value disregards this difference. Below we shall assume these two as being equal.[1]

The reader will be convinced that neglecting the above features of reality does not affect the content of the main conclusions of the theory of value. Eventually, we shall give the related generalisations in the course of the initial exposition of the theory of value. In other cases, we shall additionally analyse the circumstances originally disregarded in the course of the further positive exposition of the theory. It will usually turn out that the conclusions in given aspects are specified and developed, but their main content remains unchanged in all cases.[2]

2.3. A Mathematical Description of the Subject: Firm, Industry; Input and Output

In accordance with the above, a set E of private capitalist firms is analysed; k is the firm's index, $k \in E$. Since all firms are strictly monoproduct ones, the set E without in-

[1] This difference occurs under conditions of expanded reproduction of fixed capital, as a result of the accumulation of profit. Somewhat conventionally, it may be assumed that the appearance of this difference is a result of accumulation, representing a decrease in expenditure on reproduction.

[2] In order to make it easier for the reader to see my idea I'll note that, in the subsequent exposition, the circumstances mentioned in points (1), (4), (5), (8), (10), and (11) will be taken into consideration in one form or another; note as well that points (6), (7), (9), (12), and (13) in their formulations already contain some arguments that seem sufficient for the conclusion that it is unnecessary to include the corresponding circumstances in the analysis. For points (2) and (3) it should merely be said that some additional investigations would be necessary to take the corresponding factors into consideration.

tersections is divided into subsets E_j to represent the industries, i.e., aggregates of firms specialising in the production of a single kind of commodity j ($j = 1, \ldots, n$, where n is the quantity of all kinds of commodity classified in some way). $E_j \neq \varnothing$ is further assumed for all $j = 1, \ldots n$, i.e., in each industry there is at least one firm. Under the conditions under study, there exists a one-to-one correspondence between commodity kinds and industries, so the latter may be denoted by the commodities in the production of which they specialise, i.e., indices $j = 1, \ldots, n$ can be taken as representing not only commodities, but also industries. Note that the one-to-one correspondence does not exist for individual firms and the commodities produced by them; every firm produces one kind of commodity but, in principle, every kind of commodity is likely to be produced by many firms.[1]

It is assumed that the adopted classification covers all kinds of commodity produced and consumed in a given capitalist country over a given period of time (a year). Since we disregard the cases when some commodities stop being produced, it is assumed that all of n commodity kinds will continue to be manufactured in the future.

Set E is assumed to cover all those and only those firms that actually operated during the given year. The annual outcome of this set of firms is considered, expressed above all in commodities produced by each of them. Let Q_j^k denote the volume of output of commodity j by firm k ($Q_j^k > 0$, $k \in E_j$) Each kind of product has its own, specific physical unit of

[1] Let us mention the possible result of a mathematical description of multiproduct firms combining technologies to produce the various commodities in the hands of a single capitalist (we do not consider here technologies producing by-products). The simple correspondence between commodities and industries would then be disrupted. At least some firms would find themselves classified under different industries, if the classification were made with regard to kinds of commodity. This is, however, the only classification principle that can be applied consistently, so that no mixed industries would remain. Thus, it would be impossible to obtain the industry subsets of firms without intersections. Yet the calculations, in order to determine value, could be carried out in the same manner if the inputs of each firm were distributed in advance among its commodities. Such a distribution is the common practice in bookkeeping, so nothing substantial changes in the calculations if the existence of multiproduct firms is simply ignored; however, the assumed condition naturally simplifies the exposition by releaving it of the unnecessary details of calculations for multiproduct firms.

measure (weight, length, area, or piece, pair, etc.). Then the output of each sector is:

$$Q_j = \sum_{k \in E_j} Q_j^k > 0, \quad j = 1, \ldots, n. \tag{2.1}$$

Also, all kinds of input of reproducible material resource and of living labour by each firm are summed annually. Let A_{ij}^k be the material input of kind i made by firm k to produce amount Q_j^k of commodity j; L_j^k be the corresponding direct labour input. Let us emphasise that, in principle, one can assume a material input in the form of any commodity $\bar{n}$ produced by the given economy. If a certain firm does not consume some kind of commodity, then for corresponding i, A_{ij}^k is equal to 0. It may turn out that magnitudes $A_{ij}^k = 0$ for some i for all firms $k \in E$. In this case we are dealing with a kind of commodity of strictly nonproductive use. In a general description, however, material inputs may be assumed to have the same set of kinds as the commodities produced, so it follows that: $i = 1, \ldots, \bar{n}$.

It is assumed that

$$A_{ij}^k \equiv 0 \quad \text{for } i = j, \ k \in E, \tag{2.2}$$

this being the mathematical description of the thesis of the completely commodity character of the output of each firm (and hence of each industry).

In accordance with the analysis made in paragraph 1.1, we everywhere assume that $L_j^k > 0$ for all $k \in E$ and that, in any technology $k \in E$, some reproducible means of production are expended, i.e., there exists i such that $A_{ij}^k > 0$.

The theory of value considers labour input with regard to the difference between complicated (skilled) and simple (unskilled) labour, namely, when complicated kinds of labour are reduced to simple labour, "skilled labour counts only as simple labour intensified, or rather, as multiplied simple labour, a given quantity of skilled being considered equal to a greater quantity of simple labour. Experience shows that this reduction is constantly being made. A commodity may be the product of the most skilled labour, but its value, by equating it to the product of simple unskilled labour, represents a definite quantity of the latter labour alone. The different proportions in which different sorts of labour are reduced to unskilled labour as their standard, are established

by a social process that goes on behind the backs of the producers, and, consequently, appear to be fixed by custom".[1] The unit of measure of the labour inputs is thus the duration of labour with regard to the reduction to simple labour inputs.

Below for the sake of convenience we shall measure labour inputs by the number of annual full-time equivalent employees (reduced to those of simple labour).

For the time being, we shall not consider a method for statistically determining the coefficients for reducing complicated labour to simple one. Such coefficients are practically used and constitute, as assumed in the theory of value, dimensionless multipliers for the quantities of labour measured initially simply in terms of time (i.e. of annual full-time equivalent workers). Accordingly we may write:

$$L_j^k = \sum_{\lambda} L_{j\lambda}^k \psi_\lambda; \quad k \in E_j, \ j = 1, \ldots, n, \tag{2.3}$$

where λ is the index of the kinds of labour differing in complexity: $\lambda = 1, \ldots, \Lambda$; $L^k{}_{j\lambda}$ is the quantity of labour of kind λ expended by firm k to produce commodity j during a year (in terms of working time); ψ_λ is the coefficient (multiplier) for reducing labour of kind λ to simple (unskilled) labour; this coefficient for a given λ is the same for the whole economy.[2]

Below we mainly shall use the resultant quantities of labour input L_j^k.

We shall indicate that labour inputs, seen as the expenditure of the physical and mental forces (of the brain, nerves, muscles, etc.) of the employee may differ in intensity over time. That is why working time as the unit of measure of labour inputs is adequate only if the intensity of labour does not change during the time period under consideration. The complex problem of taking the intensity of labour into account in measuring labour inputs has not yet been solved theoretically.[3] Like in the case of reducing complicated la-

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[1] Karl Marx, *Capital*, Vol. I, pp. 51-52.

[2] It is natural to accept the labour reduction coefficient for simple, unskilled labour power as unity. Then $\psi_\lambda \geqslant 1$ holds.

[3] There is as yet no strict indicator of the intensity of labour capable of meeting the requirements of practical measurement. Usually, the intensity of labour is defined as the quantity of labour spent during a unit of working time. In this case, the quantity of labour itself needs to be measured not in terms of units of time, but by some other units

bour input to simple labour one, it may, however, be pointed out that, in the real economy, the input of more intensive labour is, in fact, equalised to that of less intensive one; this is evidenced by the exchange of commodities produced in industries with obviously different intensities of labour, although such differences cannot be measured accurately. Hereinafter, unless otherwise indicated in the text, we consider labour in all industries as being of the same intensity which in the given period is viewed as the socially normal one.

Above we have already used the notion of the output of each industry j as a whole (see formula 2.1). Material input and labour input may also be summed up, first of all on industry scales. Then we obtain:

$$A_{ij}=\sum_{h\in E_j} A^h_{ij}, \quad i,\ j=1,\ \dots,\ n; \tag{2.4}$$

$$L_j=\sum_{h\in E_j} L^h{}_j, \quad j=1,\ \dots,\ n, \tag{2.5}$$

where A_{ij}, L_j are material inputs of kind i in industry j and labour inputs in industry j respectively. Further, the inputs may be summed up on the scale of the economy as a whole. We obtain:

$$A_i=\sum_j A_{ij}, \quad i,\ j=1,\ \dots,\ n; \tag{2.6}$$

$$L=\sum_j L_j, \quad j=1,\ \dots,\ n. \qquad (2.7)^1$$

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that have not, however, been elaborated. Nevertheless, a general notion of the intensity of labour may be approximately formed if account is taken of the quantity of time during which the worker is not performing technological operations in the course of the working day, and an assessment is made of his energy expenditure and, at least by means of a point system, the exhaustion of his nervous and muscular system during the working day. If the *given* level of intensity is mentioned, this means the totality of these characteristics that remains unchanged during some period of time (for example, a year).

[1] Labour input may also be calculated without reducing labour: for the industries:

$$\tilde{L}_j=\sum_{\lambda,\ h\in E_j} L^h_{j\lambda};$$

for the economy as a whole:

$$\tilde{L}=\sum_j \tilde{L}_j$$

(the sign above symbol L indicates that the calculation was made without reduction of labour).

where A_i, L are material inputs of kind i and labour inputs respectively, within the economy as a whole (in corresponding units of measurement).

The reader should take note that, in formulae (2.1), (2.4) — (2.7), summing is carried out. Generally speaking, science requires a specific justification of the use of any mathematical operation with basic magnitudes (i.e., those obtained by means of direct measurement), including summing. In this case, the question of what the total magnitudes correspond to in reality certainly needs to be discussed.

A necessary objective prerequisite for such summing is the unity of the economy. Under capitalism, this is a sort of contradictory unity formed by the market. The existence of a unified national market is the objective condition that not only justifies, but also directly requires mathematical representation in the form of the summation of the indicators for individual private firms. First of all, this is the existence of unified social labour power.[1] Every commodity exchange sets the products of individual private enterprises on the same footing and thus represents them as an expression of unified social labour. Overall commodity production, i.e., capitalism as a social system, is based on the transformation of labour power into a commodity, at least on the national scale, that is, on the formation of at least a unified national (and, further, international) market for labour power.[2]

The summation of the commodity output of the firms in one industry implies that any consumer has a real chance to demand any kind of output j from any of the producers $k \in E_j$. In other words, each unit of any commodity j is really given in a depersonalised manner, as a unit of the to-

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[1] "All the labour power of a given society, as represented in the sum total of the values of all commodities, is one and the same human labour power. Thousands upon thousands of millions of acts of exchange prove this. Consequently, each particular commodity represents only a certain share of the *socially necessary* labour time. The magnitide of value is determined by the amount of socially necessary labour, or by the labour time that is socially necessary for the production of a given commodity, of a given use-value" (V. I. Lenin, "Karl Marx", *Collected Works*, Vol. 21, p. 60).

[2] For a detailed theoretical and statistical examination of this process and of how a unified national market is formed in general, see Lenin's work "Development of Capitalism in Russia" (V. I. Lenin, *Collected Works*, Vol. 3, 1977).

tal mass of these commodities Q_j, not as a special kind of commodity. The social mechanism depersonalising the commodities of different producers is given by the operation of merchant's capital, which buys up the commodities of all producers and sells them to any consumers. The existence of merchant's capital is known to be a historical and logical prerequisite for the dominance of capital in production. Although, at this stage in the study, the existence of all kinds of capital but industrial one may be disregarded, this consequence of the operation of merchant's capital, i.e., depersonalisation of equal commodities in the marketplace, is assumed to be given.

Any general theoretical examination certainly simplifies reality by omitting many practical circumstances. The actual existence of a unified (even national, not to speak of an international) market for *all* commodities is not achieved even in the highest stages of capitalist economic development. Suffice it to say that many commodities (services, perishable and nontransportable commodities, etc.) are inevitably sold on local markets merely owing to their use properties. We must at once emphasise that, in this case, they should be considered as different commodities with different social values, though they are equal in their use properties (for the sake of logical accuracy let us merely note that, occasionally, their value may also prove to be equal).

Also, complete unity of the labour power market is, of course, achieved nowhere in reality. To demonstrate this suffice it to note that specialised workers are not, of course, able to perform every kind of labour. That is why the labour power market is always structured, at least according to the special features of the workers' skills. Intranational, and especially international, labour power migrations encounter many obstacles as well. Yet there do exist some social mechanisms for overcoming all these internal gradations of unified labour power: the training of new workers in new occupations, retraining, reducing production operations to the simplest movements requiring a minimum of training, the development of means of transportation, relations of the simple leasing of dwelling space, etc. They generate a tendency towards unity of labour power, one that never leads to ideal unity in reality but serves as a basis for ideal theoretical analysis of the properties of the capitalist economy. In theoretical studies the conditions are assumed to be extant when, in reality, they only reflect the dominant trend,

which is modified by infinitely varied specific circumstances.[1] The theoretical consideration of the law of value itself as a trend only, and not as a directly realised law (see below), corresponds to this. It is more interesting, as will be shown later, that the law of value manifests itself in modern price-formation with rather high accuracy.

The totals obtained from formulae (2.1), (2.4) — (2.7) will be used below to find the magnitudes of socially necessary labour inputs for commodity production. This means that the items themselves have to be interpreted in some sense as quantities corresponding to the notion of necessity; namely, as the absolute maximum output volumes producible by the technologies employed by given firm k [the items of formula (2.1)]; as minimal material inputs [the items of formula (2.4)] and labour inputs [the items of formula (2.5)] under such conditions. It is not the social necessity of all technologies in use that is meant here: in accordance with the law of value, it covers not all of them. It is assumed that, since a given technology is actually used, it operates with its own minimum input-output ratios achievable under its specific properties, i.e., on the condition that no excessive expenditures or losses of output occur.

In every form of society sufficiently viable to reproduce itself for an extended period and thus to form a socio-economic system, there exists, as an objective property of production relations, some mechanism for realising the necessary (in the sense formulated) level of inputs and outputs in operating firms. Under capitalism, this is, first, the competition-induced interest of proprietors in reducing their inputs and expanding their share of the market; second, labour discipline of hunger, i.e., economically forced labour by proletarians.

Really it is only the highest limit of output (and, to note in passing, of its quality) that is, in fact, determined technologically, i.e., by the productive capacity of the firm. The real output may be less than this limit, even falling to 0, yet there is no technological determination of such a decrease. Similarly, it is only minimal standards of production inputs that are determined technologically. If production engineering is inadequate, inputs may be infinitely high:

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[1] See: Karl Marx, *Capital*, Vol. I, p. 19; Vol. III, pp. 153, 161, 175, 211, 232, 235, 238-39, 365.

there are no technological limits to its increase above the minimum.[1]

Thus, the socially necessary productive inputs are based, first of all, on the individually necessary ones (for the technologies operated by the given firms). Given this assumption, socially necessary inputs could, in first approximation, be represented by the average weighted individual input quantities, provided, as we assume here, the entire output reaches the consumer and the expediency of its production is thus socially confirmed. These average weighted input quantities could be derived simply by dividing the total inputs in the industries by the total outputs obtained in the latter:

$$a_{ij}=\frac{A_{ij}}{Q_j}, \quad i, j=1, \ldots, n; \tag{2.8}$$

$$l_j=\frac{L_j}{Q_j}, \quad j=1, \ldots, n, \tag{2.9}$$

where a_{ij} is the average coefficient of material i inputs in producing j output; l_j is the similar average coefficient of direct labour inputs.

Similar individual input coefficients by firm are as follows:

$$a_{ij}^k=\frac{A_{ij}^k}{Q_j^k}, \quad i, j=1, \ldots, n; \ k\in E; \tag{2.10}$$

$$l_j^k=\frac{L_j^k}{Q_j^k}, \quad j=1, \ldots, n, \ k\in E. \tag{2.11}$$

It can easily be demonstrated that coefficients a_{ij} and l_j are merely the average weighted ones resulting from cor-

[1] In this connection the following three circumstances should also be briefly mentioned. First, productive capacity could not be normally utilised at the level of 100 per cent in the long term, but to a somewhat lesser degree; this results from the properties of the technologies themselves. Capacity is here determined as maximum output produced in the short term by a given firm. Second, there exists a technologically determined minimum of inputs of all kinds, even if capacity is utilised to a degree below the long-term normal limit; then it is production engineering and not technology as such that determines the degree to which the minimum is exceeded. Third, there is the problem of choosing the conditions for the utilisation of technologies so as to take into account input-output ratios and levels of prices for input and output components. To solve these problems, the prices actually established in the previous periods are used and their trends extrapolated. Existing price dynamics are not, however, dependent on any one producer, even if the market is monopolised.

responding a_{ij}^k and l_j^k, the weights being output volumes Q_j^h.

The reader may examine the formation of all these magnitudes for the conventional case with a three-industry economy with three firms in each industry (see Tables 2.1-2.4).

Table 2.1

Input and Output by Firm

	Industries (j)								
	1			2			3		
	Firms (k)								
	1	2	3	4	5	6	7	8	9
Material input by kind of means of production (A_{ij}^k)									
i 1	×	×	×	6	48	18	24	56	40
2	14.4	40	15.6	×	×	×	0.8	4.8	2.4
3	7.2	24	10.8	4.8	24	7.2	×	×	×
Labour input									
$L_{j\lambda}^k$	45	80	15	24	48	8	48	76	36
L_j^k	90	160	30	36	72	12	48	76	36
Output (Q_j^k)	60	160	60	44	160	36	32	80	48

Note: Output is measured in corresponding physical units, labour input in units of working time, material input in the same units as output, i.e., in physical units of the respective means of production.

Table 2.2

Input-Output Table
(in output units)

Industries	Gross output (Q_i)	Output flows compensating for material input in the industries (A_{ij})				Net output (Y_i)
		to industry			Total (A_i)	
		1	2	3		
1	280	×	72	120	192	88
2	240	70	×	8	78	162
3	160	42	36	×	78	82

Table 2.3

Interindustry Balance of Labour Power (in units of working time)

	Labour input by industry			Total
	1	2	3	
Without reduction of labour ($\tilde{L}_j$)	140	80	160	380 ($\tilde{L}$)
Reduction of labour taken into account (L_j)	280	120	160	560 (L)

Table 2.4

Average Coefficients for Direct Material and Labour Inputs by Industry

i \ j	1	2	3
	a_{ij} (input units to produce a unit of output)		
1	×	0.30	0.75
2	0.25	×	0.05
3	0.15	0.15	×
	l_j (units of working time, with reduction of labour taken into account, to produce a unit of output)		
×	1.0	0.5	1.0

Below we shall employ this example. It is structured so that proportionality of the output of all commodities is directly observed: their volumes correspond to the demand for corresponding products. At this stage of the study, simple reproduction is considered, i.e., it is assumed that no excess of any product exists that could be used for productive accumulation, in other words, the whole surplus-product is privately used by capitalists.

Inputs of means of production (that is, those with the coefficients $a^k{}_{ij}$, a_{ij}) cover not only expenditures on raw materials, but also those in the form of the replacement of instruments of labour. At this stage of the analysis, to simplify the calculations, all means of production are assumed

to serve the same period of time, namely, exactly a year, and the production cycle in all firms is also assumed to be strictly equal to one year.

In this example, we shall disregard the intraindustry differences in the workers' skills (the complexity of labour). That is why a unified labour reduction coefficient is used for all workers of the same industry (and a specific coefficient for each one): 2 for industry 1, 1.5 for industry 2, and 1 for industry 3. The complexity of the labour of all workers is thus described by its ratio to that in industry 3, where only simple unskilled labour is assumed to be employed. Accordingly, index λ has the sense of the industry's number: $\lambda = 1, 2, 3$ (similarly i and j).

2.4. System of Industries. The Input-Output Table (Interindustry Balance)

The productiveness of the technological system. The theory of value deals with multisectoral technological systems capable of producing more output of all kinds than is required for their production. In other words, the technological system is able to produce output exceeding the material inputs in the form of the same products it consumes and therefore has to replace for the purpose of simple reproduction; it is capable of producing net product. Mathematically this means that such output structure $X = (X_1, \ldots, X_i, \ldots, X_n)$ may be obtained that $X_i > 0$, $X_i > \sum_j a_{ij} X_j$, i.e.,

$$X_i - \sum_j a_{ij} X_j > 0 \text{ for all } i = 1, \ldots, n; \qquad (2.12)$$

in vector and matrix form: there exists a vector

$$X > 0 : X > AX, \text{ i.e., } X - AX = Y > 0,$$

where $A = \{a_{ij}\}$ is the matrix of the average coefficients of direct material inputs; X is the gross product vector; Y is the net product vector. Matrix A satisfying (2.12) complies with the mathematical notion of productive matrices.[1]

[1] Matrix A with non-negative elements is called productive if and only if matrix $(I - A)^{-1}$ covers only non-negative elements. The equation

$$(I - A)^{-1} = I + A + A^2 + \ldots,$$

which we shall employ, holds.

It should not be thought that statement (2.12), which clearly describes real facts, contradicts the laws of nature. All the physical laws of conservation hold, of course, in the economy. No greater quantity of materials can be produced from a smaller quantity, this is not contradicted by the fact that more products of all kinds can be manufactured than the quantity of the same products spent on their production. It is precisely for this reason that humans inevitably require more and more materials from nature. This demand can, to a certain degree, be reduced by cutting waste, utilising secondary raw materials, etc., but the appeal to nature for resources is unavoidable.

The concept of the productiveness of a technological system was expressed by comparing input and output, so it cannot be seen immediately that this is nothing but a specific expression of the concept of the *productivity of labour*. Meanwhile, as has already been demonstrated in detail, technologies acquire specific features through labour; so the net product of a technological system must be suitably considered as the specific result of labour; the volume of the net output is merely the net gain of labour. Material means of production (both nonreproducible and reproducible) cannot themselves turn into product in general, or a net product in particular. The workers in production operate a given system of technologies not simply in order to turn means of production into output Q, but merely for the sake of gaining a net product.

The productiveness of the technological system is shown in (2.12) as potential one, which is why the vector of possible output X is taken instead of the vector of real output $Q = (Q_1, \ldots, Q_i, \ldots, Q_n)$. Even for a closed economy, it is not required that

$$Q_i > \sum_j a_{ij}Q_j \text{ for all } i, \tag{2.13}$$

In the general case of multiproduct technologies, the notion of the productiveness of a technological system means the following. Let Q_i^k be the output i, given the unit intensity of applied technology k ($k = 1, \ldots, E$); A_i^k is the material input of kind i, given the same unit intensity, $X^{max,k}$ is the maximum possible intensity of employment of method k. The system is productive if and only if the vector of intensities $X = (X^1, \ldots, X^E)$ ($0 \leqslant X^k \leqslant X^{max,k}$, for all k) exists, such that $\sum_k Q_i^k X^k > \sum_k A_i^k X^k$ for all i.

and possibly for some products i

$$Q_i = \sum_j a_{ij} Q_j \tag{2.14}$$

should be fulfilled; here, of course, there are always, in a real productive economy, products i such that condition (2.13) is fulfilled: otherwise, the economy is not productive.

The existence of products the output of which is equal to their productive consumption is not unlikely in a real economy. Suffice it to note that, if simple reproduction is meant, the whole net product is consumed by the members of society, but owing to their physical form, far from all kinds of product can be consumer goods. In such a case, all products used strictly as producers' goods are manufactured only in the quantities needed to compensate their inputs in the technological system, i.e., in accordance with expression (2.14).

From the standpoint of the theory of value it is, however, essential that any kind of product may, if necessary, be produced in excess of the amount required by simple compensation, i.e., that there are no products that are reproducible only in strictly limited volumes and the output of which cannot be expanded. Also, the inclusion of thesis $X > 0$ in (2.12) is of importance, i.e., each product under consideration can in general be manufactured in positive quantities: one may not speak of the value of products that are not actually manufactured. Condition $Q > 0$ [see (2.1)] assumed by us corresponds to this. It is known that if matrix A corresponds to the mathematical concept of productiveness, the full labour inputs in all kinds of output are strictly positive.[1]

At the same time, it is essential that, given the productive (mathematically) matrix A, on its basis an economy can be described where some of the kinds of output are produced only in quantities corresponding to the demand for compensating material inputs. In other words, if

$$X > 0 : X > AX$$

exists, then

$$Q > 0 : Q \geqslant AQ \tag{2.15}$$

always exists too, where sign $\geqslant$ means that, as far as some components are concerned, the vectors under com-

[1] For details see paragraph 2.5.

parison may be strictly equal, but the former is necessarily greater than the latter at least in relation to some components (probably all of them).[1]

The concept of the productiveness of a technological system has so far been defined so that the economically essential question of the quantity of the net output was not taken into account. Meanwhile, not every amount of excess output over inputs in its production is enough even to enable workers to survive, let alone to meet the other nonproductive needs of society. That is why it is useful to introduce the concept of *sufficiently* productive technological systems. These are systems able to produce a net output in some *socially sufficient* quantity. Net output should be enough, first, to meet the population's needs at the historically established normal level. Under capitalism, it should also be enough to accumulate means of production and, in some circumstances, increase consumption, as well as to meet certain other nonproductive requirements. Everywhere we shall assume that technological systems meet these social requirements of productiveness. Our conventional example is structured accordingly.

The fact that the economy really does exist that at least maintains a certain traditional standard of living of the members of society, demonstrates that real technological systems are based upon sufficiently productive systems of coefficients A. This is even more true if, besides the maintenance of the traditional standard of living, an expansion of production is achieved, this being characteristic of capitalism on the whole over sufficiently long periods of time. The systems based on sufficiently productive matrices A are *reproducible* ones.

The reader should note that the concept of productiveness has been formulated for the technological system as a whole and cannot be applied to any particular industry taken

[1] The correctness of this thesis may be corroborated in the following manner. Let matrix A be given and the X complying with (2.12) found; also corresponding AX are known. Let $X - AX = Y$; according to (2.12), all components of vector Y obtained are strictly positive: $Y_i > 0$ for all i. Let us consider product i', such that $\sum_j a_{i'j}X_j > 0$, and set $Y_{i'}$, equal to zero (in this new vector all components but the above are equal to those obtained earlier and for the chosen i' $Y_{i'} = 0$). Obtain now magnitudes Q for the new vector Y'. To do this, equation $Q = (I - A)^{-1} Y'$ has to be solved. $Q' > 0$ will continue to hold but, for chosen i', $Q_{i'}$ will prove to be equal to $\sum_j a_{i'j}Q_j > 0$.

outside the system of industries. When the system as a whole is productive, each of its industries is productive by definition (given the condition, accepted here, that the whole output actually reaches the consumer in the system). This matter will be considered specially below, but we must note at once that comparison of individual and socially necessary labour inputs in commodity production leads to the conclusion that, within some sector, individual non-productive technologies can exist. In a productive system, however, the industry as a whole is always productive if its output is sold. It is also useful to note that, until the concept of value is introduced, nonproductive technologies cannot be identified purely technologically.

This point is essential in order to comprehend the process that forms the socially necessary labour input. We shall emphasise the national economic nature of these quantities. It is essential that the concept of productiveness of industries itself has only the national economic sense, i.e., the industries cannot be considered outside the whole national economy formed by them. The reason is the concept of the specialised industry itself. Such an industry exists only if it renews its means of production and labour power while obtaining the corresponding products required from the other industries. At the same time, the economy as a whole cannot exist without any of its industries.[1]

This thesis could be formally expressed by the concept of the non-decomposability of the system described by the coefficients of material inputs, labour inputs, and labour power reproduction inputs.

The input-output model. Since the actual output volumes fulfil condition (2.15), an input-output table may be drawn up on their basis. The input-output model naturally arises from the mathematical representation of the economy, which implies aggregation of output into monoproduct industries and assumes the system of industries to be closed and productive.[2] It is important to emphasise this because all these

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[1] Obviously this is true only if the economy including precisely those industries the output of which is used for reproduction purposes is considered. Yet, as we have already stressed, the theory of value initially disregarded all kind of product that do not fulfil this condition (see paragraph 2.2, point 3).

[2] The input-output model can be further extended to the case when the system of industries is not closed, i.e., an economy with foreign trade relationships is meant. For its structuring, it does not necessarily

premises are always accepted in the Marxian theory of value and reproduction, irrespective of whether it employs an input-output model or some other formalisation method.[1] Should the operation of individual capitalist firms with their relationships be described, one inevitably arrives ultimately at the description of the system of interindustry relationships, i.e., the input-output balance.

Let, as before, the net output of sector i be denoted by Y_i, the vector of net output $Y = (Y_1, \ldots, Y_i, \ldots, Y_n)$. Then it holds that:

$$Q_i = \sum_j a_{ij}Q_j + Y_i, \quad i, j = 1, \ldots, n; \qquad (2.16)$$

in matrix and vector form:

$$Q = AQ + Y.$$

Here, each $a_{ij}Q_j$ is the flow of product i needed by industry j to reproduce its means of production spent; by definition, $a_{ij}Q_j = A_{ij}$. If the existing commodity and money relations ensure the necessary exchange in the economy, in terms of model (2.16) this means that, by participating in the sale and purchase of commodities, each industry j actually receives from each industry i product flows amounting at least to A_{ij}.

Strictly speaking, this initial representation of interindustry relationships does not express all output flows, but only those connected with simple replacement of expended means of production. The agents of each industry also receive commodities of the other industries to consume from net output Y (in a more sophisticated case accumulation and other expenditure are also included in Y); we shall deal with corresponding output flows below.

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require demand to coincide with supply, which is a condition assumed in the initial stages of the exposition of the theory of value and reproduction.

[1] There are well-known studies showing the input-output table to be a further development of the reproduction schemes introduced by Marx in volume II of *Capital* (cf., for example, the works of V. Nemchinov and O. Lange in: *Primeneniye matematiki v ekonomicheskikh issledovaniyakh* (Application of Mathematics in Economic Research), Moscow, Sotsekgiz Publishers, 1959). It is also known that it was Marx himself who made certain steps and proceeded from describing the economy in two divisions to a five-industry system (see: Karl Marx, "Outlines of the Critique of Political Economy", in: Karl Marx, Frederick Engels, *Collected Works*, Vol. 28, pp. 362-66).

Statement (2.16) is usually considered as the interindustry balance of production and distribution; in our book, where output volumes are assumed given and only exchange between industries is analysed, we may simply speak of the interindustry balance of distribution. Let us mention again that all products are expressed here in physical units of measurement.

Note that the condition of the marketability of all output formulated above means, in particular: $A_{ii} \equiv 0$ for all i since $A_{ii}^k \equiv 0$ for all k (all the diagonal coefficients of the matrix are, by definition, equal to 0). Output flows from any producing industry i to all the consuming industries j are represented in the I-0 model by line i; if some industry j does not consume output i as a means of production, the appropriate $a_{ij} = 0$, so $A_{ij} = 0$ as well. The summing up of all the flows will produce A_i [see formula (2.6)]. It is possible that for some i $A_i = 0$. The flows of all i output to some industry j are represented in the I-0 model by column j; if industry j does not consume the output of some i as a means of production, the corresponding flow $A_{ij} = 0$, therefore $a_{ij} = 0$. In accordance with the accepted concept of technologies, however, each of them consumes some reproducible means of production. That is why each column j contains non-zero, strictly positive A_{ij}'s. Let us consider the fact that they may not be summed: they have different dimensions of the products dealt with in relation to industry j.

In Table 2.2 the reader can see all the quantities mentioned in our conventional example.

Just as in the I-0 model for output distribution, one may speak of the I-0 model for labour power distribution. This is expressed by formula (2.7):

$$L = \sum_j L_j = \sum_j l_j Q_j$$

and, in our conventional example, is represented in Table 2.3.

Coefficients a_{ij} and l_j are given in Table 2.4.

The theories of value and of reproduction are based on a number of assumptions represented in the I-0 model. Let us formulate these assumptions to demonstrate that they are, in fact, expressed by formula (2.16).

The law of the conservation (not increasing) of output in the exchange process. The process of interindustry distribution (exchange) as such of output does not affect the vol-

ume of output in exchange; it merely represents the transfer of products from the hands of the producers to those of the consumers. This property may be called the law of the conservation of output or, more precisely, the law of the conservation of useful properties of output in the process of exchange.

This law holds strictly only for the distribution (exchange) process as such, taken in its pure form. All actual processes of finishing, packaging, maintaining of useful properties, and transportation are considered as a continuation of production, rather than distribution processes.[1] Under commodity and money relations, exchange is equivalent, so it is apt to emphasise that, in this connection, each party receives, instead of a product in one form, products in different natural forms. It is not, however, the same as the technological (productive) exchange of matter described above: in that case, one is dealing with turning some means of production into output when the form of the former is lost, whereas that of the latter regularly comes into existence for society as a whole, and not only for the party who puts this technology into operation; in exchange, only a given agent changes the form of the products in his possession, but in society as a whole the quantity of products of all kinds remains unchanged.[2] In this sense, the theory points to the *nonproductive* nature of exchange operations themselves (the distribution of products already created), although, as is clear from the whole analysis, production cannot exist without exchange, and technologies cannot be renewed unless the resources expended are replaced.

Strictly speaking, of course, not only the process of production (technological exchange of matter) but also that of the exchange of output between producers (under the given conditions, the process of commodity and money circulation) requires expenditure of some part of output and, since it takes place over time, it is connected with natural losses of output or at least partial loss of its useful properties. That is why it is more precise to speak of the

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[1] The general theory on this issue is exposed by Marx in his examination of capitalist circulation costs. See Chapter VI of Volume II of *Capital*.

[2] The author hopes that, relying on the concepts formulated above, the reader will distinguish sufficiently clearly in the text between the technological exchange of matter and that as the distribution among consumers of output already produced.

law of non-increasing output in its distribution (exchange). At this stage of the theoretical study, however, as already stated, these circumstances are disregarded. Accordingly, pure distributive operations do not form any industry in the I-O model, as we consider it here.

The law of the conservation of output is expressed by the equality of the right- and left-hand terms of formula (2.16).

The replacement fund and the net output. The process of output distribution (exchange) among industries leads to a division of each kind of output that initially, in the producing industry, seems to be some simple unity, not internally broken down, into economically different parts. Each of these is determined by how the final consumer utilises the product, and we shall be dealing repeatedly with the problem of forming the various *funds* of output on this basis. Here we merely stress the division of the gross output into the fund for replacing means of production expended and net output.

Under normal conditions, any current production first of all creates the prerequisites for renewed production in the future. This means that, while expending reproducible means of production during their technological utilisation, society should, at the same time, produce new similar items of means of production to replace those expended.[1] Hence the essential demand for output reproduction proportions: each industry should produce its output in quantities at least equal to those consumed at the same time as means of production in the using industries. Otherwise the economy would not, in the year to come, be able to repeat the output volume attained. Here we are faced with one manifestation of the already formulated dependence of the economy as a whole on each of its industries, which is reflected in the concept of the productiveness of the economy and of its industries.[2]

It may seem that the use of output for future renewal of production should be seen as the second purpose in the order

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[1] In reality, technical progress in many cases allows the means of production expended to be replaced not by new items of those in the same physical form, but by other ones. The first step in studying this subject is made, however, without considering such a possibility.

[2] It would be apt to note that the effectiveness of the strike struggle, in which proletarians of one or several industries demonstrate their ability to substantially affect reproduction of the economy as a whole, depends on this.

of priority of its uses in society, whereas the first one would be to use output for the current individual consumption. Yet the real logic of the behaviour of society under normal conditions is the opposite: it considers not the replacement fund as the residual once the individual consumption fund has been determined, but the latter as the part of the output remaining once the fund for replacing means of production expended has been formed.[1] Generally speaking, society can, on one occasion, increase its consumption at the expense of inadequate replacement of means of production expended, the physical form of output being the only constraint (but very many kinds of output may, in their physical form, serve both as producers' and consumer goods). Having once ensured such an increase in consumption, however, society would undermine its future existence and would not be able to ensure even the previous gross output, so the future consumption decrease would be substantially greater than the current increase in consumption. In any form of society able to exist for a long time, the replacement fund is the first to be formed out of the gross output; in such a society, there is always some social mechanism to carry through corresponding behaviour as a mandatory law.[2] Under capitalism, this is the mechanism of the repro-

[1] Marx based himself on this general thesis of the theory of reproduction in his criticism of Lassalle's idea of "undiminished proceeds of labour" under socialism (Karl Marx, "Critique of the Gotha Programme", in: Karl Marx, Frederick Engels, *Selected Works* in three volumes, Vol. 3, Progress Publishers, Moscow, 1970, pp. 16-18).

[2] The replacement fund is, of course, absolutely strictly fixed only under rather strong simplifying assumptions: first, the assumption of the invariability of the product mix in the economy; otherwise there would be the possibility of replacing means of production of some physical kind with those of another kind, i.e., the quantities of means of production expended and replaced would not be the same; only experience could indicate exactly how many units of new means of production would be needed to replace the old ones; until such experience exists, the replacement fund would be determined only approximately. Even more difficult would be an attempt to determine the concept of the replacement fund for producing new (not previously produced) output. Second, even given invariable product mix, the proportions between the technologies within industries vary so that if, say, a shift took place towards more material-intensive but less labour-intensive technologies, simple reproduction of output would require means of production in different proportions. These two considerations do not, in fact, settle the matter; in the general case, given *extended reproduction*, the boundary line between replacement fund and net output fund is not clear-cut since extended reproduction is, in general, associated with changes in the whole system of proportions in the

duction of capitalist propriety, i.e., capital. Each capitalist considers renewal of expended capital as the first purpose for which revenue must be used. It is theoretically substantiated that this behaviour of capitalists results from the properties of an objective law, the law of value, which makes it possible and necessary for most capitalists in each industry to renew expended capital. The bankruptcy of individual capitalists is not, of course, unlikely in this case, including when revenues do not suffice for the capitalist to renew even the constant component of his capital.

The conditions under which these requirements may actually be fulfilled have been found, however: sufficiently exactly observed proportionality in the economy, that is, the regular replacement of the expended means of production. Initially, in building the theory, these conditions are assumed given, so every buyer can find on the marketplace any required products in general, and means for renewal of production in particular. At this stage of the study, however, the question remains open as to which social mechanism in fact makes this proportionality possible. This question is to be examined below.

The I-O model represents mathematically the division of output into the two above-mentioned funds:

$$Q_i = A_i + Y_i, \quad i = 1, \ldots, n; \tag{2.17}$$

or

$$Q = H + Y,$$

where $H = (A_1, \ldots, A_i, \ldots, A_n)$ is the fund to replace the expended means of production. As already set out, here and below we shall see vector Y as the second (remainder) term of equation (2.17):

$$Y_i \equiv Q_i - A_i, \quad i = 1, \ldots, n, \tag{2.18}$$

or

$$Y \equiv Q - H.$$

In determining it, the law of the conservation of output is used.

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economy. One should not, however, think that these circumstances could have a substantial effect during the course of one year: for such a period, product mix, output volume, and proportions hardly undergo any marked change. The output volumes produced by the various industries usually increase by not more than a few tens of per cent and, on the average, by only a few per cent.

2.5. The Magnitude of Value: Socially Necessary Labour Input in the Production of Commodities

Total labour input in the production of commodities. According to the way input-output coefficients a_{ij}, and labour input-output coefficients l_j are determined [see formulae (2.8) and (2.9)], they constitute a generalised description of all kinds of input of reproducible means of production and labour power per unit output. Remember that, in the real economy, in all industries output needs living labour input: $l_j > 0$ for all sectors j; similarly there are no industries that could exist without any reproducible material resources: among coefficients a_{ij} there exist $a_{ij} > 0$ in every industry j.

In turn, in order to be produced reproducible means of production need living labour input, as well as an input of reproducible means of production. That is why, in each industry, the labour process constitutes an activity that adds a new portion to the labour expended earlier in order to transform reproducible means of production into a new output item. The properties of technologies presented above should here be comprehended from this standpoint. It is this idea of gradually adding more and more portions of labour in the process of manufacturing series of products, ending with those for nonproductive use, that forms the basis of the concept of the magnitude of the total value of a commodity.

Marx always employed the following formula for the magnitude of the value of any kind of commodity:

$$w = c + v + m,$$

where w is the total value of the commodity, c is the value of the means of production spent on its manufacture, $(v + m)$ is the value newly created by living labour, divided under capitalism into that equivalent to the value of labour power v and surplus-value m. If commodities are denoted by j, then:

$$w_j = c_j + (v + m)_j.$$

Mathematically, this formula is extremely simple, but it involves a rather strong substantial theory, and considerable space would be needed to present it. The essence of this theory is that the magnitudes of the value thus determined form the proportions in which commodities are mutually

exchanged. The theory maintains that equivalent quantities of various commodities are those that are equal with regard to the magnitudes of value so determined. In other words, the following equation should hold:

$$w_j q_j = w_{j'} q_{j'}, \tag{2.19}$$

where q_j and $q_{j'}$ are the quantities of commodities accepted as equivalent on the market; $j \neq j'$; $j, j' = 1, \ldots, n$. By comparing it with formula (1.2), the reader can see the sense in which we maintain that value is the law of prices: the market prices of commodities p_j and $p_{j'}$ are expected to make the same quantities of commodities q_j and $q_{j'}$ equivalent as those resulting from formula (2.19), i.e., from values w_j and $w_{j'}$, $j, j' = 1, \ldots, n$.[1]

Newly created value. Individual and socially necessary living labour expenditures. According to this theory "a use-value, or useful article, therefore, has value only because human labour in the abstract has been embodied or materialised in it. How, then, is the magnitude of this value to be measured? Plainly, by the quantity of the value-creating substance, the labour, contained in the article. The quantity of labour, however, is measured by its duration, and labour-time in its turn finds its standard in weeks, days, and hours".[2]

"The total labour-power of society, which is embodied in the sum total of the values of all commodities produced by that society, counts here as one homogeneous mass of human labour-power, composed though it be of innumerable individual units. Each of these units is the same as any other, so far as it has the character of the average labour-power of society, and takes effect as such; that is, so far as it requires for producing a commodity, no more time than is needed on an average, no more than is socially necessary. The labour-time socially necessary is that required to produce an article under the normal conditions of production, and with the average degree of skill and intensity prevalent at the time."[3]

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[1] Again let us recall that the theory further allows stable deviations of the average levels of prices from those directly corresponding to the value; it does not also maintain that commodities are exchanged strictly in equivalent quantities in every individual sales contract, but the equivalence thus understood constitutes a general trend of average quantities.

[2] Karl Marx, *Capital*, Vol. I, p. 46.

[3] *Ibid.*, pp. 46-47.

A short enough period of time is taken to assume the level of skills of employees and of the intensity of their labour to be invariable, i.e., historically given. Obviously skill and intensity tend to vary over time and there is, in principle, no period during which such changes would not take place. Nevertheless the theory of value everywhere disregards these changes (but it takes them into account for longer periods of time). In fact, this means that it considers the short-term changes to be insignificant in the explaining of prices for the same period.

The theory of value also disregards the difference in the intensity of labour between industries: since every employee is individually free, the movement of labour power between industries creates a social mechanism equalising the working conditions in different industries; labour power also tends to be averaged, this being characteristic of cooperation. The levels cannot, of course, be completely equalised; there is just a general tendency, continually affected by a lot of specific circumstances. In the explaining of prices, however, this fact is assumed to be insignificant.

The validity of all these and many other assessments of what should be considered significant in deriving the law of prices and, therefore, of what should be considered only insignificant fluctuations, will be tested practically. It is useful to remember, however, that the prices of commodities of the same physical kind tend, over the same period of time, on a single market, to be equal; in any case this property of real competition prompts the conclusion that differences between firms of the same industry in both labour intensity and the skills of employees make for only negligible fluctuations.

Living labour input in production also depends on the specific features of technologies used by the given firm. Even on the condition that the intensity of labour and the skills of employees in the different firms of the same industry are assumed to be equal, l_j^k would be considered as varying from firm to firm.[1] The theory of value distinguishes between the *individual* and the *socially necessary* living labour input in the production of a unit commodity and maintains that, in determining exchange proportions, the socially necessary input is essential, whereas the individual input is

[1] This is illustrated by Table 2.5 of our conventional example, where the firms within each sector have unequal indicators l_j^k.

reduced to the former. The socially necessary inputs are taken as the average of the individual inputs.

The concept employed here of the average magnitude needs to be described specifically. As Marx repeatedly emphasised, it means the labour inputs in firms producing most of the output under current socially normal production conditions. By socially normal production conditions he meant the type of technology prevailing at some time. For instance, after defining socially necessary working time as above, he continues: "The introduction of power-looms into England probably reduced by one-half the labour required to weave a given quantity of yarn into cloth. The hand-loom weavers, as a matter of fact, continued to require the same time as before; but for all that, the product of one hour of their labour represented after the change only half an hour's social labour, and consequently fell to one-half its former value".[1]

Individual labour required is thus reduced to the socially normal level, so after a new technology has become dominant in the industry, that which previously determined the latter (the labour of manual weavers in the example) proves exceedingly high and is accepted by society only in reduced, i.e., new normal quantities. For commodity producers employing obsolete technologies this reduction constitutes a real loss: the spread of firms with a higher productivity of labour in an industry does not affect the level of productivity of living labour with old technologies. Since such a spread takes place, however, an ever increasing share of the output (and particularly the increment in output in comparison with its former level) is produced by new technologies and therefore requires less labour than was typically the case previously. The former level of labour input (then the socially normal one) gradually ceases to be so and tends to be accepted by society only in the reduced amount necessary for the new technological methods.

Yet until the new technologies have spread so that most output is produced by them, i.e., until they dominate in an industry, the input of the new technologies is, on the contrary, reduced to that of the former technologies which still remain socially necessary. "If one hour's labour is embodied in sixpence, a value of six shillings will

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[1] Karl Marx, *Capital*, Vol. I, p. 47. More precisely, the value added in this case by living labour to that of expended means of production fell by half. See Marx's own example below.

be produced in a working-day of 12 hours. Suppose, that with the prevailing productiveness of labour, 12 articles are produced in these 12 hours. Let the value of the means of production used up in each article be sixpence. Under these circumstances, each article costs one shilling: sixpence for the value of the means of production, and sixpence for the value newly added in working with those means. Now let some one capitalist contrive to double the productiveness of labour, and to produce in the working-day of 12 hours, 24 instead of 12 such articles. The value of the means of production remaining the same, the value of each article will fall to ninepence, made up of sixpence for the value of the means of production and threepence for the value newly added by the labour... The individual value of these articles is now below their social value; in other words, they have cost less labour-time than the great bulk of the same article produced under the average social conditions... The real value of a commodity is, however, not its individual value, but its social value; that is to say, the real value is not measured by the labour-time that the article in each individual case costs the producer, but by the labour-time socially required for its production. If therefore, the capitalist who applies the new method, sells his commodity at its social value of one shilling, he sells it for threepence above its individual value, and thus realises an extra surplus-value of threepence. On the other hand, the working-day of 12 hours is, as regards him, now represented by 24 articles instead of 12. Hence, in order to get rid of the product of one working-day, the demand must be double what it was, *i.e.*, the market must become twice as extensive. Other things being equal, his commodities can command a more extended market only by a diminution of their prices. He will therefore sell them above their individual but under their social value..."[1]

This reasoning is noteworthy in many respects. Here we shall emphasise just two features: first, according to Marx, as long as a new technological method yields only a small part of the total output and "the great bulk of the same article" is made under the former conditions, individual input with the improved method would be reduced to those prevailing on average, which benefits the capitalist who introduces the new method. Second, the change in

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[1] Karl Marx, *Capital*, Vol. I, pp. 300-301.

the socially necessary input has already started. In order to win a greater share of the market for his commodities, the capitalist is forced to lower the price to be below the established average.[1] "...This extra surplus-value vanishes, so soon as the new method of production has become general, and has consequently caused the difference between the individual value of the cheapened commodity and its social value to vanish. The law of the determination of value by labour-time, a law which brings under its sway the individual capitalist who applies the new method of production, by compelling him to sell his goods under their social value, this same law, acting as a coercive law of competition, forces his competitors to adopt the new method."[2]

So far we have, in fact, considered situations in which some industry employs three typical technologies: an obsolete one, the most widespread one (in terms of the mass of commodities produced), and a certain new one, as yet not widespread. It has been shown how, in the use of the first and last of them, individual expenditures are reduced to the average socially necessary ones. It is certainly implied here that individual differences between firms in each of these typical technologies are eliminated: for this purpose it is enough to employ average weighted input indicators within each of the three groups of firms.[3] Table 2.1 in our conventional example shows precisely the three kinds of firms in each industry. Their individual input coefficients are given in Table 2.5. (Our conventional example also takes into account the fact that the increase in the productivity of living labour is usually associated with that in material

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[1] Note that, according to Marx, the necessity of this continues to exist for the capitalist even when the total capacity of the market remains unchanged, the purpose being only to sell as many of *his own* commodities as possible on the given market.

[2] Karl Marx, *Capital*, Vol. I, p. 302.

[3] Let us note that the determination of the average weighted magnitudes proves necessary as soon as the individual input indicators within a particular firm are calculated: strictly speaking, they differ from day to day and are dependent on the extent to which a given productive capacity is utilised, i.e., on the quantity of commodities produced, and so on. In formulae (2.10) and (2.11), these considerations have already been taken into account: first, the total input volume of each kind and the total output by a given firm per year are determined, after which only the average coefficients a_{ij}^k and l_j^k have to be determined, in this case having been determined from the very beginning as the average weighted ones.

Table 2.5

Individual Direct Material and Labour Input Coefficients by Firm

	Industries (j)								
	1			2			3		
	Firms (k)								
	1	2	3	4	5	6	7	8	9
			a_{ij}^{k}	(units of means of production spent per unit output)					
$i=$ 1	×	×	×	0.1364	0.30	0.50	0.75	0.70	0.8333
$i=$ 2	0.24	0.25	0.26	×	×	×	0.025	0.06	0.05
$i=$ 3	0.12	0.15	0.18	0.1091	0.15	0.20	×	×	×
			l_{j}^{k}	(units of working time with reduction [illegible] labour per unit output)					
×	1.5	1.0	0.5	0.8182	0.45	0.3333	1.50	0.95	0.75

input per unit output. This was at any rate true in the stage of the initial industrialisation of production.)

The question becomes more complex when technical progress offers a variety of methods for producing a certain product so that simultaneously more than three methods that are fundamentally different technologically are employed; moreover, none of them dominate in production, i.e., none of the typical technologies can produce the bulk of the commodities of the given kind. Even after the specifically individual differences in each group of firms have been eliminated, the question remains open as to which of the technologies corresponds to the concept of the socially necessary conditions of production. In this case, this concept itself seems to require generalisation: one should speak of *the group* of firms that produce *the bulk* of commodities while expending individual labour (not only living labour but also that embodied in the means of production) below the individual labour expended within *the group of the poorest* technologies, but above those in *the group of the best* technologies. Yet the question arises most acutely as to how one might actually divide the technologies into three such groups, since the unity of the technological type of production is no longer observed within these groups. The question also arises as to exactly which part of the output might be called its major part; clearly it should exceed 50 per cent of the total output in the industry Q_j, but by how much?

Note that such a general formulation of the question results in a failure to employ just the indicators of individual inputs of living labour and in the necessity of using the total input of labour, both living and that embodied in the means of production. The definition of individual and socially necessary inputs of embodied labour has not, however, been determined yet. Moreover, individual embodied labour inputs cannot be determined until the total social value of all means of production is known; the transfer of value from the means of production depends, of course, on the input standards of these means of production that are *individual* for each kind of technology and each firm, but it also depends on the *social* value of these means of production (see below). It then turns out that *until the social value of the means of production (and therefore of all commodities) has been determined, it is not possible to identify, within an individual industry, the group of technologies corresponding to the concept of socially normal conditions of production.* Yet a knowledge

of this group is apparently necessary to determine the social values of all commodities.

In fact, such knowledge is not indispensable. One has only to take into account that the social and individual values of commodities are formed by the *actual* input in production in the society for a given, sufficiently short period of time (e.g., a year), on the obligatory condition that individual technologies are employed at the level of the individually possible input minimum and output maximum. Yet we have already emphasised that this is a necessary logical condition of the structuring of the theory of value connected with the necessary real conditions of the existence of social production in general (cf. 1.1).

The actual living labour input in each industry, as determined in formula (2.5), will here be seen as socially necessary in the given year.[1] The formula (2.9) determines the average coefficient of the socially necessary living labour input. A substantial advantage of such a determination is that it is based precisely on actual productive input; accordingly, the theory of value is built on magnitudes that actually exist and can be measured in the operating firms. The average coefficient appears, in this case, as the average weighted magnitude of all real ones.

The difference between this method and that of determining socially necessary expenditures as those prevailing in most firms, dealt with above, should not be exaggerated. First, we had to resort to using the average weighted magnitude in that case too, though only among the firms of some typical technology.[2] Second, if the above technology really does yield a "great bulk" of output, the total average weighted magnitude is determined quite accurately by this typical one, since deviations from it do not have any great specific weight and at the same time to a considerable extent cancel one another out. (Remember that the typical technology is assumed to be between the worst and the best.) Thus, if the typical technology does actually exist, the calculation of the weighted average for all technologies results in

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[1] It is assumed here that the output of all firms is in demand.

[2] It is worth remembering that the magnitudes obtained as average weighted ones are likely not to coincide with the actual ones in any of the firms. This applies equally to obtaining the averages for all and for only typical firms. Yet they are real, since, in generalised form they express the actual input of the firms taken into consideration when averaging.

practice in approximately the same magnitudes as the calculation for the typical technology only. If no typical technology exists, then the calculation for all technologies remains possible. This means that the method for calculating the average weighted magnitude for all technologies is some general one, whereas that for calculating the weighted average for typical technologies is a particular one suitable only in certain cases.

It is easy to see that, in *Capital*, Marx often virtually put an equality sign between the two methods while speaking alternatively about the magnitudes corresponding to the normal conditions of production over a given period of time (i.e., the typical ones, under which the bulk of output is produced) and the magnitudes he considered to be simply the weighted averages.[1] Provided the identification of typical technologies is actually possible, these methods are, in fact, indistinguishable.[2] Note that exactly the general method takes into account the following fact referred to by Marx: although the introduction of a new and more productive method may be only just beginning, the socially necessary expenditures, even if they are not as yet affected by this, would nevertheless start to decrease.

If, of course, the formation of value may be treated without regard to the stages of the economic development, simple calculations of actual input will probably seem to contradict the concept of social necessary input; it is not impossible that actual input would include, in particular, the inputs of slack, lazy, careless workers, i.e., not socially necessary ones. As Marx showed, however, if prices are based on value, there is an incentive to reduce individual input to a level at least not higher than the average and as low as possible. To offset the excess of individual input over the average level is a direct necessity here, since in such a case the market returns to the producer, in the form of the commodity

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[1] Here is one such argument: "On the one hand, market-value is to be viewed as the average value of commodities produced in a single sphere, and, on the other, as the individual value of the commodities produced under average conditions of their respective sphere and forming the bulk of the products of that sphere" (Karl Marx, *Capital*, Vol. III, p. 178; see also pp. 182-85).

[2] In our conventional example, the average firms producing the bulk of the output are Nos 2, 5, 8. By comparing Tables 2.5 and 2.2 the reader can see:

$$1.0 = l_1 = l_1^2; \quad 0.5 = l_2 \approx l_2^5 \quad 0.45; \quad 1.0 = l_3 \approx l_3^8 = 0.95.$$

price, less labour in absolute terms than he spent on production. That is why the accepted condition under which socially necessary working time is formed, i.e., the average level of skills and intensity of labour, actually applies in society to all firms as a result of the operation of the law of value itself. *The law of value itself tends to create the conditions that ensure its operation in ever greater accordance with itself.* Therefore, as this law is accepted, any deviations of actual input from those under which the level of value is determined in accordance with the law should be disregarded.

In its most developed form, under capitalism, the law of value works as a result of the previous protracted historical process, which has formed the necessary labour discipline and culture under which major violations of the technologically necessary input minimum are essentially excluded. Accordingly, in the theory this inaccuracy is considered as insignificant. Proof of the validity of the theory is, of course, provided by practice. Before any practical proof, however, the precise theses the theory contains should be stated. The inclusion of the expenditures at the actual industry level does not contradict the theory since, if the stated law is true, the former involve excessive ones only to an insignificant extent.[1]

From all the above it follows that the magnitude of newly created value may be assumed to be L_j in all industries [see formula (2.5) and Table 2.3 in the conventional example]. In other words, it may be assumed that, in each industry

$$(v+m)_j \equiv l_j = L_j/Q_j, \quad j=1, \ \ldots, \ n. \tag{2.20}$$

Coefficients l_j^k form the corresponding magnitudes of individual input per unit output by firm—the components of the individual values of commodities.

Value transferred from means of production. The theory of value considers the labour input in creating some commodity as the sum total of labour time required in many industries for the set of products, the creation and further processing of which are required, given a certain technological system, to create that commodity. "...In determining the value of the yarn, or the labour-time required for its production, all the special processes carried on at various times and in different places, which were necessary, first

[1] The case when excessive expenditures are substantial is analysed below (p. 121, footnote).

to produce the cotton and the wasted portion of the spindle, and then with the cotton and spindle to spin the yarn, may together be looked on as different and successive phases of one and the same process... If a definite quantity of labour, say thirty days, is requisite to build a house, the total amount of labour incorporated in it is not altered by the fact that the work of the last day is done twenty-nine days later than that of the first. Therefore the labour contained in the raw material and the instruments of labour can be treated just as if it were labour expended in an earlier stage of the spinning process, before the labour of actual spinning commenced."[1]

The value of each commodity thus includes the value contained in the means of production used up in producing this commodity. It is of no importance that the concrete useful labour, for instance of the spinner, has nothing in common, in terms of the specific content of the technological processes performed, with that of the cotton-grower, machine-builder (producer of the spinning machines), builder, transport worker, and the like.

"...The commodities that take part in the process, do not count any longer as necessary adjuncts of labour-power in the production of a definite, useful object. They count merely as depositories of so much absorbed or materialised labour; that labour, whether previously embodied in the means of production, or incorporated in them for the first time during the process by the action of labour-power, counts in either case only according to its duration."[2]

Neither is anything changed by the fact that these specific kinds of labour may not be equal in complexity. The real working time of workers performing complex labour participates in forming value via a certain multiplier greater than unity, whereas the working time of workers performing simple, unskilled labour has a multiplier equal to unity. Taking these multipliers into account, all kinds of labour make up the integrated commodity value determined by the total sum of working time needed to produce the commodity, the labour being reduced.[3]

The inclusion in the total value of the commodity produced of the value of the means of production expended is called the *transfer* of the value of means of production

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[1] Karl Marx, *Capital*, Vol. I, pp. 182-83.

[2] *Ibid.*, p. 190.

[3] *Ibid.*, pp. 191-92.

to the commodity. This is carried out thanks to the fact that the means of production are actually used deliberately by the worker, processing them into a commodity, used for their proper purpose — for manufacturing the useful products for which they were originally produced and then purchased and used in really necessary amounts, i.e., not in excess of the necessary input level with the given technologies. The social mechanism making the producer carry out production under a technologically necessary minimal material input has already been considered in analysing living labour input, this mechanism being the same for all productive kinds of input. If this mechanism operates for long enough, excessive expenditures resulting from violations of technological discipline are seen to be negligible.[1] In such a case, the social material input norms may be seen, like those of labour, to be the weighted averages of the input of all firms whose output can be sold. The commodity economy reduces individual input norms of producers using different technologies to such averages with the aid of the market mechanism, which equalises the prices of similar commodities.

If some dominating technology exists, this is precisely what determines the magnitude of the above average. "...Only so much of the time spent in the production of any article is counted, as, under the given social conditions, is necessary. The consequences of this are various. In the first place, it becomes necessary that the labour should be carried on under normal conditions. If a self-acting

[1] Accordingly, the case when excessive labour and material input constitutes a substantial part of society's total may be excluded from the analysis of the problems of the theory of value. Yet it seems that the law of value would, in a certain sense, remain valid even for such a situation. In order to exist, society must replace the productive resources used up and if production is commodity production, the need for this replacement must be realised by the market and, in some sense, regulates prices. Excessive input is a deduction from the potential surplus product and is possible, of course, only within this part of the product. More precisely, (a) a potential surplus product appears partly as a component of the replacement fund covering excessive outlays of means of production; (b) another potentially possible part of the surplus product is simply not produced, owing to excessive labour-input required to produce actual output. In this case, the law of value creates incentives for making real input norms comply with the technologically necessary ones; until this is done, it constitutes an alternative to the technical progress which, generally speaking, results from the operation of the law of value.

mule is the implement in general use for spinning, it would be absurd to supply the spinner with a distaff and spinning wheel. The cotton too must not be such rubbish as to cause extra waste in being worked, but must be of suitable quality. Otherwise the spinner would be found to spend more time in producing a pound of yarn than is socially necessary, in which case the excess of time would create neither value nor money. But whether the material factors of the process are of normal quality or not, depends not upon the labourer, but entirely upon the capitalist."[1]

The transfer of value from means of production to the commodity produced is, as is clear from the above, the function of the concrete labour of production workers (proletarians, since a capitalist economy is meant). Production workers add a new value to this value, thanks to the abstract character of the labour, being the expenditure of social labour power, of social working time. Anticipating somewhat, note that the value of labour power is determined so that, by paying for it on the market, the capitalist acquires the commodity the functioning of which gives him both these results. Moreover, the first of them comes to him gratis, in the true sense of the word, merely because the means of production imbibe living labour, and are necessary objective conditions for the functioning of the worker. It is not, however, a matter of indifference to the capitalist, since he has bought the means of production and these would lose their value even if not employed (under the impact of natural processes, leading to destruction of their use-value) and, together with their value, the capitalist would lose the money he paid for them. At the same time, the value transferred from means of production is repaid as part of that of the final commodity and returns to the capitalist as money.

Thus, the magnitude of the value transferred from the means of production is determined, first, by their own value and, second, by the socially necessary input normals of these means of production. This means:

$$c_j \equiv \sum_i w_i a_{ij}, \quad i, \ j = 1, \ \ldots, \ n. \qquad (2.21)$$

Simultaneous equations for getting the total labour input in the production of unit output. Thus, taking formulae (2.20) and (2.21) into account, Marx's formula for commodity

[1] Karl Marx, *Capital*, Vol. I, p. 190.

value may be represented by the following:

$$w_j = c_j + (v+m)_j = \sum_i w_i a_{ij} + l_j; \quad i,\ j = 1,\ \ldots,\ n; \qquad (2.22)$$

or, in matrix and vector form:

$$w = wA + l.$$

Let us note precisely the identity of Marx's formula and that represented with the help of coefficients A and l, the latter formula being merely the mathematical expression of the theoretically determined corresponding magnitudes in Marx's basic formula. Yet this presentation paves the way for determining the total labour input in the production of commodities (unit value) using only direct material and labour input indicators as basic data; it expresses value by means of simultaneous equations: (2.22) is a system of n linear equations (according to the quantity of commodities) with n unknowns (total labour input, or values). Each such equation has strictly positive free terms, since it is assumed that all $l_j > 0$. There exists a unique solution to the system of equations under consideration.

It is known that, as matrix A is productive, magnitudes w_j obtained on the basis of (2.22) are strictly positive for all j.[1] The productivity of matrix A is given. The solution to (2.22) consists of the mathematical analogues of the magnitude of commodity value. In turn, system (2.22) is merely a mathematical reflection of the real process of the formation of value.[2]

The reader is familiar with the methods of solving systems of equations from mathematics textbooks. Here we shall

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[1] The following is known to be valid for productive matrices: all elements of matrix $(I - A)^{-1}$ are non-negative (where I is a unit matrix). The determination of vector w in formula (2.22) may be written in a different form:

$$l = w(I - A); \quad w = l(I - A)^{-1}.$$

Multiplying positive vector l by non-negative matrix $(I - A)^{-1}$ results in non-negative w. Since, in accordance with (2.22), all $w_j \geqslant l_j > 0$, it follows that $w_j > 0$ $(j = 1, \ldots, n)$. Since for each j there are, among the coefficients a_{ij}, strictly positive ones, $w_j > l_j > 0$ $(j = 1, \ldots, n)$.

[2] It is well known from analysing the properties of system (2.22) that the formation of the total labour expenditure appears mathematically as a chain process in which direct and indirect expenditures accumulate successively, so that the latter ultimately appears as an infinite number of cycles of the former. This conclusion may be easily drawn from the expression: $w = l(I - A)^{-1} = l + lA + lA^2 + \ldots$

simply give the solution for our conventional example:

$$w_1 = 1.69, \quad w_2 = 1.36, \quad w_3 = 2.33$$

(in labour terms, with regard to the reduction of labour, per unit output.) Table 2.6 allows one to satisfy oneself

Table 2.6

The Structure of the Total Labour Input Coefficients

	Industries (j)		
	1	2	3
Coefficients of direct input of means of production		a_{ij}	
$i=$ 1	×	0.30	0 75
2	0.25	×	0.05
3	0.15	0.15	×
Total unit labour intensity of		w_i	
commodity 1	1.69	1.69	1.69
commodity 2	1.36	1.36	1.36
commodity 3	2.33	2.33	2.33
Value transferred from the means of production		$w_i a_{ij}$	
1	×	0.51	1.26
2	0.34	×	0.07
3	0.35	0.35	×
Value added by living labour		l_j	
	1.00	0.50	1.00
Total		w_j	
	1.69	1.36	2.33

that the solution is true: using calculated values w_i in determining $\sum_i w_i a_{ij}$, i.e., the value transferred to each kind of commodity by the corresponding means of production, adding the magnitudes of the newly created value l_j, we actually obtain precisely these indicators w_j. At the same time, the Table allows the value composition of each commodity to be analysed for all components: the c_j to $(v + m)_j$ ratio, the internal structure of c_j.

Note that the total value of each commodity is higher than that newly created in producing it, since for all com-

modities $c_j \equiv \sum_i w_i a_{ij} > 0$. The reason is that each commodity requires for its production not only labour and natural resources input but also input of reproducible means of production.

The system of coefficients A and l is *necessary and sufficient* for building the system of equations allowing the value of commodities to be determined in accordance with the theory. The solution to the system (2.22) is logically just the disclosure, behind the direct material and labour inputs that lie on the surface, of a certain latent, internal magnitude representing the value expressing the former in generalised form.

To calculate the value the system of coefficients employed in I-0 tables for product and labour, i.e., coefficients A and l, is necessary and sufficient. This is possible owing to the profound internal unity of the theory of value and that of reproduction, expressed in the input-output model.

It may be apt at this point to return to a fact already considered: the input-output model and the labour I-0 cannot be suitably built unless *average* a_{ij} and l_j are used. It is necessary to use precisely average coefficients for finding commodity values too. Only by means of generalisation, the mutual demand of the industries for output to be used as means of production, as well as their demand for labour power, may be determined in accordance with reality. Yet the theory of value considers demand as an element of the market mechanism concerning the operation of the law of value.

If any particular technology, or incomplete (nonrepresentative) set of technologies, is taken in each industry, with their coefficients it is impossible to find the composition of gross output Q and the system of flows among industries corresponding to reality. This is vital for understanding why, in the theory of value, the latter also appears as some average magnitude.

Marx's formula $w = c + v + m$ and its analogue (2.22) reflect a number of main points of the theory of value that we have not yet formulated specifically, though they have, in fact, been implied. Now we shall deal with them.

The period for which the magnitude of value has to be determined. Real calculation of the magnitude of value on the basis of (2.22) implies the use of I-0 coefficients. Note

that, at least in the countries with moderate climate, where the agricultural production cycle is equal to a year (one crop is grown a year), both the I-0 model itself and the value of the reproduction of gross output must be determined for at least a year. The expenditures on the reproduction not only of agricultural produce itself, but also, correspondingly, on that of foodstuffs and light industry products, etc. cannot be determined for a shorter period of time. Neither would the calculation of the I-0 model structured for the sum total of output for a number of years correspond to the concept of the value of production: over this time, the conditions and value of reproduction tend to change; then coefficients a_{ij} and l_j express not simply the averaging of the properties of given technologies, but also a shift in the technological system itself.[1]

Representation of the expenditure of all reproducible resources as those of social labour. The theory of value as a whole is based on the concept of the special, constituting role of labour in production (see paragraph 1.1); it exposes the specific manifestation of this fact common to all production forms under the conditions of commodity-money relations.[2] The following expresses one of the specific features of the labour resource: in the combination of the I-0 model with the labour balance, all kinds of input of reproducible productive resources may be represented as expenditures of embodied labour [see (2.22)]; the opposite, however, is impossible: living labour input cannot be represented as input of material productive resources.

In other words, there actually exists, is expressed in the theory of value, and is mathematically presented in

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[1] Strictly speaking, even during a year in the industries where the production period is substantially less than a year, the cost of reproduction may tend to change. Accordingly, the value of commodities varies, as does the value of commodities in those industries where the products under consideration are used as means of production. During a year, however, value changes happen only within narrow limits. Static deterministic models, one of which is the I-O model, ignore these changes, which is a quite admissible abstraction, not affecting the quality of the conclusions.

[2] "The 'value' of a commodity expresses in a historically developed form only that which exists, albeit in *another form*, in all other historical and social forms, *that is the social character of labour*, since the latter exists as the *expenditure of social labour power*" (Karl Marx, "Randglossen zu Adolph Wagners 'Lehrbuch der politischen Ökonomie'", in: Marx/Engels, *Werke*, Bd. 19, Dietz Verlag, Berlin, 1962, p. 375).

formula (2.22) a process by which the labour embodied in means of production is transferred to its output. Yet there are no means of production embodied in labour power as such (in the human capability of purposeful productive activity itself), although the *subject*, and the *means*, of such activity are material means of production existing outside men. That is why coefficients w_j appear not as one of many possible but as the *only possible* way to transform the whole system of input coefficients a_{ij} and l_j into indicators of the input of *one* of the productive resources; in themselves, all a_{ij} have, in their numerator, particular physical units of measurement (of means of production i), so they cannot be summed and added to l_j, though all of them express expenditure per unit of one and the same commodity j. To put it briefly, only in such a transformation does all of the above expenditures appear as an integrated unity, as expenditure in general.[1]

Here we see one manifestation of labour's abstract character forming commodity value. To be products of labour is the only economically essential common property of all reproducible commodities. It makes itself felt in commodity exchange, while determining its proportions. Before this, however, it is manifested in the formation of commodity value by summing up labour input at the successive stages in the processing of primary natural material into commodities of the given type.

The law of the conservation of embodied labour in the process of its transfer to the commodity. The value of commodities as that of their reproduction. Magnitudes w_j and w_i in (2.22) are, according to the way they are determined, identically equal to $j = i$, $i, j = 1, \ldots, n$, while magnitudes w_j ($j = 1, \ldots, n$) in the left-hand part of the equations stand for having the sense of total labour input, as the latter is *formed* in the production of commodities j: magnitudes w_i ($i = 1, \ldots, n$) in the right-hand part of (2.22) have the sense of total labour input transferred from unit of means of production i.

This property of the way total labour input is calculated on the basis of the I-0 model and the labour balance is merely a mathematical expression of *the law of the conservation of embodied labour in the process of its transfer* from

[1] As may easily be seen, the possibility of calculating the coefficients of the total expenditure of each kind of means of production per unit net final output does not contradict this statement.

the means of production to the commodities produced. The transfer is expressed in (2.22) by magnitudes $w_i a_{ij}$, and the additional input of living labour by l_j. The law of conservation constitutes one of the most important attributes of the theory of value. The conservation of value is a concept relating to the process of its transfer only. In the process of commodity production, new value is created in addition to that earlier created and transferred from the means of production. This can be seen from (2.22), in accordance with which

$$w_j > \sum_i w_i a_{ij}, \quad w_j - \sum_i w_i a_{ij} = l_j > 0, \quad j = 1, \ldots, n \qquad (2.23)$$

holds. The destruction of the useful form of a commodity in the process of its consumption, as well as under the influence of natural forces, means the *destruction* of its value.

The identity of the magnitudes w_j and w_i for $i = j$ corresponds, besides, to the theoretical notion of value as that of *reproduction*. The quantity of labour transferred from the means of production to the output in the process of its use is *determined* by the simultaneous production of similar means of production, i.e., by the labour expenditures necessary for the reproduction of such means of production, for their replacement. The necessity was shown above (see paragraph 2.4) of producing new copies of means of production to replace those physically destroyed in the process of productive consumption, of producing them at the same rate as they are destroyed.

The value of the means of production changes, of course, in the industries where they and the means of their own production are produced. When their value is transferred to the commodities, however, it does not change. Yet the value is transferred only in the amount embodied in similar means of production produced at the same time, rather than in the amount embodied in them earlier, in the course of their production. This fact is vital for an understanding of many processes of the development of capitalism and the fates of individual capitals. A capitalist who has purchased means of production at their former value may be ruined or in any case suffer a substantial loss owing to the

revolution in the value of his means of production, which is out of his control.[1]

National income. Since it is accepted that relations among industries are suitably reflected in the I-O model of the distribution of output (2.16) and that of labour power (2.7), and the formation of the value of commodities in simultaneous equations (2.22), the system of premises is accepted as sufficient for concluding that the value of net output is equal to that newly created by living labour over the corresponding time period. To put it formally:

$$\sum_i w_i Y_i = L, \tag{2.24}$$

or

$$wY = L.$$

The proof of this conclusion is well known in the I-O model theory.[2] At the same time, *this conclusion plays a fundamental role in the entire Marxian theory making possible the transition from the theory of value to that of surplus-value.* Below, it will be shown that, since the total value

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[1] Indicating that initially he studied the circulation of individual capital on the assumption that the acts of purchase and sale of commodities were not "merely replacement of one commodity for another, but replacement with value-relations remaining the same", Marx goes on to say: "The values of the means of production vary. It is precisely capitalist production to which continuous change of value-relations is peculiar, if only because of the ever changing productivity of labour..." (Karl Marx, *Capital*, Vol. II, Progress Publishers, Moscow, 1978, p. 74). "The movements of capital appear as the action of some individual industrial capitalist who performs the functions of a buyer of commodities and labour, a seller of commodities, and an owner of productive capital, who therefore promotes the circuit by his activity. If social capital experiences a revolution in value, it may happen that the capital of the individual capitalist succumbs to it and fails, because it cannot adapt itself to the conditions of the movement of values. The more acute and frequent such revolutions in value become, the more does the automatic movement of the now independent value operate with the elemental force of a natural process, against the foresight and calculation of the individual capitalist, the more does the course of normal production become subservient to abnormal speculation, and the greater is the danger that threatens the existence of the individual capitals. These periodical revolutions in value therefore corroborate what they are supposed to refute, namely, that value as capital acquires independent existence, which it maintains and accentuates through its movement" (*Ibid.*, pp. 108-109).

[2] Let us give a brief mathematical demonstration of (2.24). $Q = AQ + Y$ [see (2.16)] is given. Hence $Y = (I - A)Q$, $Q = (I - A)^{-1}Y$. Then $L = lQ = l(I - A)^{-1}Y$. Yet, at the same time, $w = wA + l$ [see (2.22)], from this $l = w(I - A)$, $w = l(I - A)^{-1}$. Then it is true that: $l(I - A)^{-1}Y = wY$, therefore $L = lQ = wY$.

of the whole net output is equal to the living labour newly expended by the workers in the process of production and the workers receive only part of the net output, while the rest goes to the capitalists, we are dealing with relations of capitalist exploitation.

The conclusion is based on the following property of the total labour input coefficients w_j derived from (2.22): they include the living labour input l_j as constant:

$$lQ = wY = L. \tag{2.25}$$

This property expresses the second aspect of *the law of the conservation of labour* employed in the theory of value: the commodity embodies, as part of its value, the input of living labour in its production. This applies, first of all, to each j commodity. At the same time, on the scale of society as a whole, the total value of net output is just such an embodiment. It is the value of net output that forms the *national income*.

The national income is a category of value (superficially, of prices). Net output is a vector of output in physical units of measurement. The formation of both is characterised by inner unity: in the latter case the replacement fund of expended means of production in physical terms is subtracted from the total volume of output in physical terms; in the former case, the value transferred from the same expended means of production, i.e., the replacement fund in value (superficially—in prices) is subtracted from the total value of the gross output. That is why the newly created value turns out to be nothing but the total value of the net output of society. In other words, since the discussion applies to net output, its total value (which includes, of course, in each unit of each commodity the value transferred from the means of production) appears on the scale of society as an embodiment of the total labour spent by the workers, and of that labour only. This fact is in complete compliance with the principle, common to all forms of production, that the final goal is to create precisely the net output that remains after the necessary material input have been replaced, rather than simply to create the gross output.

The law of the conservation of the value of commodities in the process of exchange and distribution. The output distribution process, as it is described in the I-O model, does not change the amount of labour embodied in the products.

Since this distribution is performed in the form of commodity circulation (exchange), this means that exchange does not alter the value of the products exchanged. In other words, once exchange has been completed, the commodities remain of the same value as before. The following formula is valid:

$$wQ = wH + wY. \qquad (2.26)$$

This formula expresses the law of the conservation of the labour embodied in output during its exchange process.[1] It relies on the law of the conservation of output; since it is more correct to formulate the latter law as one of non-increasing output in its exchange process, it would be more correct to speak of the law of the *non-increase* of the value of commodities in their exchange process. Exchange requires the expenditure of products, i.e., diminishes the quantities of them to be directly consumed for productive and nonproductive purposes; accordingly, it requires the expenditure of the value embodied in those products, as well as a more living labour expenditure. This does not, however, increase the value of the commodities. On the contrary, it constitutes a deduction from society's working time fund, which might otherwise have been used for productive labour.[2] Let's, however, get down to the analysis of expenditure on performing the exchange process.

The operation of the given law applies to all the processes of distribution of commodities as such, i.e., those that do not augment the masses of real use-values at society's disposal. Besides the interindustry distribution (exchange) processes, these also include the processes of output distribution between the classes in society.[3] All of these theoreti-

[1] "Turn and twist then as we may, the fact remains unaltered. If equivalents are exchanged, no surplus-value results, and if non-equivalents are exchanged, still no surplus-value. Circulation, or the exchange of commodities, begets no value" (Karl Marx, *Capital*, Vol. I, pp. 160-61). "...Exchange itself be it in the form of barter or in that of circulation leaves the values put in it intact adding to these no new value" (Karl Marx, Friedrich Engels, *Gesamtausgabe* (MEGA), II, Band 3, Teil 1, S. 22).

[2] This applies only to genuine circulation operations as such, i.e., ones connected with the change in the form of value.

[3] "...The separation and resolution of new value annually added by new labour to the means of production, or to the constant part of capital, into the various forms of revenue, viz., wages, profit and rent, do not at all alter the limits of the value itself, the total value to be

cal points imply one and the same assumption, namely, that commodity value is created (and transferred from the means of production) only during the creation of use-values and is a specific expression of productive labour expenditure under the conditions of commodity production; accordingly, it is lost by commodities only together with their loss of the use-value (it is also true of its transfer). Since some economic processes (exchange, distribution) do not change the useful properties of commodities, neither do they change the values of these. This is the connection between the law of the conservation of output in the distribution (exchange) process and that of the conservation of value in the same process. Really productive labour is always labour creating real social wealth, use-values.[1] This fundamental general thesis of historical materialism underlies all Marxian economic theory.

The individual value of commodities. The value of commodity, as determined by formula (2.22), expresses the average, socially necessary input of labour on reproduction. It is based on average (for each industry) direct material and labour input coefficients. These averages do not, however, actually exist as such. They are only a generalised expression of actual inputs by actual firms, i.e., the individual material and labour productive inputs that can actually be measured. These inputs are not usually equal for the firms of an industry; as a rule they differ in the composition of the resources spent and in the magnitude of the coefficients (normals per unit output). They were expressed above by coefficients a_{ij}^{k} and l_{j}^{k}.

Along with the average (social) value, the individual value of commodities arises. It is the sum of labour spent to produce commodities in a certain private firm. It varies depending, first, on the varying *input norms* of means of production (the value of the same means of production being *the same* for all firms employing them), and second, on the varying input norms of living labour. The formula

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distributed among these various categories; any more than a change in the mutual relations of these individual parts can change their total, this given magnitude of value" (Karl Marx, *Capital*, Vol. III, p. 858).

[1] The opposite, however, is false: not all labour creating use-values is seen by society as productive. To create useful products is a necessary but not sufficient indication of productive labour. The determination of it depends on the social form of production.

Calculation of the Individual Value of the Unit Commodity by Firm

	Industries (j)								
	1			2			3		
	Firms (k)								
	1	*2*	*3*	*4*	5	6	7	8	9
Direct material input-output coefficients a_{ij}^{k}, i = *1*	×	×	×	*0.1364*	0.30	0.50	0.75	0.70	0.8333
2	*0.24*	*0.25*	*0.26*	×	×	×	0.025	0.06	0.05
3	*0.12*	*0.15*	*0.18*	*0.1091*	*0.15*	0.20	×	×	×
Value transferred from the means of production $w_i a_{ij}^{k}$, i = 1	×	×	×	0.2305	0.507	0.845	1.2675	1.183	1.4083
2	0.3264	0.34	0.3536	×	×	×	0.034	0.0816	0.068
3	0.2796	0.3495	0.4194	0.2542	0.3495	0.466	×	×	×
Sum total $\sum_i w_i a_{ij}^{k}$	0.606	0.6895	0.773	0.4847	0.8565	1.311	1.3015	1.2646	1.4763
Living labour input coefficients l_j^{k}	1.5	1.0	0.5	0.8182	0.45	0.3333	1.50	0.95	0.75
Individual value in total w_j^{k}	2.11	1.69	1.27	1.30	1.31	1.64	2.80	2.21	2.23

of the individual value of unit commodity j is, thus, for firm k:

$$w_j^k = \sum_i w_i a_{ij}^k + l_j^k, \quad k \in E_j, \quad j = 1, \ \ldots, \ n. \qquad (2.27)$$

Given the vector w and the indices a_{ij}^k, l_j^k, and Q_j^k for individual firms on the basis of which matrix A, vectors l and Q are calculated, individual values are determined in the *unique* way and as *strictly positive* quantities.

The total individual value of the whole amount of commodities produced by a given firm is:

$$W_j^k = w_j^k Q_j^k. \qquad (2.28)$$

In such a case, the total value of all commodities in each industry is the sum total of the individual values:

$$W_j = \sum_{k \in E_j} W_j^k, \quad (j = 1, \ \ldots, \ n). \qquad (2.29)$$

Indeed, from (2.27) and (2.28) it follows that:

$$\sum_{k \in E_j} W_j^k = \sum_{k, i} w_i Q_j^k a_{ij}^k + \sum_k Q_j^k l_j^k;$$

and from this, with regard to (2.8) and (2.9), that:

$$\sum_{k \in E_j} W_j^k = \sum_i w_i A_{ij} + l_j Q_j = Q_j \sum_i w_i a_{ij} + l_j Q_j = w_j Q_j = W_j.$$

Accordingly, social value w_j appears as the weighted average of individual values:

$$w_j = \frac{\sum_k w_j^k Q_j^k}{\sum_k Q_j^k}, \quad j = 1, \ \ldots, \ n, \ k \in E_j. \qquad (2.30)$$

It should be stressed that the individual values of commodities w_j^k do not exist until w_i, i.e., the social values of commodities, are determined. Indeed, the latter should be given when calculating by means of formula (2.27). This complies with the theoretical concept of the formation of individual value: it includes the values transferred from the means of production; but it is only the input norms a_{ij}^k that are individual for each firm k, while the values of means of production i to be spent are determined "outside" the given consumer k, in the same way for all

consumers, as unified social values and are not an expression of the specific individual features of firm k.

The method for determining individual values allows the following conclusions to be drawn:

(a) the *national economic* nature of the indicators of total labour input w_j: these cannot be determined within sector j by averaging individual w_j^k, because the latter are not determined before w_j; the latter can be determined only from the national economic calculation using equations (2.22).

(b) the *national economic* nature of the differences between individual technologies within industry j in the total labour input coefficients: indicators w_j^k are determined only if all w_i are known.

In this latter respect, w_j^k differ fundamentally from the indicators of direct labour intensity l_j^k, which are determined simply within the industry.

The formation of individual values by means of formula (2.27) is illustrated in our conventional example by Table 2.7. The reader can, if he wishes, himself perform additional simple calculations to convince himself that statements (2.29) and (2.30) are true.

An examination of Table 2.7 allows us to return to our discussion of the issue of the character of the averages forming the social value of output. Each of our three industries has a technology that produces most of the output (firms 2, 5, 8 in Table 2.1). Now it is clear that, given the established social values, the individual value of the commodities of firm 2 is strictly equal to the social value of the corresponding commodities:

$$w_1^2 = 1.69 = w_1.$$

In this case, we have a complete concurrence between the two averages: the weighted average and the modal average, the latter corresponding to the most widespread technology (i.e., to the statistical concept of the mode as an average).

The individual value of the commodities of firm 5 turns out to be very similar to that in the best firm, 4, of industry 2 (though the latter produces a relatively small part of the output). Both firms together produce the overwhelming share of the output of the industry. That is why the average social value is, despite a rather considerable devia-

tion of the individual value in firm 6 from the typical one, very similar to this modal magnitude:

$$w_2^6 = 1.31 \approx w_2 = 1.36.$$

Industry 3 presents a similar picture, with the one difference that the individual value in the firm with the largest scale of production (firm 8) proves to be the smallest in the industry as a whole; the individual value in firm 9 and the social value of the unit commodity are rather close to it:

$$w_3^9 = 2.21 \approx w_3 = 2.33.$$

The value of labour power. So far we have dealt with the value of usual commodities. Under the conditions of overall commodity production, i.e., capitalism, as already said, labour power is also a commodity.

"The value of labour-power is determined, as in the case of every other commodity, by the labour-time necessary for the production, and consequently also the reproduction, of this special article. So far as it has value, it represents no more than a definite quantity of the average labour of society incorporated in it. Labour-power exists only as a capacity, or power of the living individual. Its production consequently pre-supposes his existence. Given the individual, the production of labour-power consists in his reproduction of himself or his maintenance. For his maintenance he requires a given quantity of the means of subsistence. Therefore the labour-time requisite for the production of labour-power reduces itself to that necessary for the production of those means of subsistence; in other words, the value of labour-power is the value of the means of subsistence necessary for the maintenance of the labourer. Labour-power, however, becomes a reality only by its exercise; it sets itself in action only by working. But thereby a definite quantity of human muscle, nerve, brain, etc., is wasted, and these require to be restored. This increased expenditure demands a larger income. If the owner of labour-power works today, tomorrow he must again be able to repeat the same process in the same conditions as regards health and strength. His means of subsistence must therefore be sufficient to maintain him in his normal state as a labouring individual. His natural wants, such as food, clothing, fuel, and housing, vary according to the climatic and other physical conditions of his country. On the other hand, the number and extent of his so-called necessary wants, as also the modes of satisfying them, are

themselves the product of historical development, and depend therefore to a great extent on the degree of civilisation of a country, more particularly on the conditions under which, and consequently on the habits and degree of comfort in which, the class of free labourers has been formed. In contradistinction therefore to the case of other commodities, there enters into the determination of the value of labour-power a historical and moral element. Nevertheless, in a given country, at a given period, the average quantity of the means of subsistence necessary for the labourer is practically known."[1] At a certain, historically given level the value of labour power involves that of the means of subsistence needed to maintain the worker's family (reproduction of working generations), including the costs of education, training, and acquiring some skill.

"The value of labour-power resolves itself into the value of a definite quantity of the means of subsistence."[2] Marx expresses this by the mathematical formula presented here, employing the symbols accepted in this book:

$$w^l = \sum_i w_i a_i^l. \tag{2.31}$$

Here a_i^l is the amount of means of subsistence of kind i necessary on average per year to reproduce a unit of simple labour power;[3] w^l is the annual value of simple labour power.

The value of each worker's labour power is far from equal, skills differences being taken into account. The differences are due to the historically established, traditionally fixed consumption standards of the categories of workers differing in their skills; behind these standards lie differences in their habits and living requirements. Even so, Marx believed that the value of the labour power of the different categories of workers, if reduced to that of simple, unskilled labour, may be accepted as equal in magnitude. Certainly, like all other economic magnitudes common to society as a whole, this only tends to be equal, with greater or smaller deviations, depending on the overall level of development of capitalism and on a lot of actual circumstances.[4] The thesis of

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[1] Karl Marx, *Capital*, Vol. I, pp. 167-68.

[2] *Ibid.*, p. 169.

[3] If a commodity i is not consumed by workers and their families, the appropriate coefficient $a_i^l = 0$.

[4] "...In the creation of surplus-value it does not in the least matter, whether the labour appropriated by the capitalist be simple unskilled

the unified value of labour power is just as true as that of the unity of social labour power, the unity of its market.

Thus, if w^l is the annual value of labour power per worker of simple labour, the annual value of labour power per worker of category λ, reckoning for his skill, is:

$$w_\lambda^l = w^l \psi_\lambda, \ \lambda = 1, \ \ldots, \ \Lambda. \qquad (2.32)$$

It would be wrong to conclude from formula (2.32) that consumption standards for workers of category λ and their families could be determined simply as those for unskilled labourers (i.e., a_i^l), multiplied by the corresponding ψ_λ for each i. Formula (2.32) states only that the value of the labour power as a whole increases proportionally to the rise in their skills, the value items being left alone. At the same time, inevitable changes are observed in the pattern of consumption, given the increase in the overall consumption level. The production and reproduction of skilled labour power demands some special expenditures on education and, more broadly, on satisfying the personal requirements of developed people, this being a necessary condition for the maintenance of the general intellectual potential serving as a basis of higher skills. The point of importance is, precisely, this concept of the reproduction of labour power as the multilateral reproduction of standard conditions necessary for its maintenance and renewal. It would be wrong to reduce the matter simply to careful calculation of expenditures on training, ignoring the notion of a historical (and moral) element in the value of labour power.

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labour of average quality or more complicated skilled labour. All labour of a higher or more complicated character than average labour is expenditure of labour-power of a more costly kind, labour-power whose production has cost more time and labour, and which therefore has a higher value, than unskilled or simple labour-power. This power being of higher value, its consumption is labour of a higher class, labour that creates in equal times proportionally higher values than unskilled labour does" (Karl Marx, *Capital*, Vol. I, pp. 191-92). "For instance, if the labour of a goldsmith is better paid than that of a day-labourer, the former's surplus-labour produces proportionately more surplus-value than the latter's. And although the equalising of wages and working-days, and thereby of the rates of surplus-value, among different spheres of production, and even among different investments of capital in the same sphere of production, is checked by all kinds of local obstacles, it is nevertheless taking place more and more with the advance of capitalist production and the subordination of all economic conditions to this mode of production" (Karl Marx, *Capital*, Vol. III, p. 142).

Replacement of relatively low-quality consumer goods by ones of relatively high quality is a process characteristic of more prosperous families. For example, the share of the consumption of the best sorts of meat, of quality clothes and furniture, etc. increases. The value of a labour power unit of kind λ can, therefore, be defined as

$$w_\lambda^l = \sum_i w_i a_{i\lambda}^l, \ \lambda = 1, \ \ldots, \ \Lambda \tag{2.33}$$

where, $a_{i\lambda}^l$ are the historically established average annual consumption standards by worker category λ and their families.[1] Magnitudes $a_{i\lambda}^l$ tend, historically, to establish themselves within limits such that (2.32) is simultaneously observed; hence:

$$\sum_i w_i a_{i\lambda}^l = w^l \Psi_\lambda, \ \lambda = 1, \ \ldots, \ \Lambda. \tag{2.34}$$

(2.34) as such expresses, of course, what is, in fact, only a tendency.

The logics of the consideration of the given issue are as follows. First, the concept of the unity of the value magnitude of labour power reproduction (reduced to simple labour power[2]) is, in effect, derived from the concept of the unity of the labour power market. Then the problem of the real consumption pattern of the workers of differing skills and their families is considered within the value of the corresponding labour power. In turn, this pattern is, in fact, established as an expression of the reproduction conditions of labour power of the given skill level and it is treated accordingly in the theory. Moreover, account should be taken of the fact that the actual level of the necessary expenditures on labour power reproduction is formed under the

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[1] Note that magnitudes w_λ^l are unified for all workers of category λ. Meanwhile, the family composition and other labour power reproduction conditions are not equal, so real consumption standards per family member are not equal either. Here the individual differences similar to those in the individual production cost of commodities make themselves felt.

[2] Or to average. If the reduction coefficients are determined, it is possible to reduce the labour expenditures of each of the skill categories to the working time of any of them. It would possibly be more convenient to employ the average magnitude in considering the value of labour power but, in order to unify our description of all reduction processes, we shall employ the reduction of the value of labour power to simple, unskilled labour power, in the same way as that of labour itself.

influence of the necessary demands made by capitalism for a change in labour specialisation, in particular it has to enable the workers to improve (and not simply maintain) their skills, if this is necessary for the development of production. The latter has, at any rate, been true of the developed capitalist nations since World War II.

Thus, the determination of value is completed for all commodities entering capitalist commodity circulation in relation to which there are sufficient grounds for speaking of the social labour input in their reproduction, i.e., of the existence of value itself.

2.6. Surplus-Value

A logical corollary of the concepts of the value of ordinary commodities and of that of labour power as a commodity is the concept of surplus-value.

Newly-created value and the value of labour power. Let us assume, as before (see paragraph 2.3), that the intensity of the labour of workers of all categories is equal. Let us also assume that their working day is equal in length (the trend towards its equalisation again results from the unity of the labour power market). The number of yearly full-time equivalent workers of category λ is then measured simply by their actual numbers on the scale of society as a whole:

$$\widetilde{T}_\lambda = \sum_{k,\, j} L^k_{j\lambda}; \; k \in E_j; \; j = 1, \; \ldots, \; n, \; \lambda = 1, \; \ldots, \; \Lambda. \qquad (2.35)$$

With regard to the reduction of labour:

$$T_\lambda = \widetilde{T}_\lambda \psi_\lambda, \; \lambda = 1, \; \ldots, \; \Lambda. \qquad (2.36)$$

T_λ is merely the sum total of the values created by the workers of category λ per year. In formula (2.36), it is equal to the number of yearly full-time equivalent workers of category λ, multiplied by their labour reduction coefficient. Remember that we take as the unit of labour (and therefore of value) the amount of time spent by a full-time equivalent worker of simple labour. This is why expression (2.36) is so simple.

The ratio of value created by a worker of simple labour annually to that of his labour power is $1/w^l_k$. Let us show that, given our assumptions, the ratio of these magnitudes

is the same for any category of worker λ:

$$\frac{T_\lambda}{w_\lambda^l \widetilde{T}_\lambda} = \frac{\widetilde{T}_\lambda \psi_\lambda}{w^l \psi_\lambda \widetilde{T}_\lambda} = \frac{1}{w^l}, \quad \lambda = 1, \ldots, \Lambda. \qquad (2.37)^1$$

The magnitudes of the value newly created by the worker and that of his labour power do not coincide: the former is the quantity of his labour (as reduced to simple one) put in production; the latter is the quantity of labour embodied in his means of consumption. The quantity of labour is determined by the working time and skill of the workers, whereas that of the goods to be consumed—by the traditions of a given nation, and the value of these goods by social labour productivity. Of course, the workers' skills depend on their overall consumption level, especially specific expenditures on acquiring and maintaining these skills. In any event, however, given this level of consumption and of skills, the worker may spend more or less time on production. The value created by him does not, therefore, simply depend on the value of labour power.

Necessary product and necessary labour. Workers' labour inputs cannot be less than the value of their labour power, so this value constitutes the lower limit of their working time.

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[1] The fact that ratio $T_\lambda / w_\lambda^l \widetilde{T}_\lambda$ is the same for all categories of workers underlies Marx's statement, quoted above, that the norms of their exploitation are unified. We shall return to this question below.

It may be stressed immediately, however, that strict invariance of the ratio discussed in (2.37) for all kinds of labour power is not obligatory. There are grounds for assuming that more skilled labour distinguishes itself with a higher ratio $T_\lambda / w_\lambda^l \widetilde{T}_\lambda$. Indeed, in order to acquire and maintain a certain skill it is, of course, necessary to spend more on all kinds of consumption, but certainly not in equal proportion, the greatest share of additional expenditure being spent on education and other cultural requirements that form, in fact, a relatively small component of the value of labour power as a whole. If expenditures on food, clothing, housing, and other essentials are assumed to rise more slowly than workers' skills, taking into account the considerable share of these items in the total value of labour power it would be justifiable to conclude that the labour power reproduction value increase due to the increase in the skill is somewhat slower than that of the value created by labour power. Such a lag should, however, be counteracted by the trend towards equalisation resulting from the unity of the labour power market. That is why the lag can hardly be great for mass categories of skilled workers. Some actual data for evaluating the importance of these opposite processes under current conditions will be found in Chapter 4.

The total value of all labour power is:

$$W^l = \sum_{\lambda} w_{\lambda}^{l} \widetilde{T}_{\lambda} = \sum_{i,\lambda} w_i a_{i\lambda}^{l} \widetilde{T}_{\lambda} = \sum_{i} w_i Y_i^l, \qquad (2.38)$$

where Y_i^l is the quantity of goods of kind i to be consumed unproductively by the working class, or:

$$Y_i^l = \sum_{\lambda} a_{i\lambda}^{l} \widetilde{T}_{\lambda}, \quad \lambda = 1, \ldots, \Lambda; \; i = 1, \ldots, n. \qquad (2.39)$$

By vector $Y^l = (Y_1^l, \ldots Y_n^l)$ we shall denote the total quantity of goods acquired by the workers for consumption by them and their families.

The amount of output Y^l constitutes *the necessary product* created by the workers, and the labour embodied in it wY^l — *the necessary labour* of the workers. Given a corresponding modification affected by the division of labour, this applies to every worker. "...The labourer, during one portion of the labour-process, produces only the value of his labour-power, that is, the value of his means of subsistence. Now since his work forms part of a system, based on the social division of labour, he does not directly produce the actual necessaries which he himself consumes; he produces instead a particular commodity, yarn for example, whose value is equal to the value of those necessaries or of the money with which they can be bought. The portion of his day's labour devoted to this purpose, will be greater or less, in proportion to the value of the necessaries that he daily requires on an average, or, what amounts to the same thing, in proportion to the labour-time required on an average to produce them. If the value of those necessaries represent on an average the expenditure of six hours' labour, the workman must on an average work for six hours to produce that value. If instead of working for the capitalist, he worked independently on his own account, he would, other things being equal, still be obliged to labour for the same number of hours, in order to produce the value of his labour-power, and thereby to gain the means of subsistence necessary for his conservation or continued reproduction. But as we have seen, during that portion of his day's labour in which he produces the value of his labour-power, say three shillings, he produces only an equivalent for the value of his labour-power already advan-

ced by the capitalist*; the new value created only replaces the variable capital advanced. It is owing to this fact, that the production of the new value of three shillings takes the semblance of a mere reproduction. That portion of the working-day, then, during which this reproduction takes place, I call '*necessary*' labour-time, and the labour expended during that time I call '*necessary*' labour.** Necessary, as regards the labourer, because independent of the particular social form of his labour; necessary, as regards capital, and the world of capitalists, because on the continued existence of the labourer depends their existence also."[1]

The consumption of the working class is, at any rate, limited by the net product:[2]

$$Y_i^l \leqslant Y_i, \quad i=1, \ldots, n. \tag{2.40}$$

Therefore (see 2.24)

$$W^l = \sum_i w_i Y_i^l \leqslant \sum_i w_i Y_i = L$$

is valid.

Note that $w^l = W^l/L$. From $L \geqslant W^l$ it follows that:

$$1 \geqslant w^l; \ 1/w^l \geqslant 1. \tag{2.41}$$

The value of labour power does not, in any case, exceed that created by this labour power. This applies to all categories of employee:

$$w_\lambda^l \leqslant T_\lambda/\tilde{T}_\lambda, \ \lambda = 1, \ \ldots, \ \Lambda. \tag{2.42}$$

* [Note added in the 3rd German edition.—The author resorts here to the economic language in current use. It will be remembered that on p. 182 (present edition, 174) it was shown that in reality the labourer "advances" to the capitalist and not the capitalist to the labourer.—*Engels*].

** [In this work, we have, up to now, employed the term "necessary labour-time", to designate the time necessary under given social conditions for the production of any commodity. Henceforward we use it to designate also the time necessary for the production of the particular commodity labour-power. The use of one and the same technical term in different senses is inconvenient, but in no science can it be altogether avoided. Compare, for instance, the higher with the lower branches of mathematics] (Note by Marx.—*Author*).

[1] Karl Marx, *Capital*, Vol. I, p. 208.

[2] In fact, it is certainly only a part of it. See below. Note also that up to Chapter 5 the term *consumption* is applied to goods currently received only in order to simplify the terminology. In reality, it also applies to goods accumulated previously.

Surplus-product and surplus-value. Actually, under capitalism,

$$w^l < 1, \ w_\lambda^l < T_\lambda / \tilde{T}_\lambda, \ \lambda = 1, \ \ldots, \ \Lambda. \qquad (2.43)$$

In other words, the working class works more time than is necessary for its own consumption. This is a necessary condition for the existence of capitalism as a social form of production.

The capitalist class appropriates part of the net output of society, using it, first, for its personal consumption and that of its servants, second, for accumulation of capital (expansion of means of production, the hiring of additional labour power), and, third, for maintenance of the exploitative state and covering its expenses.[1] This is an observable fact and is possible only because the net output in which the national income is embodied exceeds the necessary product:

$$Y > Y^l.$$

The theory calls this difference the surplus-product. We shall denote it by Y^s. It is defined as the difference between the two vectors:

$$Y^s = Y - Y^l; \ Y^s = (Y_1^s, \ \ldots, \ Y_n^s). \qquad (2.44)$$

Thus

$$Y = Y^l + Y^s. \qquad (2.45)$$

$Y^s \geqslant 0$ is necessary, but not sufficient, for the existence of capitalism.

Generally speaking, the division of Y into Y^l and Y^s, where both vectors contain positive components, existed prior to capitalism (beginning with the period of the decay of primitive production forms) and remains after capitalism has been replaced by socialism.[2] The existence of Y^s in itself not only does not imply capitalism, it does not imply exploitation either. If social relations are based on public ownership of the means of production, the surplus-product, though not to be used for the personal consumption of direct producers, remains their collective property. In this case, there is no

[1] As long as the theory deals with the economy as such, this third use of the net output surplus is disregarded.

[2] "Surplus-labour in general, as labour performed over and above the given requirements, must always remain. In the capitalist as well as in the slave system, etc., it merely assumes an antagonistic form..." (Karl Marx, *Capital*, Vol. III, p. 819).

exploitation. If, however, direct producers receive only the necessary product while the surplus-product is appropriated by the class of owners of the means of production, there are relations of *exploitation*. Exploitation is *capitalist* if direct producers are deprived of ownership of the means of production and form a class of wage workers and the owners of the means of production acquire those means of production together with labour power for money which, in this case, is *capital*.

By multiplying both terms of equation (2.45) by the vector of total labour inputs (and with regard to 2.24), we obtain equation:

$$wY = wY^l + wY^s = L,$$

or, the same thing:

$$L^l + L^s = L. \qquad (2.46)$$

It is characteristic that $wY^l \equiv L^l$ is a mathematical representation of the theoretical concept of *necessary labour* on the scale of the economy as a whole, and $wY^s \equiv L^s$ is that of the concept of *surplus-labour*. The sum total of necessary and surplus-labour is equal to the total expended labour L.

Since we are dealing with the capitalist economy which transforms all products into commodities, the labour embodied in them constitutes their total value. Accordingly, under capitalism surplus-labour creates *surplus-value*. On the scale of the whole economy, surplus-value is the total value of the surplus-product.[1] It is equal to wY^s.

Surplus-value is thus called since it is created by workers in excess of the value equivalent to that of their labour power. It is precisely this part of the newly created value that is the direct objective of production under capitalism.

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[1] "This surplus-labour appears as surplus-value, and this surplus-value exists as a surplus-product" (Karl Marx, *Capital*, Vol. III, p. 819).

The value of every commodity consists, of course, in that transferred to it from the means of production and that created by living labour in producing the commodity, whereas the surplus-value is only a part of the latter. On the scale of the closed economy, however, one may distinguish a share of commodities (the replacement fund equal to $Q - Y$) the total value of which is equal to that transferred from all the means of production expended; some part of commodities (necessary product Y^l) the total values of which is equal to that created by the necessary labour of all workers; finally, part of commodities (surplus-product Y^s), the total value of which is equal to the surplus-value created by all workers.

"During the second period of the labour-process, that in which his labour is no longer necessary labour, the workman, it is true, labours, expends labour-power; but his labour, being no longer necessary labour, he creates no value for himself. He creates surplus-value which, for the capitalist, has all the charms of a creation out of nothing. This portion of the working-day, I name surplus labour-time, and to the labour expended during that time, I give the name of surplus-labour. It is every bit as important, for a correct understanding of surplus-value, to conceive it as a mere congelation of surplus labour-time, as nothing but materialised surplus-labour, as it is, for a proper comprehension of value, to conceive it as a mere congelation of so many hours of labour, as nothing but materialised labour. The essential difference between the various economic forms of society, between, for instance, a society based on slave-labour, and one based on wage-labour, lies only in the mode in which this surplus-labour is in each case extracted from the actual producer, the labourer."[1]

A necessary precondition for the existence of any form of surplus-labour is a certain development level of labour productivity, such that the workers could spend less time than their working day on creating their necessary means of subsistence. Such productivity is a result of a long historical process and the technical progress. Capital, while carrying through technical progress, develops this precondition further. Yet it certainly does not follow from this that surplus-value is generated by technologies as such, by the totality of their objective and subjective factors. It is apt at this point to recall the concept of technologies set out in Chapter 1. The means of production do not themselves yield any output; the constituent element of production is labour power. There exist no *technological* reasons that would make workers work more time than is necessary to reproduce their means of subsistence. But this reproduction is a prime social requirement, so the time spent on it is *prime* time, a necessary part of total working time. Only labour *in excess* of this produces surplus-product, accordingly forming surplus-value under capitalism.

Constant and variable capital. As shown above, thanks to the specific useful character of workers' labour, the value of

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[1] Karl Marx, *Capital*, Vol. I, p. 209.

the means of production is conserved as a component of that of the commodities produced. As the input of working time, this labour creates the new value added to that embodied in the means of production, since, in the process of the productive employment of the means of production, the value embodied in them remains quantitatively constant (only the physical form in which it is embodied changes). Marx called this part of the value of the capital advanced *constant* capital.[1] At the same time, the capital advanced to acquire labour power is *variable.*

No *transfer* of the value of labour power to the commodity may take place. Its value is that of the consumer goods for the workers and their families, and these goods are not involved in the technological process of creating a new commodity.[2] By their labour workers create a new value, first of all that equivalent to the value of their consumer goods (accordingly, to the capital advanced by the capitalists to purchase their labour power). Yet they work longer than is necessary to create such an equivalent, which is why the new value created by them exceeds the corresponding capital value. This all provides grounds for considering the latter to be variable. This concept expresses the variable nature of labour time itself, which is spent again and again by the working people in the process of labour and which, at the same time, disappears together with the useful form of the finished products of this labour—the goods of nonproductive consumption.

Rate of surplus-value. Since surplus-value appears as the increase of variable capital in excess of its equivalent, i.e., the value created by surplus working time, its level is determined as a ratio to the variable capital generating it. Usually, the following percentage is employed:

$$m' = \frac{m}{v} \cdot 100 \qquad \text{(as Marx denoted it)}$$

[1] The value of constant capital does not change during its productive use, but like any other component of value it changes during the production of corresponding commodities. If, while certain means of production are employed, the value of their reproduction changes, it is not the previous magnitude of their value that is transferred to the commodity, but rather the value of their reproduction (see paragraph 2.5), though this is transferred in a constant amount.

[2] In the process of consumption by workers and their families, the value of commodities is ultimately lost, together with the useful properties of the consumer goods.

Table 2.8

Calculation of the Rate of Surplus-Value

Kind of output i	Value of unit w_i	Net output: in physical terms Y_i	Net output: in value terms w_iY_i	Necessary product: in physical terms Y_i^l	Necessary product: in value terms wY_i^l	Surplus-product: in physical terms Y_i^s	Surplus-product: in value terms $w_iY_i^s$
1	1.69	88	149	—	—	88	149
2	1.36	162	220	136	185	26	35
3	2.33	82	191	41	95	41	96
Total	×	×	560	×	280	×	280

$$m' = \frac{280}{280} \cdot 100 = 100\%.$$

or, the same thing:

$$m' = \frac{L - w^l L}{w^l L} \cdot 100 = \frac{wY^s}{wY^l} \cdot 100. \qquad (2.47)$$

Marx called this the rate of surplus-value. It can easily be seen that

$$m' = \frac{L^s}{L^l} \cdot 100. \qquad (2.48)$$

The magnitudes of the necessary and surplus-product, and surplus-value and its rate are calculated on the basis of our conventional example (see Table 2.8). Here the following consumption rates are accepted:

$$a_1^l = 0.0$$

$$a_2^l = 0.243$$

$$a_3^l = 0.073$$

(units of the corresponding product per unit labour power reduced to simple). In our example, the rate of surplus-value is thus 100 per cent.

Chapter 3

FORMS OF MANIFESTATION OF VALUE AND SURPLUS-VALUE

The concepts of value and surplus-value express the essence of the capitalist economy (the former—of any commodity production in general). On the surface of the phenomena neither value nor surplus-value are observed directly, but they determine phenomena so as the latter appear on the surface, i.e., they represent the *laws* governing these phenomena.

The inherent laws of the objective world are generally fairly simple. This also applies to those of the economy. It proves, however, difficult to find out that, behind the various and very complex phenomena, the simple laws that most affect them are to be found.

We began considering the capitalist economy with typical features as they appear superficially. Then we exposed the concepts of value and surplus-value. To combine these two aspects of the discussion of capitalism is to reveal the characteristic properties of the phenomena actually to be determined by the value of commodities and surplus-value; the external forms of capitalism are thus seen as the forms in which its inherent laws operate. These forms can thus be explained by means of the laws, whereas the latter have to be proved and the mechanism by which they operate understood.

3.1. Value as the Law of the Prices of Reproducible Commodities

The law of expenditure on the reproduction of commodities. The social value of ordinary commodities and that of the specific commodity—labour power [see formulae (2.22), (2.31)-(2.34)] appears directly as a generalisation of individual labour input in the production of these commodities, as the average level of input. As the commodities of each kind are

themselves depersonalised in relation to their producers, like a single mass, the social value of their units is related to each such unit. Society sees each unit commodity as an average result of social labour as a whole, rather than a result of the individual labour of its producer.

In other words, each unit of commodity of a given kind has a social value which is an *invariant*, at least in the following three respects:

(a) as regards the individual input of labour in the production of commodities produced simultaneously;

(b) as regards the average labour input in the production of commodities produced at different times, but offered on the market simultaneously;[1]

(c) as regards the specific commercial transaction of sale and purchase by which the commodity is handed over by the producer to the buyer.

Real invariants (constants) are always seen in science as the *laws* governing corresponding phenomena. Social *value* is primarily the law (law-governed level) of labour input in the production of commodities, individual labour input being in this case random variations.

A fundamental property of magnitudes w_j, absolutely essential for understanding them correctly as magnitudes of social value (socially necessary, law-governed labour input in the production of commodities), is that they are based on *mathematical expectations* of the technological coefficients of material and labour inputs. These are coefficients a_{ij} and l_j. In fact, according to (2.8)

$$a_{ij}=\frac{A_{ij}}{Q_j};\ i,\ j=1,\ \ldots,\ n,$$

but $A_{ij}=\sum_{k\in E_j} A_{ij}^k$, where $A_{ij}^k=a_{ij}^k Q_j^k$

[see formulae (2.4) and (2.10)]. Then

$$Q_j=\sum_{k\in E_j} Q_j^k. \qquad \text{[see (2.1)]}.$$

[1] Remember that, according to formula (2.22), the process of averaging the input forming the value of commodities involves only that for their reproduction. If the commodities offered on the market result from previous production, they are considered as products of the current year.

It follows:

$$a_{ij} = \frac{\sum_h A_{ij}^h}{\sum_h Q_j^h} = \frac{\sum_h a_{ij}^h Q_j^h}{\sum_h Q_j^h}; \ k \in E_j, \ i, \ j = 1, \ \ldots, \ n.$$

Similarly, with regard to formulae (2.5) and (2.11):

$$l_j = \frac{L_j}{Q_j} = \frac{\sum_h l_j^k Q_j^k}{\sum_h Q_j^h}; \ k \in E_j, \ j = 1, \ \ldots, \ n.$$

Thus, according to the way w_j are formed, they are seen to be the mathematical expectations of the total labour input in the production of commodities. This is also expressed in formula (2.30), where w_j appear as the mathematical expectations of magnitudes w_j^k. Weights Q_j^h are represented in all three formulae as probabilities, i.e., the frequencies with which specific technological coefficients a_{ij}^k and l_j^k, and individual values w_j^h actually occur. Note that these are frequencies derived from observations covering the entire totality.

The mathematical expectation of some varying (randomly) magnitude is usually considered in science as its *law-governed* level, in contrast to its specific values, which are random. In our case, this general scientific approach is at once substantially corroborated: interindustry commodity flows, the replacement fund, and net output can be correctly determined on the basis only of average a_{ij} for the economy as a whole (see above), the interindustry distribution of living labour—on the basis of l_j, and so forth. If any system of specific technological coefficients a_{ij}^k and l_j^k is used, instead of a_{ij} and l_j (so that, from each set E_j, a single technology k is selected), in the general case the balances corresponding to the actual state of affairs, i.e., actual interindustry flows, net output (if the whole real commodity output was taken as vector Q), etc., would be missed. Concurrence with the real input-output table can occur by chance, but is unlikely. The point is, however, that, on the basis of mathematical expectations (coefficients a_{ij} and l_j), the adequate I-O table and balance of labour occur regularly, owing to the very mode by which these coefficients are formed.

The *law-governed* level of total labour input in the production of commodities and socially *necessary* total labour

input are, in economic theory, equivalent concepts, i.e, have the same content. The actual problem of determining the socially necessary input is that of determining the law-governed level of total labour input in production. If it has been proved that magnitudes w_j express the law-governed level of input, it has also been proved that, under the conditions of commodity production, they are the social values of the commodities.

Individual inputs (a_{ij}^k, l_j^k, w_j^k), taken as such, without averaging, are of economic importance only for the individual producer. In his activities (but only within their limits), they perhaps express even a certain law of the technological method of production (if a_{ij}^k, l_j^k really corresponds to the minimum input characteristic of technology k). On the market, however, all commodities of a given kind are seen as an aggregated, totalled supply. That is why individual inputs are of no importance for the buyer of the commodity, nor, therefore, for society as a whole, as the commodity's consumer. It is average coefficients a_{ij}, l_j, and w_j summarised on their basis that are *society's* input in it. This means that w_j is the law of the magnitude of input, whereas w_j^k are fluctuations around the law.]

Prices and the reproduction of conditions of production. The theory of value maintains, at the same time, that the above w_j form the law of proportions of commodity exchange, i.e., the *law of prices* for all commodities to which, in general, the concept of value relates. Accordingly, the price is seen as an expression of value in monetary terms.

The relation of price and value cannot be observed directly. It is discovered through scientific analysis. Social value is, in general, not a familiar concept to the private producer: in his firm, he deals with the individual input level, on the market with the system of average prices and fluctuations around them, whereas the formation of social value, i.e., the averaging of productive input, is completely beyond the control of any individual producer. It happens without his knowledge, as a spontaneous resultant force of the labour process in society as a whole.

"What, first of all, practically concerns producers when they make an exchange, is the question, how much of some other product they get for their own? In what proportions the products are exchangeable? When these proportions have, by custom, attained a certain stability, they appear

to result from the nature of the products, so that, for instance, one ton of iron and two ounces of gold appear as naturally to be of equal value as a pound of gold and a pound of iron in spite of their different physical and chemical qualities appear to be of equal weight. The character of having value, when once impressed upon products, obtains fixity only by reason of their acting and re-acting upon each other as quantities of value. These quantities vary continually, independently of the will, foresight and action of the producers. To them, their own social action takes the form of the action of objects, which rule the producers instead of being ruled by them. It requires a fully developed production of commodities before, from accumulated experience alone, the scientific conviction springs up, that all the different kinds of private labour, which are carried on independently of each other, and yet as spontaneously developed branches of the social division of labour, are continually being reduced to the quantitative proportions in which society requires them. And why? Because, in the midst of all the accidental and ever fluctuating exchange-relations between the products, the labour-time socially necessary for their production forcibly asserts itself like an over-riding law of Nature. The law of gravity thus asserts itself when a house falls about our ears.* The determination of the magnitude of value by labour-time is therefore a secret, hidden under the apparent fluctuations in the relative values of commodities. Its discovery, while removing all appearance of mere accidentality from the determination of the magnitude of the values of products, yet in no way alters the mode in which that determination takes place."[1]

The assertion that value is the law of prices is directly seen to be a hypothesis, a conjecture that can be tested subsequently by practice, i.e., checking of the expectations it originates of the properties of observed facts. Yet this is a hypothesis that can benefit from the substantiations stated at the very beginning and resulting from the consideration of the properties and functions of the commodity exchange

* "What are we to think of a law that asserts itself only by periodical revolutions? It is just nothing but a law of Nature, founded on the want of knowledge of those whose action is the subject of it" (Friedrich Engels, "Umrisse zu einer Kritik der National-Ökonomie", in *Deutsch-Franzosische Jahrbücher*, edited by Arnold Ruge and Karl Marx. Paris, 1844). [Note by Marx *Author.*]

[1] Karl Marx, *Capital*, Vol. I, pp. 79-80.

process. In the scientific consideration of these properties and functions, in fact no reasonable alternative remains to the assumption that exchange proportions are determined, on average, by the value of commodities.[1]

The equalisation of heterogeneous use-values to one another in exchange is a fact, moreover, one observed billions of times. "...We also equate, as human labour, the different kinds of labour expended upon them. We are not aware of this, nevertheless we do it."[2]

Surely not only products of labour but also such things as honour, conscience, posts on elective organs, etc. are, of course, bought and sold as well as objects the ownership of which is realised in the given social relations in specific revenues (rent). The prices of the latter cannot be derived from the law of value.[3] Value is considered as the law of prices strictly only in relation to reproducible commodities, moreover, such that their reproduction in a large number of pieces really corresponds to the social demand.[4] Arguments

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[1] The class interests of the bourgeoisie at a certain stage in the evolution of capitalism make one disregard such a consideration and propound conceptions that are wide open to scientific criticism as conflicting with the theory of value. The power of survival and the wide spread of such conceptions in the literature only testifies to the fact that purely scientific arguments are not enough to overcome them: it is necessary to abolish the class interested in maintaining social ignorance.

[2] Karl Marx, *Capital*, Vol. I, pp. 78-79.

[3] "The price-form, however, is not only compatible with the possibility of a quantitative incongruity between magnitude of value and price, *i.e.*, between the former and its expression in money, but it may also conceal a qualitative inconsistency, so much so, that, although money is nothing but the value-form of commodities, price ceases altogether to express value. Objects that in themselves are no commodities, such as conscience, honour, etc., are capable of being offered for sale by their holders, and of thus acquiring, through their price, the form of commodities. Hence an object may have a price without having value. The price in that case is imaginary, like certain quantities in mathematics. On the other hand, the imaginary price-form may sometimes conceal either a direct or indirect real value-relation; for instance, the price of uncultivated land, which is without value, because no human labour has been incorporated in it" (Karl Marx, *Capital*, Vol. I, pp. 104-105).

[4] Let us note that *unique* products of labour (for example, works of art, scientific and technological ideas, etc.) to which the relations of ownership may be extended also become commodities, are specifically priced as well, the price not being regulated by labour value. This does not apply to their production in large amounts (for example, book printing is subject to the law of value, as opposed to the copyright for the work to be published).

to substantiate the assertion that their prices are regulated by value do not come down to indication of the fact that their sole common property of economic importance is that they are products of labour.

Reproducible commodities are products of labour that can meet human requirements (both productive and personal) merely because they are produced in large numbers, since a single piece is not enough to meet the needs of society. After being used, they in fact lose their useful qualities, so need to be reproduced in ever greater quantities[1] (not necessarily in the same form, but in order to meet the demand to which they previously corresponded). Accordingly, the industries making the corresponding products for personal and productive use should be constantly renewed. In other words, the relevant technological systems should be renewed (possibly with variants), i.e., expended reproducible means of production and labour power should be replaced. As already shown, under commodity production such replacement (the universal social exchange in general) is carried out as the process of equivalent exchange. *It is the proportions of such exchange that obey the necessity of reproducing technological systems*, i.e., of reproducing the system of the social division of labour. Without systematic reproduction, the system simply cannot exist. Yet its reproduction is nothing but the replacement, in each industry, of the expenditure on production (in exchange for its products) and in sufficient amounts to continue production, i.e., to renew the *conditions of production*.

Since the social division of labour is, in fact, reproduced, it is the real exchange of products that performs the function in question, and the proportions of exchange correspond to it. In this sense, the theory of value is a *generalisation of the observed facts*.

When commodity exchange is carried out by means of money (commodity production), the receipts of the industries for their commodities (the sum total of their prices) form the source of assets for buying the commodities needed to replace expended means of production and labour power. Here lies the reproductive function of price as its primary and main function. The content of this function is expressed by the

[1] This distinguishes them from ideas and works of art that either do not lose their qualities in use or, if they do, cannot be replaced by producing new pieces of them.

requirement of the *profitability* of all industries:[1]

$$p_j \geqslant \sum_i p_i a_{ij} + l_j \sum_i p_i a^l_{ij}, \quad i, j = 1, \ldots, n \tag{3.1}$$

where

$$a^l_{ij} = \frac{\sum\limits_{\lambda=1}^{\Lambda} a^l_{i\lambda} L_{j\lambda}}{L_j}, \tag{3.2}$$

i.e., the average consumption standards of the workers and their families in industry j, per unit of simple labour power (with regard to the ratio between the skill groups),[2] $L_{j\lambda}$ being the sum total (by firms):

$$L_{j\lambda} = \sum_{h \in E_j} L^h_{j\lambda}. \tag{3.3}$$

The sense of formula (3.1) is: the prices of the products of all industries should be such that the receipts should be at least enough for purchasing, on the market, the means of production and labour power in sufficient quantities, on average, for renewing production in each industry. In this case, the price of annual labour power is, in industry j:

$$p^l_j = \sum_i p_i a^l_{ij}, \quad i, j = 1, \ \ldots, n. \tag{3.4}$$

Formula (3.4) represents the price of labour power on the basis of one annual full-time worker of simple labour[3]. This is

[1] Profitability is defined here as economic activity without losses; strictly positive profitability is a particular case.

[2] Let us note one conclusion from formula (3.1), which is not of importance to our exposition but is sociologically interesting. Standards a^l_{ji} are seen as the average consumption standards of real categories of workers reduced to simple labour. As weights $L_{j\lambda}$ are, on average, not equal in different j, such averaged standards, even if calculated on the basis of simple labour, are *not equal in different industries*. Here, apparently, is the basis of the existence of the *consumption stereotypes of industries*. This is one of the factors behind the formation of social groups within the working class by industry, those with special interests; it is well known that the conscious overcoming of alienation of the workers of different industries is one of the most important problems in the development of their class struggle.

[3] On the basis of a worker with average skills in industry j the price would be:

$$\tilde{p}^l_j = \frac{\sum\limits_{i,\lambda} p_i a^l_{i\lambda} L_{j\lambda}}{\tilde{L}_j}. \tag{3.5}$$

why its multiplication by l_j, in formula (3.1), is justified: l_j is also the coefficient of the input of labour, reduced to simple labour.

Let us note that, if total wages in industry j are equal to $p_j^l L_j$, it follows from (3.2) and (3.4) that

$$p_j^l L_j = \sum_i p_i a_{ij}^l L_j = \sum_{\lambda=1}^{\Lambda} p_i a_{i\lambda}^l L_{j\lambda}; \quad i, \ j = 1, \ \ldots, \ n. \quad (3.6)$$

The total wage $p_j^l L_j$ is thus enough for each category of workers to buy consumer goods according to standards $a_{i\lambda}^l$, i.e., to reproduce labour power.[1]

Keeping to condition (3.1) is enough to renew (replace) the expended conditions of production in every industry j as a whole and thereby create, in general, the necessary prerequisites for renewing production in each whole industry at least on the previous scale.[2] This follows from the way in which averages a_{ij} and a_{ij}^l are formed [see formulae (2.8), (3.2)].

Expenditure on reproducing the conditions of production are shown in formula (3.1) to be those of one and the same resource, represented by money. Expenditure of the conditions of production themselves on the production of commodities, as they are shown in (2.22), are again those of one and the same resource, represented by social labour (see paragraph 2.5). At first sight we seem to be dealing here with merely superficial similarity. In reality, it is based on the fundamental fact that money is nothing but value that begins to act as an autonomous social factor, and acquires a special, corresponding form.

The first requirement of the law of value. The producer of a commodity is compensated for its value on the market by receiving the value in the *form of other commodities*. When exchange is said to be performed strictly according to value, this means that, for each producer, the social value of the commodities received is, in the final stage of the formula $C—M—C$, equal to the social value of the commodities

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In calculating p_j^l and $\tilde{p}_j^l$, the numerator is the same, the difference between these magnitudes being due to the difference in the denominator in calculating them.

[1] We shall not discuss here the problem of the price of labour power of each category λ.

[2] The question inevitably arises here as to whether it would be enough to renew production in the specific firms that make up the industry. We shall deal with this question below.

he initially offered.[1] Exchange can proceed with deviations from such strict equivalence; moreover, the total number of producers can receive on the market less value than they put into it (if some of the commodities have been acquired by nonproductive forces of society). Yet it is always true that, if all industries are profitable (in the precise, i.e., monetary, sense), that is, (3.1) is observed, it directly follows that:

$$w_j^R \geqslant \sum_i w_i a_{ij} + w^l l_j, \quad j = 1, \ \ldots, \ n, \tag{3.7}$$

or, if the right-hand term is given in Marx's symbols:

$$w_j^R \geqslant c_j + v_j.$$

Here, w_j^R is the average volume of the commodity j value realised on the market. In effect, if (3.1) is fulfilled, the producers receive, together with commodities obtained from the market according to standards a_{ij} and a_{ij}^l, their value, i.e,

$$\sum_i w_i a_{ij} \equiv c_j \text{ and } \sum_i w_i a_{ij}^l \equiv v_j.$$

The above implies that the necessary commodities i can, in fact, be bought. Up to now, however, we have had a situation where supply covers demand.

In turn, formula (3.7) is merely an expression of the first and main requirement of the law of value: the proportions of the exchange of commodities should be such that the conditions of production might be reproduced in each industry. This requirement expresses the nature of value as the magnitude of social labour input in the reproduction of commodities.

Any system of prices corresponding to formula (3.1) realises *thereby* the requirement of *the law of value* (3.7). Profitability of industries is in fact the replacement of their labour expenditure on reproducing their conditions of production (with a surplus which is at least non-negative). This indisputable fact is what is primarily meant when it is said that the price is the monetary expression of value and that value is the law of prices. The mechanism of prices is considered directly as the way the law of value is realised.

[1] If this is observed, it does not matter whether or not the value of M was equal to that of the initial and final C; generally speaking, the real mediator can have no value at all.

In other words, anybody who acknowledges that the necessity of the profitability of all industries is the law of the system of prices, thus acknowledges that prices fulfil the requirement of the law of value and are the outward, market form of realisation of this law. One can only repeat that the theory of value appears here as a direct conclusion drawn from observed facts, so that no scientific alternative exist to it.

The following objection is possible: there are sufficient grounds for speaking of the existence of (accordingly, of the necessity of replacing) total input not only of social labour, but also of all the various reproducible means of production on the production of commodities. This total input is, in every case, the sum total of direct and indirect input (for example, direct and indirect expenditure of electricity, metal, wood, etc. on the production of each kind of commodity). Even if the coefficient for the direct input of some means of production i in sector j is equal to zero ($a_{ij} = 0$), given a sufficiently developed division of labour, the overwhelming majority of kinds of means of production, if not all of them, prove to be spent on each commodity kind [all or almost all the coefficients b_{ij} as elements of matrix $(I - A)^{-1}$ are positive for the kinds of output i that are means of production]. Why, then, are just w_j selected from all of the coefficients of the total input to be considered as the law of value?

This objection has, in fact, already been rejected. Coefficients of total input of labour w_j distinguish themselves from the whole system of b_{ij} by the fact that, in a generalised form, they represent all kinds of input, both of means of production and of living labour (see paragraph 2.5). Meanwhile, any coefficient b_{ij} merely expresses that of means of production i.[1] For their producers, however; the prices of commodities are the resource for buying not only means of production, but also labour power: the component of the price of the produced commodities that is received by the workers in the form of wages is, for them, the resource for buying consumer goods. The conclusion should be drawn that no coefficient b_{ij} can, according to the sense, be the law of prices.

Formula (3.7), as can easily be seen, is not, of course, identical with (2.22). If the requirement of the profitability

[1] This is clear from the way b_{ij} are determined as elements of matrix $(I - A)^{-1}$.

of all industries is fulfilled [the prices actually correspond to condition (3.1)], each industry thereby realises on the market at least the part of the value of its commodities that is equal to the value of the reproduction of its conditions of production. The total value received by industry j from the market in the form of other commodities may, however, turn out to be either greater or smaller than that of its own commodities. Mathematically, this is represented as follows:

$$w_j^R \lesseqgtr w_j, \quad j=1, \ \ldots, \ n \tag{3.8}$$

the deviations in both directions being dependent on circumstances.

A direct correspondence of prices to value would mean the following:

$$\frac{p_j}{w_j} = const \text{ for all } j=1, \ \ldots, \ n; \tag{3.9}$$

or:

$$p_j = hw_j, \quad h > 0, \tag{3.10}$$

where, under gold money circulation,

$$h = 1/w_{gold}. \tag{3.11}$$

If prices are direct proportions of the change of commodities for fiat money, h is simply a certain invariant (a coefficient of proportionality) depending on the mass of paper money in circulation. It is equally clear that, if (3.10) is fulfilled, (3.1) is implicitly fulfilled too.[1] Therefore the *precise fulfilment of the law of value* (strict proportionality of prices

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[1] From (2.22) it follows that:

$$hw_j = h\left(\sum_i w_i a_{ij} + l_j\right)$$

where $l_j \geqslant w^l l_j$ (see 2.41); in turn, it is true that:

$$w^l = \sum_i w_i a_{ij}^l.$$

Hence:

$$hw_j \geqslant h\left(\sum_i w_i a_{ij} + l_j \sum_i w_i a_{ij}^l\right);$$

$$p_j \geqslant \sum_i p_i a_{ij} + l_j \sum_i p_i a_{ij}^l.$$

This is precisely what was to be demonstrated.

to the value of commodities) *is enough to make all industries profitable.*

Conditions of direct operation of the law of value. When (2.41) is fulfilled in the form of a strict equality ($1/w^l = 1$), the precise fulfilment of the law of value is not only sufficient for reproducing the conditions of production normally, but is also *necessary* for this.

In this case $w^l l_j = l_j \sum_i w_i a^l_{ij} = l_j$, therefore

$$hw_j = h\left(\sum_i w_i a_{ij} + l_j \sum_i w_i a^l_{ij}\right), \quad i, \ j = 1, \ \dots \ n. \quad (3.12)$$

Let, for some commodity j' (i'), the price $p_{j'} \neq hw_{j'}$, for example, $p_{j'} > hw_{j'}$. Now let sector j' obtain profit:

$$p_{j'} > \sum_i p_i a_{ij'} + l_{j'} \sum_i p_i a^l_{ij'}.$$

In the economy in question, however, there are no commodities that are not taken into account in calculating by means of formula (3.1). In fact, the full volume of commodity output $Q = H + Y$, where $H = AQ$, and net output Y in the economy under consideration is equal to the consumption fund of productive workers ($Y_i = Y^l_i = \sum_j a^l_{ij} L_j$ for all i). Hence it follows that if $p_{j'} > hw_{j'}$, at least one industry j'' can be found, such that

$$p_{j''} < \sum_i p_i a_{ij''} + l_{j''} \sum_i p_i a^l_{ij''}.$$

The existence of credit and the possibility of mobilising money accumulated earlier being excluded, however, the latter expression means precisely that it is impossible to reproduce the conditions of production in sector j'' normally. (First of all, the workers will be unable to consume according to standards a^l_{ij}.) Yet it is impossible to live forever on credit or at the expense of treasure accumulated previously.[1]

The conclusion is as follows: in the case under consideration, prices in all industries tend, for a sufficiently long period of time, to correspond to value. This is not only the sufficient but also the necessary condition for reproduction where $Y = Y^l$.

Of course, the situation considered here ($w^l = 1$ or, the same thing, $Y = Y^l$) is particularly conventional. Capital-

[1] Were it possible, such an industry would simply have halted production.

ism is, in this case, excluded, because there is no surplus-product here.[1]
Yet only capitalism converts all production into commodity production. Here we are discussing merely some universal commodity production.

It is, however, logically correct to take this situation as the starting point in studying the question of the extent to which a precise correspondence is required between prices and value. The answer is: it is implicitly necessary only in the case where $Y = Y^l$. Even then, it is not necessary for prices to conform to value in each industry every year. It is sufficient that the following requirement be fulfilled on the average for several years:

$$p_j \to hw_j \text{ for all } j. \tag{3.13}$$

Then, short-term (for a year or two) deviations of prices in excess of value create, in the industry involved, a treasure reserve fund; or they allow it to return the credit taken earlier as a result of previous deviations of prices below value.

It is important, however, to discuss the situation where $w^l = 1$, not only in logically elucidating the properties of the law of value. The situation has two historical practical analogues.

First, simple commodity production, in which there are no capitalists, the workers are the proprietors of small private enterprises, net receipts being used for their personal consumption.[2]

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[1] It may seem that, since capitalism is not present, it would be unjustified to employ the symbols w^l and a^l_{ij}, introduced in discussing the value of labour power: in the case under study, it is not a commodity and has no value. The qualitative content of the notions behind the above symbols does, of course, undergo a change, but account should be taken of the fact that the value of labour power is merely a historically specific form of what has an economic sense in all modes of production: of labour input in the reproduction of consumer goods for the workers and their families; in any form of society there exist historically established, traditional consumption patterns expressed here by coefficients a^l_{ij}, which are inevitably related to the difference in the categories of workers' skills. That is why it is justified to employ a^l_{ij} to multiply them by the total labour input w_i, to sum such magnitudes, and obtain w^l which, in this case, is considered simply as the total social labour input in reproduction of consumer goods for workers (adjusted to workers of simple labour).

[2] Since prices are proportional to value, net income is the higher, the higher the value created by the labour of the worker at the given skill level. Accordingly, net income *per worker* are higher in industries with more skilled labour. The traditional level of their consumption,

As follows from the analysis, if a society of simple commodity producers could exist, the law of value would be felt directly. In effect, simple commodity production cannot be a stable socio-economic formation, as follows just from the law of value. Nevertheless, if it plays an important part in the production of commodities, it generates a tendency toward a levelling of prices at value.

Simple commodity production is the historical forerunner of capitalism and develops into it in conformity with the objective laws. Marx exposed the foundations of the law of value with reference to the conditions of simple commodity production (see Part 1 of the first volume of *Capital*) and proceeded to study its action under capitalism. We follow his logic.

Second, the prices might also tend to be directly proportional to the commodity values under capitalism if the owners of capital, being receivers of profit, used it only or mainly for personal consumption. In this case, reproduction of a simple type holds. In some sense, this brings the situation closer to the conditions of simple commodity production: the income is largely assumed to be privately consumed by the production agents (though, in this case, not only by the workers, but also by the capitalists).

The goal of capital is not, of course, personal consumption by the capitalist, but rather it is expansion of capital itself.

Even so, in his analysis, Marx used to study all problems relating not only to extended, but also simple reproduction. The extent to which this case may prove practical, can be seen from the current general trend towards a decrease of economic growth rates in developed capitalist countries.[1]

If prices are proportional to values, i.e., comply with formula (3.10) the equivalence of commodities on the market is evidently determined by their values.[2]

given the conditions of simple commodity production, is formed within the limits of average financial resources thus arising for purchasing consumer goods. Formula (2.34) assumes the following form:

$$\sum_i w_i a_{ij}^{l} = l_j Q_j / \tilde{L}_j.$$

The magnitude of newly created value (per worker) represents a general constraint on which norms a_{ij}^{l} depend.

[1] It will be shown in Chapter 4 that at the same time the ratios of prices draw closer and closer to those of values.

[2] Remember that the quantities of commodities q_j and $q_{j'}$ appear on the market as equivalent ones if

$$p_j q_j = p_{j'} q_{j'} \quad (j,\ j' = 1,\ \ldots,\ n)$$

Let us return to our conventional example, recalling that we distinguish skill categories only by industry, i.e., $\{\lambda\} = \{j\}$. Let industry 1 be the production of gold,[1] the latter being a money commodity. Then $1/h = 1.69$ man-years per unit weight of gold. The prices proportional to value are:

$$p_2 = \frac{1.36}{1.69} = 0.805 \text{ gold units per commodity unit 2;}$$

$$p_3 = \frac{2.33}{1.69} = 1.379 \text{ gold units per commodity unit 3.}$$

The reader can make sure on his own that, given these prices, all industries reproduce the conditions of their production. To simplify the calculations, it may be assumed that the consumption standards, as reduced to simple unskilled labour power, are the same in all industries. Then $a^l_{ij} = a^l_{i\lambda} = a^l_i \psi_j$ for all j. The standards are given in the final section of Chapter 2. The total amount of labour power in our example is equal to 560 (as reduced to simple labour power). Then, the components of the vector Y^l

$$Y^l_1 = 0 < 88 = Y_1$$

$$Y^l_2 = 136 < 162 = Y_2$$

$$Y^l_3 = 41 < 82 = Y_3$$

The reader will also see that, given the prices under discussion, all three industries are profitable.

The possibility of redistributing value between industries by means of prices. The second requirement of the law of value.

[see formula (1.2)]. Then let us consider quantities of commodities q_j and $q_{j'}$ that are equivalent in their value, i.e.,

$$w_j q_j = w_{j'} q_{j'}.$$

For any number h, it holds that

$$h w_j q_j = h w_{j'} q_{j'};$$

subject to $p_j = h w_j$ $(h > 0)$, we obtain:

$$p_j q_j = p_{j'} q_{j'}.$$

Prices p_j, $p_{j'}$ make the same quantities of commodities equivalent as values w_j, $w_{j'}$.

[1] In this industry, as can easily be seen from Table 2.3, the rate of reduction $\psi_j = 2$ is the highest for all industries. From Chapter 4 (see, for example, Table 4.2) it may be concluded that, at present, the highest rate of reduction of labour should be in the mining industry, including gold mining.

Now consider the situation where $w^l < 1$ and accordingly $Y^l \leqslant Y$, without the whole surplus of net product being used by capitalists for personal consumption.[1] Let us disregard the latter.

In the case under consideration, it remains true that the direct fulfilment of the requirement $p_j = hw_j$ is sufficient for all industries to be profitable, the profitability of the economy as a whole being, in this case, strictly positive (the sum total of profit in society as a whole is defined as $\sum_i p_i (Y_i - Y_i^l)$). It is by no means obligatory for capitalism, however, that all of them are *equally* profitable. In particular. it is anything but obligatory that the following be observed:

$$p_j = \sum_i p_i a_{ij} + l_j \sum_i p_i a_{ij}^l (1+s), \quad i,\ j=1,\ \ldots,\ n, \quad (3.14)$$

where $s > 0$ is the coefficient, common to the whole economy, expressing the profit to wage ratio. In order that the first requirement of the law of value be fulfilled, it is quite sufficient that:

$$p_j = \sum_i p_i a_{ij} + l_j \sum_i p_i a_{ij}^l (1+s_j), \quad s_j \geqslant 0, \ i,\ j=1,\ \ldots,\ n. \quad (3.15)$$

This is obvious from (3.1). The case $s_1 = \ldots = s_j = \ldots = s_n = s$ is merely one of many particular cases satisfying (3.15), but it is only in this case[2] that $p_j = hw_j$ is valid.

If, then, quantities $s_j \neq const$, part of the value is evidently redistributed from some industries to others.

Below we shall describe, in particular, one kind of price that appears when $s_j \neq const$—the price of production. The properties of prices corresponding to (3.15) will be commented on additionally using these as an example. Here let us merely note the following very simple but theoretically fundamental fact.

Whatever the system of prices might be, if these comply with requirement (3.1) or, the same thing, requirement

[1] The inequality sign $\leqslant$ between the *vectors* is treated here and below as follows: if $a \leqslant b$, the components of the vectors are divided into two and only two subsets: first, a subset of components that, in vector a, are strictly smaller than in vector b; second, a subset of components that, in both vectors, are strictly equal; both subsets may be non-empty and at least one of them is.

[2] The requirement $s_j = const$ for all j is necessary and sufficient for (3.10) to be fulfilled. We omit here the strict demonstration of this obvious fact.

(3.15), i.e., if all industries are profitable, $s_j \geqslant 0$, the value is redistributed between industries within the value of that part of the net product that remains after the necessary product Y^l has been subtracted. This part of the net product is called the surplus-product. Its definition is

$$Y_i^s = Y_i - Y_i^l, \quad i = 1, \ \ldots, \ n, \text{ or } Y^s = Y - Y^l. \qquad (3.16)$$

Here Y^s is the vector of the surplus-product and Y_i^s is the surplus-product of kind i in society as a whole (component of vector Y^s).

It is maintained that the quantity of value redistributed among industries (subject to the fact that all industries are profitable) is limited by the sum total of the value of the surplus-product, i.e., does not exceed

$$\sum_i w_i Y_i^s = wY^s.$$

To prove this, it is sufficient to refer to the law of the conservation of value in the process of commodity exchange, formulated above. According to (2.26) and taking into account that $Y = Y^l + Y^s$:

$$wQ = wH + wY = wH + wY^l + wY^s. \qquad (3.17)$$

It follows from (3.7), however, that each profitable industry can realise on the market at least $w_j^R \sum_i w_i a_{ij} + w^l l_j$ and the sum total of such magnitudes is, in the economy as a whole, equal to $wH + wY^l$, as can easily be seen. This means that the magnitude of value not exceeding wY^s remains to be redistributed.

This is the *second property* (second requirement) of the law of value which, in effect, is merely a logical result of the determination of the value magnitude and of the first requirement. It is, in its sense, a derivation from observation of the facts and there is no alternative to it.

In formulating it, Marx wrote "...the capitalist cost-price of the commodity differs in quantity from its value, or its actual cost-price. It is smaller than the value of the commodity, because, with $C = k + s$, it is evident that $k = C - s$. On the other hand, the cost-price of a commodity is by no means simply a category which exists only in capitalist book-keeping. The individualisation of this portion of value is continually manifest in practice in the actual production of the commodity, because it has ever to be re-

converted from its commodity-form by way of the process of circulation into the form of productive capital, so that the cost-price of the commodity always must repurchase the elements of production consumed in its manufacture".[1]

"If a commodity is sold at its value, a profit is realised which is equal to the excess of its value over its cost-price, and therefore equal to the entire surplus-value incorporated in the value of the commodity. But the capitalist may sell a commodity at a profit even when he sells it below its value. So long as its selling price is higher than its cost-price, though it may be lower than its value, a portion of the surplus-value incorporated in it is always realised, thus always yielding a profit. ...There is obviously an indefinite number of selling prices possible between the value of a commodity and its cost-price. The greater the surplus-value element of the value of a commodity, the greater the practical range of these intermediate prices."[2]

Actually, not more than 10 to 20 per cent of the total value of all commodities seems to be involved in the redistribution among industries. This fact is quantitatively important: although prices deviate from value, they are, on average, sufficiently similar to it because most of the price (apparently 80-90 per cent an average) proportional to the corresponding commodity value part. As far as some industries are concerned, of course, much greater portions of the value quantities than 10 to 20 per cent can be redistributed in their favour (or withdrawn).

The law of value as the general law of commodity production. The two properties formulated above: the requirement of profitability of all industries and the possibility of value redistribution only within the value of the surplus-product, are enough to explain the effect of value on prices. By and large, prices do not thus follow simply from value; they may deviate from it, and, in this sense, possess a certain autonomy. "Magnitude of value expresses a relation of social production, it expresses the connexion that necessarily exists between a certain article and the portion of the total labour-time of society required to produce it. As soon as magnitude of value is converted into price, the above necessary relation takes the shape of a more or less accidental exchange-ratio between a single commodity and another, the money-

[1] Karl Marx, *Capital*, Vol. III, pp. 26, 28.
[2] *Ibid.*, p. 37.

commodity. But this exchange-ratio may express either the real magnitude of that commodity's value, or the quantity of gold deviating from that value, for which, according to circumstances, it may be parted with. The possibility, therefore, of quantitative incongruity between price and magnitude of value, or the deviation of the former from the latter, is inherent in the price-form itself. This is no defect, but, on the contrary, admirably adapts the price-form to a mode of production whose inherent laws impose themselves only as the mean of apparently lawless irregularities that compensate one another."[1]

There exist two types of deviation of price from value: random fluctuations resulting from a specific demand to supply ratio and stable shifts in the average level of prices in relation to value, i.e., value modifications. The former may, for some short time, even violate the first requirement and make an industry unprofitable. This inevitably results in difficulties in reproduction, at least for the producers of most of the output (and should the downward deviation of price from value be sufficiently great—for all producers). Further, two processes that usually take place in some combination are likely: technological improvement of production can make it profitable even given existing prices; difficulties in reproduction amounting to a reduction in output of an industry result in a shortage of commodities on the market, i.e., demand is not fully met at the given prices. In such a case, the prices for the output of the given industry increase in comparison with those for the output of others, at least some of the existing technologies thus being made profitable. They would persist, whereas the unprofitable ones would be superseded. In some way or other, the requirement of profitability is fulfilled, the above ways being nothing but the mechanisms by which the law of value operates.

Long-term shifts in average prices in relation to value are possible, but they do not affect the requirement of the profitability of industries and are reduced to a partial redistribution of the value of the surplus-product.

The law of value operating so that only its two above properties are really necessary complies with the conditions of every form of commodity production. This is the general law of commodity production. Moreover, it is *the form*

[1] Karl Marx, *Capital*, Vol. I, p. 104.

through which (under the conditions of commodity production) the general properties of social production in general come into effect, which go back to the constituting role of human labour in production.

It can easily be shown that the conditions discussed above (paragraph 2.2) with which the theory of value is initially constructed are not necessary to deduce the law of value. They were introduced only to simplify the logical constructions, without affecting the conclusions.

In accordance with the intention expressed above, we shall gradually extend the concept of the law of value to demonstrate that it covers the points we initially disregarded.

Reimbursement of circulation costs. According to point 5 of paragraph 2.2 we have so far taken no account of the genuine costs of circulation. Exchange does not change the amount of values exchanged, but it requires input—of material and labour—to be carried out. Let the employees in the sphere of circulation consume according to the same standards as those in the sphere of production. Then at least part of the surplus-product will be spent to replace material inputs in the sphere of circulation and to maintain its employees:

$$Y^c = (Y_1^c, \ldots, Y_i^c, \ldots, Y_n^c); \; Y_i^c = \sum_j a_{ij}^c Q + a_{ic}^l \tilde{L}^c, \tag{3.18}$$

where a_{ij}^c are the average material input coefficients of kind i for carrying out the sale and purchase of the commodity of kind j; at least some $a_{ij}^c > 0$; natural losses may be taken into account by means of standards a_{ij}^c; a_{ic}^l are the average consumption standards of employees in the sphere of circulation; $\tilde{L}^c = \sum_j \tilde{l}_j^c Q_j$ is the number of workers employed annually in operations in the sphere of commodity circulation; $\tilde{l}_j^c$ is the average labour input in the sale and purchase of commodities of kind j; $\tilde{l}_j^c > 0$ for all j; here $\tilde{L}^c$ is assumed to be part of social labour force, exceeding the number of the workers employed in production L. Note also that the rates of material circulation costs involve some coefficients for the wear and tear of the monetary commodity.

In reality, therefore, any commodity production is completely impossible without a surplus-product, the existence of which (at least in the amount necessary to replace the true

costs of circulation) is a necessary condition for the evolution of commodity money-relations. At the same time, we are dealing with the first constraint on the redistribution of the value of the surplus-product among industries producing commodities: in any case, each industry should receive (by means of the price system) the means to replace the genuine costs of distribution it has to bear. On the other hand, all commodity producing industries, if they make use of the services of a specialised marketing company to bring their commodities to the consumer for sale, should cover the latter's costs; the aggregate of producers thus receive, for this single reason alone, *less* value from the market than they introduce into it, having to pay for the nonproductive functions inherent in the given society.

The theory thus takes account of the fact that the real law characteristic of value is, strictly speaking, one of non-increase, rather than its conservation in the process of exchange.

3.2. The Inevitable Results of the Operation of the Law of Value

Since the sales prices of equal commodities tend to be the same for all producers and the purchase prices the same for all consumers, being basically determined by the social value of these commodities, a number of conclusions may be drawn to express the properties of an economy in which the law of value operates. They are logically inevitable and, at the same time, well-known in practice. The law of value discovered in the theory here plays the role of a scientific statement capable of explaining the facts observed en masse. At the same time, it is itself proved by these facts.

Unequal profitability of various technologies and firms. We have already dealt with at least one case of equal profitability of commodity producing *industries*—zero profitability of the industries of simple commodity production. It should be stressed that the concept of equal profitability concerns only industries, and not the firms constituting them. Profitability may, in some sense, be equal, but is not obligatorily so. The equal profitability of industries does not preclude, however, the profitability of firms to be unequal.

Let us introduce the concept of the necessary monetary costs of reproduction, denoting them by CP_j^h. This is a rat-

ed magnitude to show the amount of money needed, given the prices of all commodities, by some technology k to carry out normal reproduction of its conditions—objective as well as subjective; $k \in E$. Let us emphasise that it arises when it is assumed, in particular, that the labour power employed in this technology is reproduced at a normal level. Then we obtain:

$$CP_j^k = \sum_i p_i A_{ij}^k + \sum_{i,\lambda} p_i a_{i\lambda}^l L_{j\lambda}^k \text{ for all } j = 1, \ldots, n; \ k \in E. \tag{3.19}$$

Per unit commodity:

$$cp_j^k = \frac{CP_j^k}{Q_j^k} = \sum_i p_i a_{ij}^k + \sum_i p_i a_{i\lambda}^l l_{j\lambda}^k. \tag{3.20}$$

Given prices p_i, cp_j^k depend on the individual material input standards a_{ij}^k and labour input coefficients $l_{j\lambda}^k$ for each technology. There exist no *a priori* circumstances, i.e., ones not connected with this technology, given from outside, that would limit the magnitudes of a_{ij}^k and $l_{j\lambda}^k$ from above, the single *a priori* constraint from below being the non-negativity of these magnitudes. That is why, given any system of *unified* prices,[1] it is totally improbable that:

$$cp_j^k/p_j = const \text{ for all } k \in E, \ j = 1, \ \ldots, \ n.$$

It is also unlikely that such a constant would become established within any industry j if the latter actually employs more than one technology.

We are going to use the concept of the *profitability of output*: the ratio of profit to the costs of production in money terms. If magnitudes cp_j^k are assumed to be the costs, this means:

$$\frac{p_j - cp_j^k}{cp_j^k} \neq const \text{ for } k \in E_j, \ j = 1, \ \ldots, \ n \quad (3.21).$$

Different technologies used to manufacture one and the same commodity are virtually inevitably, owing to their own specific characteristics, unequal with regard to the profitability of output.

[1] Note that the conclusion holds even if the difference between the producer's price and that of the final consumer is taken into account. If the producer's prices are the same for all producers and the consumer prices for all consumers, the conclusion is still valid.

If, as we have so far assumed, each firm employs one technology, the above implies a different profitability of output in different firms within an industry. Yet even if it is assumed that the firms combine several technologies to produce one commodity or even combine several kinds of commodity, this inevitably leads to different profitabilities of output of the firms on the whole.

Profitability was here determined with reference to the rated magnitude of cp_j^k covering the *standardised* total input in the reproduction of labour power. A further analysis should be made with regard to the difference between simple and capitalist commodity production.

The differentiation and decay of petty commodity production. If the profitability of industries overall is equal to 0, it follows from the statement expressed in formula (3.20) that, in each industry, there are small businesses the necessary costs of which are higher than the output prices and ones where they are lower than these. The first of these groups cannot reimburse the costs from the price at the required level. First of all, this leads to underconsumption by the owners, and thus to violation of the normal reproduction of their labour power, to a drop in it. Reproduction of material conditions of production in normal quantities corresponding to a given technology may also become unattainable. The second group receives a certain surplus over the necessary costs, which can be used both to raise consumption above the usual standards and expand production.

Directly, this state of affairs is merely a *differentiation* of petty producers with regard to the conditions of reproduction in their businesses in general and the reproduction of their labour power in particular. For some period of time, both technologically advanced and backward businesses may coexist. The latter achieve this, above all, on the basis of underconsumption. With time, however, the natural differentiation ultimately leads to the *decay* of simple commodity production and to the emergence of two opposite social classes—proletarians and capitalists.

In this sense, simple commodity production with its inherent law of value may be described as a system with a positive feedback displayed over time: rich people become richer and richer; poor ones become even poorer. The difficulties in reproduction resulting from technological backwardness make such producers either reduce their output directly or fall into debt in order to survive, which can

postpone, but ultimately merely increases the probability of their complete ruin. The monetary surplus of the technologically advanced enterprises, on the contrary, yields a resource not only for exploiting their potentialities, but also for up-to-date technical renewal, as well as for extending the scale on which the given technologies are used.

The latter requires that the scope of production be wider than is determined by the family resources of labour power, i.e., that there should be hiring of workers. Such workers are offered by the first process—that of proletarianisation of small private owners. Eventually, *most* of them inevitably become proletarians, the minority—capitalists.[1]

These two spontaneous processes of the polarisation of society to form two classes, as a result of the operation of the law of value, may and have usually been, greatly accelerated by external intervention (for example, enclosure and the laws against vagrancy in England). Here we cannot discuss in detail the whole system of the so-called primitive accumulation of capital.[2] Let us merely indicate that such accumulation is an inevitable result of the operation of the law of value, even when there exists small commodity production of owners who do not employ wage labour. That is why that type of production cannot form a social formation: it is unstable, it does not create the conditions for regular self-reproduction. The laws of equivalent exchange inevitably lead to appropriation based on the owner's own labour being replaced by appropriation based on the exploitation of wage labour.

Technical progress. The actual ruin of simple commodity producers is based on backward (low-productive) technologies being superseded and on relatively more efficient ones becoming widespread. This is also a logical result of the operation of the law of value, which forms a spontaneous *economic mechanism* of technological progress.

As already indicated in paragraph 2.4, in a productive technological system all industries are productive. This problem must now be considered at the level of the firms making up the industry.

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[1] Here, the general factor limiting this process is the total limit to labour power, including that which has become proletarian.

[2] For more detail, see Marx's *Capital* and Lenin's "Development of Capitalism in Russia" (Karl Marx, *Capital*, Vols. I, II, III; V. I. Lenin, *Collected Works*, Vol. 3, 1977).

The productivity of all industries means, in terms of the law of value, that requirement (3.7) is observed for all of them, which is the same as that of profitability. Since an industry is, on the whole, productive, it follows that there exist unit productive firms (technologies) k in it. For some time, however, nonproductive technological methods may also exist; these are such methods k that $w_j < \sum_i w_i a_{ij}^h$ (I). One may also speak of methods the productiveness of which is non-negative, but not enough to support their employees: $0 \leqslant w_j - \sum_i w_i a_{ij}^h < \sum_i w_i a_{ij}^l l_j^h$ (II), where a_{ij}^l are the consumption standards per unit of simple labour; of methods providing such support, but not resulting in a surplus-product (III); finally, of methods yielding a surplus-product: $w_j > \sum_i w_i a_{ij}^h + \sum_i w_i a_{ij}^l l_j^h$ (IV). In all cases, the evaluation of the productiveness of method k is based on the quantities of total labour input (which can be seen from the formulae above).

The relative efficiency of technologies within an industry is a phenomenon of the general economic order; it is determined by the system of technologies in the economy as a whole. Whether some technology k, within industry j, belongs to the group of the best, medium, or worst ones, is determined not only by its own parameters, nor even by the parameters of the other technologies of the same sector, but by the whole technological system of society. Hence the historical nature of the position occupied by a given technology k with regard to efficiency: it can be advanced under some conditions and backward under others. The opposite trend is not excluded when in society the value of those means of production which distinguish the given technology k from the others in the same industry is decreasing.

The law of capitalism (and of commodity production in general) is surely the replacement of nonproductive methods. Particular nonproductive technologies may exist for a time at the expense of the net product of the other technologies. In any society there are, however, mechanisms generating the replacement of nonproductive technologies. In commodity production this is a price system such that $\sum_i p_i A_i^h > p_j Q_j^h$. Such a technology k is unprofitable even without regard to labour remuneration. As for prices, as already

stated, on the average 80 to 90 per cent are determined by the value of the commodities.

Consider the situation where $p_j = hw_j$. If the methods belonging to the worst group with regard to the total labour input completely replace their material input from the receipts from their commodities at average prices p_j (p_i), they will not be able to create the average socially normal conditions of reproduction of labour power for their employees. Without replacement of their material input, however, these methods cannot technologically carry out reproduction anyhow, but only on a continually decreasing scale. In one way or another (owing to deficient replacement of material input, worse conditions of labour power reproduction, or to both), *these methods are superseded.* A similar reasoning can easily show that, on the contrary, methods k can, at the expense of receipts $p_jQ_j^k$, bring about an *extension* of production.

This means that one regular result of the law of value is, first of all, technical progress: the worst methods are superseded, while the best ones spread. If we are dealing with any form of private ownership of the means of production (i.e., not only simple commodity production, but also capitalist production), technical progress is equal to the superseding, and usually the ruin, of the producers employing the worst methods and to the enrichment of those who employ the best ones.

The differences in the methods with regard to the above groups of technology regulate, of course, renewal of technologies only on the condition that $p_j = hw_j$. Yet since deviations of price from value cannot, on average, be very great, the ratios of the individual and social values of commodities are the *main* reason, they determine *in the main* the superseding of some technological methods and the spread of others.

In this respect, the operation of the law of value is again characterised by a positive feedback displayed over time: the technological system determines the productivity of social labour and, thereby, the magnitudes of the value of products; through the price system these magnitudes make more productive technological methods spread; the more the latter spread, the lower the social value of the commodities, so the more probable is the removal of less productive methods from the system; the superseding process again reduces the social value, etc.

Regulation of the proportions of production. The mechanism of the law of value. According to its definition, the value of commodities is a generalised expression of the proportions of social production.

First, the newly created value is merely a direct expression of the distribution of living labour (reduced to simple labour) among industries. This is clear from the expression:

$$L = \sum_j L_j.$$

Further, the average value added by living labour is, per unit commodity:

$$l_j = L_j / Q_j,$$

i.e., it expresses the proportion between living labour input and output.

Second, the formation of value also reflects all similar proportions between embodied labour input and output (ratios $w_i A_{ij}/Q_j$), hence, in such a form, the ratios of the input of reproducible material resources to output ($a_{ij} = A_{ij}/Q_j$) and, therefore, finally, the actual distribution of the various means of production among the industries.

Let us recall at this point that the average coefficients l_j, a_{ij} themselves depend on output Q_j, and so on the proportions between the outputs in the various industries.

The totality of these characteristics of value is reflected in the concept of value as the distribution of living and embodied labour among industries. Here the formation of value according to formula

$$w = l(I - A)^{-1} = l + lA + lA^2 + \ldots$$

appears as an infinite series, where direct labour inputs immediately expressing its distribution among industries are gradually accumulated and summed up.

By leading to technological development, the law of value changes the system of technological proportions (coefficients l_j, a_{ij}). Ultimately, technological development also changes (through the system of the other social development processes) the proportions of nonproductive requirements (see Chapter 5).

The market mechanism *demand—supply—prices*, as that of the realisation of the law of value, leads to changes in the proportions of output of commodities in accordance with those in demand, and thus to a corresponding redistri-

bution of social labour. Here, our previous assumption that demand and supply coincide must be dismissed. A discrepancy between them must be allowed for some period of time. Then their equality will not be considered as directly given, but as a trend coming into effect only through nonfulfilment, through violations of the balance of these magnitudes, which (violations) generate each other, have opposite signs and tend to cancel each other out.[1]

Demand is defined as the quantity of commodities that buyers are prepared to purchase (directly in exchange for money or on credit) over a certain period of time, given the prices established in the foregoing process. Demand thus expresses their requirements with regard to cash and expected monetary resources. In turn these requirements change largely irrespective of the prices for each specific commodity, being affected primarily by shifts in technological systems. In changing over from one technology to another, the decisive role is, of course, played by the sum total of the prices of input and output, and not simply by the prices of the particular kinds of commodity constituting, for instance, means of production. Similarly, in nonproductive consumption, it is the sum total of the prices of the range

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[1] "In reality, supply and demand never coincide, or, if they do it is by mere accident, hence scientifically =0, and to be regarded as not having occurred. But political economy assumes that supply and demand coincide with one another. Why? To be able to study phenomena in their fundamental relations, in the form corresponding to their conception, that is, is to study them independent of the appearances caused by the movement of supply and demand. The other reason is to find the actual tendencies of their movements and to some extent to record them. Since the inconsistencies are of an antagonistic nature, and since they continually succeed one another, they balance out one another through their opposing movements, and their mutual contradiction. Since, therefore, supply and demand never equal one another in any given case, their differences follow one another in such a way—and the result of a deviation in one direction is that it calls forth a deviation in the opposite direction—that supply and demand are always equated when the whole is viewed over a certain period, but only as an average of past movements, and only as the continuous movement of their contradiction. In this way, the market-prices which have deviated from the market-values adjust themselves, as viewed from the standpoint of their average number, to equal the market-values, in that deviations from the latter cancel each other as plus and minus. And this average is not merely of theoretical, but also of practical importance to capital, whose investment is calculated on the fluctuations and compensations of a more or less fixed period" (Karl Marx, *Capital*, Vol. III, pp. 189-90).

of commodities forming a certain consumption pattern (shaping a certain living standard) that is basically fundamental, and not simply the prices of individual commodities as such. (Below it will be shown statistically that the structure of nonproductive consumption changes regardless of the changes in relative prices; see Chapter 5.)

The law of value thus affects the patterns of demand in some indirect way—through the structure of technological input and consumption, with their characteristic internal, autonomous complexity. It would, therefore, be incorrect to place the demand for some kind of commodity simply into the isolated dependence on its own value (price).

Supply is defined as the quantity of commodities vendors are prepared to sell (directly in exchange for money or on deferred payment—on credit) over a certain period of time, given the prices established in the foregoing process. Supply is determined by current output and the commodity owners' policy in relation to forming reserves (accumulation, clearance sales). Usually, stocks are accumulated when the prices of the corresponding commodities have, for some period of time, been rising. It is reckoned that they might be sold in the future at an even higher price. Stocks are sold off if their holders expect prices to fall (if the fall in prices has already begun and has been stable for some time, if the commodity is expected to become obsolete, etc.). In total, over a sufficiently long period of time, supply is virtually equal to output, because all commodities are ultimately produced for sale. In theory, it is also assumed that all the commodities produced will eventually, sooner or later, be demanded and *in this sense* demand and supply achieve equilibrium for a sufficiently long period of time.[1]

The demand for and supply of each commodity take shape independently at the given prices. Moreover, they form as a result of many isolated decisions by sellers and buyers, the sum totals of these appearing only on the market and being,

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[1] It does not follow from this that any *primary* demand is usually met, i.e., that, on the market, there are always enough commodities to meet any demand.

Note, also, that while a commodity is in storage, it may, of course, completely or partly lose its useful properties and will, therefore, never be sold (or be sold at a very low price that does not cover the costs). In constructing the general theory, however, the ruining of commodities, as well as their deliberate destruction, are disregarded.

by definition, random functions.[1] That is why their direct coincidence is virtually excluded.[2] If they do not coincide at the given prices, this, in turn, causes a change in the latter.

When demand exceeds supply, there is no actual competition between sellers, whereas the competition between buyers is inevitable. Such a situation is called *a sellers' market*. Buyers' competition is expressed in that those who can afford to, offer a higher price than that established earlier, which shaped the given demand and supply. That is why the price tends to rise.

In contrast, when supply exceeds demand, competition between the sellers is inevitable. (This is a *buyers' market*.)

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[1] Let us give a conventional example of this. Let there be an economy in which sellers and buyers are agents of capitalist production only (there are no special agents of the sphere of circulation and no forms of nonproductive consumption, except that of workers and capitalists). Then, given the system of average prices $\{p_j^t\}$, each capitalist $k \in E$ draws up, for the subsequent period of time, an output plan $\overline{Q}_j^k(t+1)$; the supply of the commodities produced earlier being $S_j^k(t+1) \leqslant Z_j^k(t)$, where $Z_j^k(t)$ is the stock of commodities accumulated by the end of the time period t; demand for means of production is $D_i^k(t+1)$ $(i=1, \ldots, n)$ and for labour power is $D_\lambda^k(t+1)$ $(\lambda=1, \ldots, \Lambda)$, the demand depending on the output plan and accumulated stocks of means of production, as well, possibly, as on the plan for stocks over and above current production needs. Moreover, each capitalist and worker, independently of each other, determines his own demand for consumer goods. Obviously, the total demand on the market D_i, D_λ and supply S_i $(i=1, \ldots, n)$, S_λ $(\lambda=1, \ldots, \Lambda)$ are merely random functions of a set of independent variables.

[2] This consideration involves the substance of the reply given by the Marxian theory of capitalism to the conceptions and models of the so-called general competitive equilibrium. To refute these, it is *sufficient* to point out that both demand and supply actually respond to changes in prices at a finite speed (whereas, in these conceptions, infinite speed—the instantaneous nature of this response—is accepted, as well as the instantaneous response of prices to changes in demand and supply). A detailed analysis of the best-known competitive equilibrium models can be found in the following works: K. Valtukh, "The Arrow—Debré Theorem of Competitive Equilibrium, and Problems of Economic Theory", in: *Problems of National Economic Optimum*, Novosibirsk, Nauka Publishers, 1973 (in Russian); V. Petrova, "Analysing the Conditions of the MacKenzie Theorem of Competitive Equilibrium", in: *National Economic Models. Trends in the Development of the USSR Economy*, Novosibirsk, Nauka Publishers, 1974 (in Russian).

This competition is expressed in that the attention of buyers is attracted by price reductions.[1] Prices tend to fall.

If we are considering an economy where fiat money is employed in circulation, processes of a purely inflationary nature (the emission of soft money) are likely to induce a rise in the prices of commodities, irrespective of the demand to supply ratios. The buyers' market should then be said to cause a fall in relative prices, i.e., the price rise is less than the simultaneous one on a sellers' market. It is thus a matter of comparing *price indices.*

Yet a situation is possible in which the market for all, or almost all, commodities is a buyers' market or, the inverse case, a sellers' market dominates. In such cases, the price indices seem to be dependent on the intensity (degree) to which supply exceeds demand in the former situation and demand exceeds supply in the latter. The general case is usually expressed as follows: the price index is the higher, the higher the intensity with which demand exceeds supply (this also applies to a negative excess, i.e., the real excess of supply over demand). In other words, the price index is assumed to be a direct function of the demand to supply ratio:

$$Ip_j(t) = f(D_j(t)/S_j(t)), \quad j = 1, \ \dots n \qquad (3.22)$$

where I is the sign of the index, $D_j(t)$ is the demand for commodity j over time period t, $S_j(t)$ is the supply of this commodity over the same time period. It is assumed that t is a sufficiently short span of time for demand and supply not to change, in turn, as a result of changes in the price system.

So, it is assumed (on the basis of observations), on the

[1] A distinction is often drawn between so-called non-price and price competition. An increase in the useful qualities of commodities at the given price is called non-price competition. In the theory, however, the price is taken as the proportion of the exchange of a given commodity, with given useful qualities, for money. With this definition of price, so-called non-price competition is price competition, i.e., both competitions are based on prices.

Note, also, that competitive equilibrium models neglect the fact that the price is a necessary but not unique sphere in which competition manifests itself. By assuming the prices at each given moment to be strictly unique, the models demand from the agents on the market a social discipline that is incompatible with real competition. In contrast, the theory of value considers the price to be unique only as a trend and not as a directly given fact.

one hand, that the vectors of demand and supply are functions of the established price system with regard to its trends:

$$D_j(t+1) = D_j(t+1)(\{p_j(t)\}, \{p_j(t-1)\}, \ldots, \{p_j(t-\delta)\}); \quad j=1, \ldots, n \qquad (3.23).$$

$$S_j(t+1) = S_j(t+1)(\{p_j(t)\}, \{p_j(t-1)\}, \ldots, \{p_j(t-\delta)\}); \quad j=1, \ldots, n \qquad (3.24)$$

At the same time, it is assumed (also on the basis of observations) that the price system, in turn, is directly dependent on demand and supply:

$$p_j(t+1) = p_j(t+1)(D_j(t)/S_j(t)), \; j=1, \ldots, n.$$

It is important to see that these interactions within the *demand-supply-price* system do not in themselves give a definite answer to the questions as to what the price ratios are and in what way they change. Indeed, a rise in the relative price of a certain commodity may induce, as a trend, first, an accelerated growth of its production and supply, second, a relative decrease in consumption and demand, these cancelling out the initial excess of demand over supply and able even to engender an excess of supply over demand. In any case, they lead to a decrease in the relative price. We are thus dealing here with a system with negative feedbacks which, by itself, can generate only *stability* of the price ratios or, more precisely, *fluctuations* of them around a certain level. They cannot, however, throw light on this level itself.[1] Moreover, the price ratios actually *tend to change* over sufficiently long period of time. These changes cannot, after all, in any case, be derived from the actual interaction between demand, supply, and price.[2]

[1] The answer must not be accepted that this price level is simply a certain price of a purely market equilibrium of demand and supply. The point is that, as has been shown, the equalisation of demand and supply is not a function of prices, the latter being, by the very mechanism of their formation, not oriented on such a short-term equilibrium. Equilibrium is ultimately achieved, but the mechanism of that equilibrium covers not only the market, but also, above all, production.

[2] "Supply and demand determine the market-price and so does the market-price, and the market-value in the further analysis, determine supply and demand. This is obvious in the case of demand, since it moves in a direction opposite to prices, swelling when prices fall, and vice versa. But this is also true of supply. Because the prices of means of production incorporated in the offered commodities determine the demand for these means of production, and thus the supply of com-

The theory of value is not, however, confined to observations of the surface of phenomena and to the statement that such observations cannot explain long-term trends. It *does explain* these trends, though in a by far from obvious way.[1]

Both demand and supply themselves are, in the long run, an expression of changes in the development of production, as is clear from their definitions.[2] If this fundamental circumstance is taken into account, a transition is required from analysing market relations alone to considering the interrelated production and market development processes.

Then above all the understanding of the effect of relative prices on supply must be more precise: an accelerated growth of production and, on this basis, of supply is natural not only when the relative price of a commodity is rising but, more strictly, when the whole price system renders production of this commodity more *profitable* for its producers than the production of other commodities.[3] The rise in the relative prices increases the *probability* of such a situation, but it does not always result in it: even if the relative commodity price is rising, its output may for some

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modities whose supply embraces the demand for these means of production. The prices of cotton are determinants in the supply of cotton goods.

"To this confusion—determining prices through demand and supply, and, at the same time, determining supply and demand through prices—must be added that demand determines supply, just as supply determines demand, and production determines the market, as well as the market determines production" (Karl Marx, *Capital*, Vol. III, p. 191).

[1] This may be shown mathematically, as an expression of the characteristics of a model of capitalist production. A much more complex model is obtained, however, than the static input-output one. It cannot be analysed here. The main ideas of such a model may be found in the paper by K. Valtukh, E. Yershov, "A Model of Free Competition Capitalist Commodity Production", in: *National Economic Models. Trends in the Development of the USSR Economy.*

[2] Let us stress once again that this refers to the formation not only of supply, which is sufficiently obvious, but also of demand, and, what is more, not only for producer's, but also for consumer goods.

[3] Here we do not specify in what precise sense the concept of profitability is employed. Anticipating somewhat we shall note here that, under the conditions of premonopoly capitalism, it should be defined as the ratio of profit to the capital advanced. Then the industry that achieves a higher than average rate of profit (i.e., where the commodity sales price is higher than that of production) is taken to be relatively more profitable. Yet any positive indicator of profitability implies, above all, a positive absolute quantity of profit.

time remain less profitable than that of the other commodities.

Accordingly, the fall in the relative price of some commodity merely makes it more *likely* that the profitability of its production, the given system of technologies being retained, becomes, on average, relatively lower, though this is not obligatory.[1] Meanwhile, a decrease in the growth rates of production, especially a direct reduction in it, is a function of relative profitability and not simply of price movements.

As for the profitability of industries, as already shown, it is nothing but an expression of the price to value ratio and not simply that of the price system. It is regulated by prices and coefficients of production costs (the latter in general form is expressed in the commodity value). Profitability is, in fact, determined by whether the industry replaces (or exceeds) the value of its production conditions.

It then turns out that the *demand-supply-price* mechanism determines prices, but only in that sense that it realises the internal law of price, i.e., it is the mechanism by which the law of value operates.

If the price system renders a certain industry especially profitable, the *probability* increases that resources allowing a growth of production will be drawn into it. It is essential here, first, that increased profitability directly offers an additional development resource and, second, that it encourages its investment in the proper industry.[2] That is why its output is highly likely to grow rapidly. There seem to be grounds for maintaining that the higher the industry's profitability, the higher its production growth rates tend to be. Accordingly, the lower its profitability, the higher the probability of lower rates (including the possibility of

[1] The firms in one and the same industry differ in their individual profitability. Even an industry with a low average profitability may include *firms* the profitability of which is higher than average level of profitability of some other, highly profitable *industry*. In this part of text, however, the ratios of profitability of whole industries, rather than of the particular firms within them, are being considered. It may be added that the more profitable the industry as a whole is, the higher the share of the firms in it with a profitability exceeding the average social level.

[2] The existence of such an incentive engenders a direct flow of capital from outside to the corresponding industry (by means of credit, the construction of new plants, etc.) so it comes out that rapid growth is ensured not only by additional internal development resources.

a decrease in the volume of output, moreover, not only in the short term, but also the long term, up to a complete halt to the production of the given commodity).

It then turns out, however, that *demand-supply-price* mechanism regulates, above all, the distribution of resources, particularly living labour, among industries in accordance with changing productive and nonproductive requirements. To be precise, through numerous fluctuations (i.e., not directly but spontaneously, through nonrealisation), *prices* tend to be formed so that the mentioned conditions of profitability be created that make output growth rates (*supply*) accord with the growth of requirements (*demand*). This implies precisely a tendency towards a demand and supply equilibrium, which is thus nothing but the resultant tendency of the change in the production structure in accordance with the consumption pattern, i.e., the tendency for production proportions to be regulated by the law of value.

So the true sense of the assertion concerning the trend towards a balance of demand and supply boils down to the following. First, it may be assumed without any great error that the entire output produced will ultimately be demanded; second, in this case, the output structure gradually changes in accordance with the changes in the effective demand pattern. The concept of the demand and supply equilibrium does not really imply anything more. In particular, it is not true that, for any short time period, the whole primary effective demand is met, or the real commodity supply is necessarily sold.

At some stages of the capitalist reproduction cycle, demand on the whole exceeds supply (revival, boom); at the other stages, on the contrary, supply exceeds demand (crisis, depression). In this work, however, we cannot analyse the cyclical character of the operation of the law of value.

Together with the process of the balancing of demand and supply, prices tend to become equal at their normal level. In this case, the market mechanism *can equalise the prices of products of industries at a level according to any principle*, and this is why this mechanism itself does not determine the equalisation principle, i.e., the centre of gravity of prices. This centre is determined by the law rooted in the relations of production. Under the conditions of simple reproduction, be it simple commodity or capitalist production, when the technological system, and thus the system of requirements, are stagnant, this equalisation is at the level

of value: for all industries it creates equal conditions of reproduction of output on a constant scale. Under extended reproduction, this equalisation is at the level of a certain modification of value (when all industries replace the value of the reproduction of their production conditions and, at the same time, surplus-value is, according to some principle, partly redistributed among them).

So the trend of the *demand-supply-price* mechanism consists, first, in an equalisation of demand and supply; second, in an equalisation of prices at a normal level corresponding directly to value, or its modification. In this sense, the theory of value maintains that the normal price level is that at which demand and supply cover one another. *The tendency of prices towards the normal level corresponding to value results in demand and supply tending to be balanced.*

The fundamental difference between this thesis and the main points of the theory of price equilibrium is obvious. It is especially felt in analysing price movement problems.

When the market mechanism is analysed in isolation from production, the effect of the demand to supply ratio on price movements may be interpreted in only one way: the more rapidly demand grows, the faster the rise in relative commodity prices will be. Then the price index should be *directly* connected with the rate at which demand is growing and, therefore, the volume of real sales as well.[1] The theory of value leads to a different conclusion. In the short term, output growth is possible only on the basis of existing technologies, and, if not always, then often, is due to an increase in the material and labour input coefficients. This is usually characteristic of growth in the neighbourhood of the productive capacity of firms. An increase in average value is possible only if the relative price of the commodity in question rises. In the long term, however, the output growth is based not so much on mobilisation of the potentialities of existing

[1] It is characteristic that the abundant western literature dealing with price theory and price-formation models does not dwell in detail on the question of the ratio of price indices and output indices: as far as possible this literature usually avoids conclusions that may be tested by verifying. Nevertheless, it may be indicated that the idea set out in this text has been reflected in this literature, by which we mean the following formula by Paul Samuelson:

$$dp/dt = k\,[D\,(p) - S\,(p)].$$

(Cf. P. A. Samuelson, *The Stability of Equilibrium: Comparative Statics and Dynamics*, Econometrica, Evanston, 1941, pp. 97-120).

technologies as on the introduction of new ones (additional capacities). On the whole, the share of new technologies should be the greater, the more rapidly output grows.[1] As far as technical progress is concerned, these new technologies have relatively low social labour input per unit output. The more rapidly their share in the industry grows, the more rapidly the average value of output decreases. Then, in accordance with the law of value, the relative price of this output should fall more rapidly on the market.

The relative prices of growing product should fall for a sufficiently long period of time, because its relative value should decrease. At the same time, these *prices should remain higher than value* (*or some other, modified, centre of fluctuations of prices*), otherwise the producers would not be able to increase their output rapidly. Accordingly, the prices of slowly increasing product should (for a sufficiently long period of time) be *lower* than value (or its modification), but the *relative prices of such output should rise.*

This is precisely the conclusion we meant in saying that the inferences of the theory of value are not trivial. In Chapter 4 it will be shown that it is justified statistically with a great precision.[2]

Marx considered the effect of value on price movements to be the strongest characteristic of the law of value. "Whatever the manner in which the prices of various commodities are first mutually fixed or regulated, their movements are always governed by the law of value. If the labour-time required for their production happens to shrink, prices fall; if it increases, prices rise, provided other conditions remain the same."[3]

According to Marx, in particular, changes in value are a decisive factor behind the change in the prices of produc-

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[1] New technologies are, of course, introduced to one extent or another in all industries, this being a general property of the law of value.

[2] The conclusion is, however, by nature a probable trend rather than a deterministic dependence. Suffice it to point out that the speed at which new technologies spread is still not an indicator of their progressiveness from the point of view of the decrease in output value. It is not excluded that, in a certain rapidly developing industry, value may fall less than in some slowly developing one, for example, due to the fact that new technologies are not sufficiently cost-effective.

There are many other circumstances due to which the above trend acts only in a diffused manner and not strictly uniquely.

[3] Karl Marx, *Capital*, Vol. III, p. 177.

tion although the latter, by themselves, can deviate rather strongly from value.[1] The price of production changes, first, as a result of changes in the total rate of profit, behind which changes in surplus-value are to be found and, hence, in the value of labour power and in newly-created value. Second, it is affected by the production costs of a given commodity, which depend on the same factors as its value. Of course, the price of production of a given commodity can change even if its value remains the same. The value of commodity can change while its price of production remains stable. But the probability of such cases, while being greater than 0, is nevertheless rather low.[2]

3.3. Forms of Surplus-Value

The price of production as a converted form of value. An important place in the theory of value and surplus-value, in the history of its formation,[3] and in the current discussion around it belongs to the question of the correlation of value to the price of production, which is the name given to the average commodity price (being the centre of gravity of specific prices), if it yields an equal rate of profit on capital advanced in all industries.

In this book, whenever possible we employ modern mathematical methods to set out and demonstrate Marx's theory. As the reader will see, it is very important in this case.

Economic science in Marx's times did not possess the mathematical techniques of linear algebra, and this left its imprint on the language of *Capital*, in which Marx did not use models requiring the solution of simultaneous linear equations and inequalities (although a number of simple mathematical models expressed in the form of individual equations was used). Often, when a modern economist will use a system of linear equations, Marx gives only a numerical sample in tabular form. The difference between such samples and models is fundamental. The former serve only

[1] The concept of the price of production will be given below. It is useful, however, to describe this kind of price dependence on changes in value at this point in order to avoid returning to this issue.

[2] For details see: Karl Marx, *Capital*, Vol. III, pp. 205-206.

[3] We shall not consider this here. Let us note only that David Ricardo was inclined to identify value and price of production, which resulted in a lot of contradictions. Marx showed that the price of production does not coincide with value, but is a converted form of it.

to illustrate the conclusions from the logical analysis, while the latter are secondary subjects of study, the primary ones being the actual economy itself. The properties of the models are examined with the help of sophisticated mathematical methods, which is an important means for understanding (and not just illustrating) the properties of the economy.

To give a more graphic exposition of his theory of prices of production as a converted form of value Marx first of all used numerical examples in tabular form.[1] In this form, he succeeded in presenting some fundamental points of the issue, in particular the deviation of the price of production from value in industries where the organic composition of capital differs from its average composition in the economy as a whole. The differences between industries in the speed of circulation of the capital advanced in them were also represented in tables: the part of constant capital *c* consumed during the year forms an unequal share in the volume of it advanced; however, the differences between industries

The Numerical Example Presented in Chapter IX of Volume III of Marx's *Capital* to Illustrate the Transition of Value to the Price of Production

Capital	Rate of surplus-value, %	Surplus-value	Rate of profit, %	Used up c	Value of commodities	Costs of production	Total rate of profit, %	Price of production of commodities	Price-from-value deviation
I. 80*c*+20*v*	100	20	20	50	90	70	22	92	+2
II. 70*c*+30*v*	100	30	30	51	111	81	22	103	—8
III. 60*c*+40*v*	100	40	40	51	131	91	22	113	—18
IV. 85*c*+15*v*	100	15	15	40	70	55	22	77	+7
V. 95*c*+5*v*	100	5	5	10	20	15	22	37	+17
In total 390*c*+110*v*	100	110	22	202	422	312	22	422	0

Source: Karl Marx, *Capital*, Vol. III, pp. 156-57.

[1] See: Karl Marx, *Capital*, Vol. III, pp. 154-72.

in the speed of circulation of variable capital were not shown (although this could, in principle, be illustrated in tabular form as well). Of fundamental importance is Marx's attempt to take into consideration industries that differ in the speed of circulation of capital.

Let us present a calculation, made by Marx himself, combining two his tables in one (see the Table above).

An essential shortcoming of this illustrative calculation is as follows: the capital advanced, and, accordingly, the costs of production, are presented in value terms. At the same time, prices of production differing from value were obtained, but the productive capital advanced (both its constant and variable components) is formed in all industries by purchasing appropriate commodities—means of production and labour power. Since commodity prices tend to approach the prices of production, the constant capital advanced should be presented not as value, but as the sum total of the prices of production of the appropriate means of production, and the variable capital advanced not as value, but as the sum total of the prices of production of consumer goods serving to reproduce labour power. The price of production is not the sum total of capitalist costs in value terms plus average profit. It is made up of the sum total of costs in terms of the prices of production and of average profit. Such a calculation cannot, however, be presented by a simple illustrative table. It presupposes the solution of simultaneous equations (moreover, strictly speaking, non-linear ones), and so a transition to a mathematical model of the phenomenon.

It should be emphasised that the analysis of the illustrative tables was, to Marx himself, the very point of departure of the study. Moreover, he pointed out directly, unequivocally and repeatedly that shortcoming of this stage referred to above. He began to make the necessary corrections by Chapter IX of Volume III of *Capital*. In particular, Marx said: "We had originally assumed that the cost-price of a commodity equalled the *value* of the commodities consumed in its production. But for the buyer the price of production of a specific commodity is its cost-price, and may thus pass as cost-price into the prices of other commodities. Since the price of production may differ from the value of a commodity, it follows that the cost-price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value

derived from the value of the means of production consumed by it."[1]

Bearing in mind the effect of the deviations of prices of production and of the means of labour power existence from the corresponding values, Marx made a general conclusion: "Under capitalist production, the general law acts as the prevailing tendency only in a very complicated and approximate manner, as a never ascertainable average of ceaseless fluctuations."[2]

This is precisely how, i.e., only rather approximately, the equality of profits and surplus-value is achieved in capitalist society. The idea of the existence of their strict equality resulting from a consideration of illustrative tables is but the initial approach to determining their real ratio.

The next step in the analysis becomes possible if the formation of the price of production is represented in the form of simultaneous equations. Although Marx himself did not do this, he led the study to the verge of this step by concluding that capitalist costs in all industries should be evaluated in terms of the price of production of the commodities making up these costs.

Now let us consider the problem in that form. Here we shall use the entire set of conditions under which the formula for the magnitude of value (2.22) was obtained. At this stage of the analysis, the very existence of the prices of production as the level of average prices towards which they actually tend to gravitate is assumed as given, i.e., is taken as an axiom resulting from observation of facts. The question of why the centre around which prices fluctuate must be clarified specifically, by analysing the characteristics of free competition capitalism. Here only the following question has to be discussed: how the theory according to which profit is based on exploitation of the working class (the theory of surplus-value) can be compatible with that according to which the centre around which prices fluctuate is formed by the prices of production. This is Marx's logic.

The time of production differs in the different industries. In baking, let us say, the technological process takes just

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[1] Karl Marx, *Capital*, Vol. III. pp. 164-65.

[2] Karl Marx, *Capital*, Vol. III, p. 161.

The law of value, the law of the general rate of profit etc. operate only approximately on the average. See "Engels to Conrad Schmidt in Zurich", in: Marx, Engels, *Selected Correspondence*, Progress Publishers, Moscow, 1975, p. 457.

hours, whereas in plant-growing in countries with a temperate climate—almost a year. Yet everywhere it is more than 0. That is why any technology requires means of production and labour power to be advanced as preconditions for obtaining output. Moreover, as already stated, many kinds of instruments of labour are advanced for a number of successive acts of production, rather than for a single one. The same is true with some objects of labour.

At the same time, the bulk of kinds of objects of labour, some instruments of labour and labour power have to be advanced all over again for each act of production. Under capitalism, these requirements of the technologies are met by advancing capital, different elements of capital advanced being withdrawn and replaced at different speeds: some of them are completely expended in one act of production and, therefore, completely replaced physically after each act; others—after a number of such acts, but not less than once a year; the rest only during a series of acts of production that takes several years. These differences between the physical components of the capital advanced can be represented with the aid of a special system (matrix) of coefficients a_{ij} to characterise the speed at which element i in industry j is physically replaced (the number of events yearly); similar indicators also exist for labour power, here denoted by a^l_j. Coefficients a_{ij}, and a^l_j obviously depend on the specific features of each industry j (in particular, on the time of production in it) and on the specific features of the way the resource in question is used technologically.

Besides the matrices and vectors mentioned in Chapter 2, one more matrix $a = \{a_{ij}\}$ and vector $a^l = (a^l_1, \ldots, a^l_n)$ are thus employed to describe social production technologically and economically.

Magnitudes Q_j, Y_j and, accordingly, coefficients a_{ij}, l_j and, therefore, coefficients a_{ij}, a^l_j have physical units of measurement. It is possible to turn them into monetary units only with the aid of prices.

The prices of production of usual commodities will be denoted by vector $\bar{p}$, $\bar{p} = (\bar{p}_1, \ldots, \bar{p}_n)$. To denote the price of labour power, we shall employ symbol $\bar{p}^l_j$, assuming that

$$\bar{p}^l_j = \sum_i \bar{p}_i a^l_{ij} \quad j = 1, \ldots, n. \qquad (3.25)$$

By definition, the prices of production are equal to capitalist costs plus average profit, the costs themselves being measured by the same prices of production. Expressing this definition formally, we obtain:

$$\bar{p}_j = \sum_i \bar{p}_i a_{ij} + \bar{p}_j^l l_j + \frac{(C_j + V_j)}{Q_j} r, \quad j = (1, \ldots, n). \qquad (3.26)^1$$

Here $C_j = \sum_i \frac{\bar{p}_i a_{ij} Q_j}{\alpha_{ij}}$ is the capital advanced in industry j and embodied in the means of production (constant capital); $V_j = \frac{\bar{p}_j^l l_j Q_j}{\alpha_j^l}$ is the capital advanced in industry j and embodied in labour power (variable capital); r is the rate of profit, the same for all industries (in relation to the capital advanced).

Formula (3.26) implies, in accordance with the sense of (3.25), that coefficients l_j have the following dimension: annual workers per unit output.

The relation between Marx's theory of surplus-value and the theory of average profit could be understood by comparing expressions (2.16), (2.25), (3.25), (3.26), these being comprehended socially.

It has been shown that, on the scale of the economy as a whole, surplus-value is nothing but the value of surplus-product produced by the class of proletarians [see expression (2.46) and the comments on it]. At the same time, the sum total of the profit received by the class of capitalists is merely the sum total of the prices of the same commodities making up the surplus-product on the scale of the economy as a whole. This is true of any price system, including the prices of production $\bar{p}_j$.

Multiply both terms of expression (3.26) by the quantity of commodities Q_j, $\bar{p}_j^l$ being transformed according to (3.25):

$$\bar{p}_j Q_j = Q_j \sum_i \bar{p}_i a_{ij} + Q_j l_j \sum_i \bar{p}_i a_{ij}^l + (C_j + V_j) r;$$

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[1] This is a non-linear system because the unknown r is, in the last term, multiplied by the unknown $\bar{p}_i$.

A solution to the system can be obtained for all $\bar{p}_i$ and r if vector $\bar{p}$ is considered to be a positive eigen-vector and r to be the root of the matrix built upon the coefficients of system (3.26). In accordance with this, let us note that this matrix is determined for an actually existing capitalist economy as a productive one, which is why a solution (the sole one) exists such that $\bar{p} > 0$, $r > 0$.

now sum up the commodities produced in price terms:

$$\sum_j \bar{p}_j Q_j = \sum_j Q_j \sum_i \bar{p}_i a_{ij} + \sum_j Q_j l_j \sum_i \bar{p}_i a^l_{ij} + r\sum_j (C_j + V_j). \tag{3.27}$$

Compare this expression with expression (2.16), the components of vector Y being represented here in accordance with (2.45) and both terms being multiplied by the vector of the prices of production:

$$\sum_i \bar{p}_i Q_i = \sum_i \bar{p}_i \sum_j a_{ij} Q_j + \sum_i \bar{p}_i Y^l_i + \sum_i \bar{p}_i Y^s_i. \tag{3.28}$$

Since $i, j = (1, \ldots n)$, it is true that:

$$\sum_j \bar{p}_j Q_j = \sum_i \bar{p}_i Q_i;$$

$$\sum_j Q_j \sum_i \bar{p}_i a_{ij} = \sum_i \bar{p}_i \sum_j a_{ij} Q_j.$$

Magnitudes Y^l_i have been determined above as the quantities of goods of kind i to be consumed by the working class as a whole [see (2.39)]. Then:

$$\sum_j Q_j l_j a^l_{ij} = Y^l_i, \quad i = 1, \ldots, n, \tag{3.29}$$

hence

$$\sum_j Q_j l_j \sum_i \bar{p}_i a^l_{ij} = \sum_i \bar{p}_i Y^l_i.$$

Then, it follows from (3.27) and (3.28) that:

$$r\sum_j (C_j + V_j) = \sum_i \bar{p}_i Y^s_i. \tag{3.30}$$

The mass of profit on the scale of the economy as a whole is equal to the sum total of the prices of the surplus-product in which the surplus-labour of the working class is embodied, i.e., the mass of the surplus-value created in society as a whole. It is this that underlies the concept of average profit as converted surplus-value and, accordingly, of the price of production as converted value. The logical mutual compatibility of the concepts of exploitation of proletarians and of the prices of production is expressed by a precise mathematical formula: the sum total of the magnitudes of average profit embodied in the prices of production of commodities equals that of the prices of production of commodities produced by the working class's surplus-labour.

The mass of surplus-value in a capitalist economy is the summed value of the surplus-product. The mass of profit is, in this society, the total price of the same surplus-product, the latter being nothing but the embodiment of surplus-labour. This means that profit is but a form assumed by surplus-value.

The following objection may be raised here. The concept itself of the total expenditure of resources per unit final output exists not only in reference to labour: there exists a mathematically similar total input of any kind of material resource. The total specific volume of any kind of capital advanced (i.e., capital advanced in the whole economy per unit net output of any individual industry) can also be calculated. In any case, it is true that

$$d_R Q = D_R Y, \tag{3.31}$$

where d_R is the vector of the coefficients expressing direct input or direct advancing of resource R to produce a unit of output; D_R is the vector of the corresponding total coefficients.

Formula (2.25) is a particular case of (3.31), where R are resources of labour power. Why is the concept of exploitation applied precisely to this, and only to this resource?

This question has already, in fact, been answered, since exploitation has been shown to be a social phenomenon expressing relations of ownership. We shall, however, give a direct answer to the above objection.

Although an equality of type (3.31) exists for all resources, it does not follow that they are replaced from net output Y. On the contrary, every kind of material resource without exception is, in fact, replaced from a component of output $H = Q - Y$. In accordance with this, the following is first of all true of the whole *input* of material resources R:

$$d_R Q = D_R Y = H_R.$$

In other words, the expenditure of these resources is embodied in H rather than in Y, although it is carried out ultimately to obtain Y. In such a case, Y represents the part of Q remaining after subtraction of the part in which the expenditure of all kinds of material productive resource is em-

bodied. Just for this reason, Y appears as the embodiment, exclusively, of social labour input.[1]

As for the *capital advanced*, insofar as it is not expended, it is not, of course, embodied in output. To the same extent, however, it remains embodied in its original physical form. That is why no special part of output is produced to replace it. If Y^s is capitalised, i.e., turned into the capital advanced, the latter's amount can only increase. The absence of any part of output to embody the capital advanced as such (i.e., taken in the part not expended on the production of commodities) is thus demonstrated quite obviously.

So the capital embodied in the product is completely replaced, and returns to the class of capitalists, while the labour embodied in it is not completely replaced for return to the working class: the total labour input in the commodities received by the working class is lower than the amount of labour this class donates to production. The rest of this class's labour is embodied in commodities received by the capitalists over and above replacement of full capital input in production, while they retain ownership of the elements of the capital advanced but not expended. It is precisely these relations between the two classes that imply exploitation.

Formula (3.30) demonstrates very clearly the validity of Marx's well-known conclusion, drawn from the transformation of surplus-value into average profit: in this form, the profit of each capitalist appears in its dependence on exploitation not only of the workers he himself hires, but also of the working class as a whole by aggregate capital.[2] "Here, then, we have a mathematically precise proof why capitalists form a veritable freemason society vis-à-vis the whole working-class, while there is little love lost between them in competition among themselves."[3]

Two conclusions follow from (3.30) specifying the concept of average profit as transformed surplus-value.

First, if surplus-labour is identically equal to 0, the average

[1] See Marx's similar analysis of the concept of exploitation in connection with his study of the expression of the value of output in relative shares of it (first in paragraph 2, Chapter IX, Volume I of *Capital*, then, in reference to the social product as a whole, in Part III, Volume II of *Capital*). Note that the turning of Y^s embodying surplus-labour L^s into the property of capitalists is considered to be exploitation, rather than the appropriation of Π.

[2] Karl Marx, *Capital*, Vol. III, pp. 168-70.

[3] *Ibid.*, p. 198.

rate of profit r must be identically equal to 0. Indeed, surplus-labour

$$L^s \equiv wY^s \equiv 0$$

if, and only if, all components of Y^s equal 0 (all $Y_i^s \equiv 0$), since the vector of total labour input $w > 0$, i.e., all $w_i > 0$.

If, however, all $Y_i^s \equiv 0$, the right-hand term of expression (3.30) equals 0. As the advancing of capital for productive activities is unavoidable [i.e., $(C_j + V_j) > 0$ for all j], it follows that $r = 0$.

This all means that, if surplus-labour is identically equal to 0 (i.e., society does not experience surplus-labour, because there is no surplus-product), *surplus-value is also identically equal to zero and, at the same time, there are zero profits as well.* Yet, by definition, capital exists only on the condition that (on the scale of society as a whole) profit is strictly greater than 0. That is why, given the former condition, capitalist production does not exist, nor do the prices of production. In other words, the very problem under consideration itself no longer exists if $Y^s \equiv 0$, i.e., $L^s \equiv 0$.

Second, if surplus-labour is greater than 0, the average rate of profit r is strictly greater than 0 as well. Indeed, surplus-labour

$$L^s \equiv wY^s > 0$$

if, and only if, at least some $Y_i^s > 0$. Then, however, we obtain, in the right-hand term of (3.30):

$$\sum_i \overline{p}_i Y_i^s > 0,$$

which is why, in the left-hand term, $r > 0$.

This all means that profit cannot be equal to zero, i.e., it is positive if surplus-value is identically positive. In these circumstances, capitalist production exists, together with the price of production (free competition being an additional condition). The very existence of the prices of production implies that $r > 0$, i.e., $r > 0$ is involved in the concept of such prices, in their definition. This concept is thus based on the concept of exploitation of the proletariat.

All this merely means that average profit is nothing but transformed surplus-value, the prices of production being, accordingly, the transformed labour-value of commodities.

The transformation can and should, in practice, be associated with the differences in the proportions in which the

sum of the prices of production $\bar{p}Q$ and that of the values of output Q (represented by wQ) are divided into the replacement fund and national income, and the latter into the share of the working class and that of capitalists. In other words, in the general case

$$wH : wY^l : wY^s \neq \bar{p}H : \bar{p}Y^l : \bar{p}Y^s, \qquad (3.32)$$

as a proportionality of the prices of production of various commodities to their values is virtually impossible. This fact follows sufficiently clearly from comparing expression (2.22), describing the formation of the total labour intensity of commodities, and expression (3.26), describing the formation of the prices of production. If the capital advanced per unit output, i.e., $(C_j + V_j)/Q_j$ were, in each industry, proportional to the direct labour intensity of output l_j, then, as can easily be seen from (3.26), the prices of production calculated by (3.26), would, of course, be proportional to the total labour intensities calculated by (2.22). Yet the specific features of the technologies in the industries, expressed through those in the relations between C_j and V_j in the capital advanced, render such a proportionality virtually impossible.[1]

Marx maintained precisely this in demonstrating that the prices of production regularly and steadily diverge from values owing to the specifics of the organic composition of capital in different industries.[2]

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[1] A comparison of the above formulae for C_j and V_j shows that these specific features are determined by the relations of a_{ij}/α_{ij} and l_j/α_j^l, i.e., they depend on the technological structure of the capitalist costs and speed of circulation of the various elements of capital, which ultimately determines the differences in the organic composition of capital in industries. As the physical forms of the means of production employed in different industries differ, the duration of the technological process also being unequal, it is completely improbable that

$$\frac{C_j+V_j}{Q_j} = \alpha l_j \bar{p}^l,$$

where α is the proportionality coefficient common to all sectors. Yet, since this is true, in the general case there also exists no h such that $\bar{p}_j = hw_j$, i.e., there is no proportionality of the prices of production to the value of commodities.

[2] In his analysis, Marx initially disregarded various industries' specifics of the circulation of capital, which meant that the differences in the organic composition were dependent only on industries' specific ratios of the amounts of the means of production to labour power employed in the technological process; differences in the speed of circula-

Expression (3.32) means that, given the evaluations in terms of the prices of production and value, in the general case there is no coincidence, first, between the two estimates of the share of national income in the gross social product and, second, between the two estimates of the shares of the classes in the national income. In particular,

$$\frac{L^s}{L^l}=\frac{wY^s}{wY^l}=m'\neq\frac{\bar{p}Y^s}{\bar{p}Y^l}, \qquad (3.33)$$

i.e., the rate of surplus-value m' finds no exact quantitative reflection in the evaluations obtained on the basis of the prices of production. The reason is that the physical structures of Y^s and Y^l are different (suffice it to point out that Y^s usually involves the means of production, while Y^l does not, the physical structure of the personal consumption of workers and capitalists also being different), which, subject to $\bar{p}_i \neq hw_i$, means that the deviations of the prices of production of commodities affect the sum total of the wages of the working class and the sum total of the profits of capitalists differently.[1]

Nevertheless, the approximate equality

$$\frac{L^s}{L^l}=\frac{wY^s}{wY^l}=m'\approx\frac{\bar{p}Y^s}{\bar{p}Y^l} \qquad (3.34)$$

should be observed.

The point is that Y^s is unlikely to consist only of commodities the prices of production of which deviate from value in only one direction (say, upwards) and Y^l of commodities, the prices of production of which deviate from value only in the opposite direction (i.e., downwards). In fact, both Y^s and Y^l, in the general case, involve commodities with prices of production deviating from value both upwards and downwards. Among the industries producing consumer goods there are ones with a higher or lower than average organic composition of capital advanced. The same

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tion were then involved in the study as an additional factor. In the general case, however, it is true that the organic composition of the capital advanced depends on this entire set of technological specifics of production, as can be seen from formulae for C_j and V_j.

[1] Note that, if the prices are proportional to values ($p_i = hw_i$), then (3.33) and, accordingly, (3.34) become strict equations irrespective of the differences in the structures of vectors Y^s and Y^l.

is true for capital goods industries.[1] Yet in such a case, the deviations of prices of production from values partly cancel themselves out both as to the sum total of $\bar{p}Y^s$ and to that of $\bar{p}Y^l$. Obviously, these magnitudes cancel themselves out in practice to different degrees, but it does happen and it thus approaches the evaluations of profits to surplus-value and those of wages to the value of the necessary product. The approximate correspondence expressed by (3.34) should thus be the case.[2] Yet just such, and only such, an approximate correspondence expresses Marx's own standpoint on this issue, which has already been demonstrated in the text.

Having now finished our formal exposition of the main points of Marx's theory of the price of production as transformed value, we shall additionally dwell on the following point.

The theory of value and of prices of production can be further developed provided the simplifying assumptions used to formulate it initially are successively removed.

One of these assumptions is that there is a one-to-one correspondence between the kinds of output and industries. The general and realistic case is that particular capitalist firms, while being narrowly specialised, are not, nevertheless, monoproduct ones. Accordingly, there are no monoproduct industries. Moreover, the whole set of firms cannot, in fact, be broken down into non-overlapping subsets representing industries; there inevitably exist firms representing a transition from one industry to another (although such a transition does not, of course, exist between every two industries). With regard to this circumstance, the Marx's remark quoted above acquires particular validity, pointing out that general laws tend to act in a very complicated and approximate manner, which applies especially to the prices of production. The concept of average profit is now modified: the average profit in the price of a given commodity is such that its sum total for all kinds and for the total quantity of commodities produced by each industry ensures equality of rates of profit. In formalised form:

$$\bar{s}_i = s_i \text{ so that } \sum_{i \in G_q} s_i Q_i/(C_q + V_q) = const \text{ for all } q, \qquad (3.35)$$

[1] These two sets of industries partly overlap.

[2] Below we shall present data for Austria, Hungary and USA to demonstrate that this expectation is justified.

where s_i is the profit in the price of commodity i; $\bar{s}_i$ is the average profit, as a component of the average price of the same commodity; q is the symbol representing the industry that now is no longer in a one-to-one mutual correspondence with a particular kind of commodity; C_q is the set of indices i that form industry q.

The above obviously applies not only to the formation of average profit and, accordingly, the price of production, but also to that of prices in general. Formula (3.35), in particular, demonstrates the principle on the basis of which the theory of value solves the problem of so-called co-products (the production of which is not divisible technologically; for example, cattle-breeding for slaughter simultaneously yields meat of various sorts, hide, horns, hooves, etc.). As applied to such industries, the concept of value as the law of prices (if it is assumed to act directly) boils down to the following:

$$\sum_{i \in G_q} p_i Q_q^i = hW_q.$$

Here, Q_q^i is the quantity of co-products of kind i produced by industry q; W_q is the total socially necessary labour input in the commodities of industry q; h as above. This requirement can obviously be met by various price systems; in other words, the law of value does not determine the price level of co-products uniquely (nor, for this reason, of those for which these co-products serve as means of production). Here we are again, this time from a specific angle, faced with the fact that value is not something strictly fixed, but fluctuating and changing. At the same time, it determines rather narrow limits for the sum total of the prices of the products of a certain industry in the short term.

Taking into account the existence of co-products, the specific value of which cannot be determined in a definite way,[1] it should be concluded that the essential, most general expression of the law of value is that referring not to the whole value, but to the newly created value. Then, the prices p_i directly corresponding to value are such that:

$$\frac{l_q Q_q}{\sum_i p_i Q_q^i - \sum_i p_i a_{iq} Q_q} \to const \text{ for all } q, \qquad (3.36)$$

[1] Value W_q can be divided among the quantities of commodities Q_q^i proportionally to the sums of their market prices $p_i Q_q^i$, but this is, of course, purely a method of computation.

where Q_q is the quantity of the sets of co-products of industries q, l_q and a_{iq} are, correspondingly, the living labour input reduced to simple labour and the material inputs per set. (One set may, of course, include a single kind of commodity as well.)

We shall not return to the problem of co-products again.

It is obvious that, in the general case, as given in formula (3.35), average profit not only does not coincide with surplus-value, but it is not even determined uniquely as an element of the price of production of particular commodities. In each industry, any cut (within or even below costs) of the price of some commodity at the expense of a reduction in the profit can be cancelled out by raising the price and, accordingly, the profit from another commodity. It becomes especially clear that the price of production represents only the centre of fluctuations, the general law of prices, but certainly does not determine the specific prices of commodities.

The patterns of these specific prices require further study, but this cannot affect the main tenets of the theory of prices of production as transformed commodity value.

The profit of merchant capitalists, interest, taxes, ground rent. The study of the costs of circulation leads immediately to a conclusion essential for understanding the theory of value in general: this theory does not maintain that the sphere of production replaces, on the market, the total expenditure of labour embodied in the commodities created in it. The totality of commodity producers actually inevitably receives less value from circulation than it puts into it at least by magnitude wY^c [see (3.18)].

Particular kinds of capital specialising in the sphere of circulation exist in capitalist society: merchant and loan capital (industrial capital is, in one way or another, also engaged in these activities). These two special spheres first of all reimburse the circulation costs and, besides, provide a profit. Without giving a detailed description of the formation of the costs of loan capital, let us merely note that (similarly to those of circulation) they are covered by surplus-value. A part of surplus-value created in the sphere of production is also redistributed in favour of merchant and loaning capitalists to form their profit. The mechanism of these processes (the multi-stage prices from wholesale prices of production to retail prices, and loan interest) is well known. It is important to us here only that D (the profit

of merchant capitalists), P (interest, which serves to cover the expenses of loaning capitalists and forms their profit), everything being expressed in terms of labour, are formed from surplus-value besides wY^c:

$$L^s \geqslant wY^c + D + P. \tag{3.37}$$

To a certain extent, the government taxes are similar in nature since they are used, ultimately, to purchase commodities; for this reason:

$$L^s \geqslant wY^c + D + P + U, \tag{3.38}$$

where U is the total labour input in the commodities bought out of taxes. At the given stage of the analysis, the form in which taxes are paid, and the fact that they are levied not only on the agents in the sphere of production, but also on those of the sphere of circulation, are of no importance.

The totality of *producers* thus, in fact, regularly obtain strictly less value from circulation than they put into it, the upper limit of this difference (the source for covering corresponding nonproductive costs) being surplus-value. This even applies to simple commodity production.[1] The difference between capitalist and simple commodity production is thus that the branches of capitalist *production* extract more value from the market, in the form of commodities, than is necessary simply to renew their conditions of production.

Above we have derived a formula to express, in a first theoretical approximation, the fact of the transformation of surplus-value into average profit, the prices being formed according to the principle of prices of production:

$$r \sum_j (C_j + V_j) = \sum_i \bar{p}_i Y_i^s,$$

where r is the average rate of profit. The formula was derived subject to the assumption that wholesale and retail prices do not differ, credit and rent being ignored. If these economic phenomena are taken into account, the formula should be presented as an inequality

$$r \sum_j (C_j + V_j) < \sum_i \bar{p}_i Y_i^s. \tag{3.39}$$

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[1] Note that, even if capital does not dominate production, it exists in circulation: its merchant and usury forms have existed since money appeared, at which moment commodity production came into being. That is why capital is inevitable with any form of private commodity production.

Yet this inequality also means that average profit is transformed surplus-value (the surplus-labour embodied in the surplus-product), though not a single, but one of a certain number of such forms that are limited in their totality by the amount of surplus-value.

Already in its precapitalist forms, commodity production turns out to be associated with specific rent relations between producers and landowners. We shall omit here a formal description of those initial forms, which can easily be carried out by the reader himself if desired. The study of the law of value requires, however, that such a description should be presented at least for the conditions of capitalist production.

Logically essential to an understanding of capitalist ground rent is the fact that the law of value, in the general case, allows for incomplete replacement by the market of the value of commodities produced by some industries (but not less than the value of the reproduction of means of production and labour power in these industries). This, in contrast, enables the other industries to extract more value from the market than they put into it. If such a redistribution is regularly enforced by certain stable circumstances, it is related to steady deviations of prices from the average value of commodities. In reference to industries that technologically essentially employ nonreproducible natural resources,[1] such a steady deviation (upwards) means the appearance of a steady additional profit and its transformation into ground rent, the concept of value here being itself modified as that of the socially necessary labour input in production.

The demand for the commodities of these industries (farming, mining) cannot be regularly met if natural conditions of only one single *quality* are employed, i.e., multigrade natural conditions must be involved in production. (The fact that these may include ones of low efficiency that could well be done without is disregarded.) Then, the whole set of natural conditions actually employed should be seen as socially necessary and, accordingly, the whole set of real technologies k, as soon as their differences are determined precisely by natural conditions. To analyse the problem in pure

[1] The concept *technologically essential* has the following sense: the differences in the natural resources employed affect technological coefficients a^k_{ij}, l^k_j and, for this reason, the individual productive input.

form it is assumed that, from the point of view of reproducible means of production and labour power, the firms employing these technologies are all at a normal social level.[1] In particular, marginal cost technologies α_j, (i.e., ones with the highest individual costs in the industry) are socially necessary ones such that

$$\alpha_j = k', \text{ then } w_j^{k'} = \max_k w_j^k, \ k \in E_j, \ j = 1, \ldots, n. \qquad (3.40)$$

Since this is the case, the market shall replace productive input and the capitalist market of the age of premonopoly competition—production costs together with average profit, i.e., realise the commodities according to the corresponding prices of production. Given unified prices of similar output, this means the appearance, for the rest of technologies k of farming and mining, of a steady extra profit, which is, to a certain degree of accuracy extracted by landowners as rent payments, which form differential ground rent.

In this case the marginal costs, whilst being socially necessary, exceed the average ones. Therefore it is true:

$$w_j^{\alpha_j} Q_j > w_j Q_j \qquad (3.41)$$

where j are the indices of farming and mining; $w_j^{\alpha_j}$ is the value of the output answering the social properties of private commodity production. The rental component of this value is, in fact, not a materialisation of some actual labour by the employees in the above industries; it is but a result of the redistribution of surplus-value created in other industries in favour of the agents of these ones. That is why Marx pointed out that we are dealing here with one form surplus-value assumes in society, and called the

[1] Let us point out briefly that the worst technologies (i.e., such k that $w_j^k > w_j$) in this case differ from those discussed above: the worst *natural* conditions, as far as their use is needed to cover demand, are not ousted from the technological system of society (at least over a fairly long period of time). Society does not reproduce the best and medium-grade natural conditions (as, by definition, it does not reproduce natural resources at all). As for the *reproducible* conditions of production, precisely as such they can be relatively quickly diffused and, for this reason, the producer's demand switches constantly over from the worst technologies to the best and medium-grade ones. This means that the worst technologies (if their deficiency is merely a consequence of backwardness with regard to reproducible resources) are not among the socially necessary ones, even if their products are bought,

corresponding component of the value of the commodities of these branches "false social value".[1]

The total mass of value making up differential rent in α_j industry j is:

$$DR_j = (w_j^{\alpha_j} - w_j) Q_j, \tag{3.42}$$

where j are the above industries. For a particular firm k, the differential rent is:

$$DR_j^k = (w_j^{\alpha_j} - w_j^k) Q_j^k, \quad k \in E_j. \tag{3.43}$$

Obviously, for $\alpha_j \in E_j$ $DR_j^{\alpha_j} = 0$.

Under certain conditions known in economic theory, there also exists absolute rent, which is nothing but a redistribution of part of the surplus-value created in capitalist society in favour of landowners. Let us denote it by AR_j. Then the total value of the whole rent (in value terms) is:

$$R_j = DR_j + AR_j \tag{3.44}$$

(j are the indicated industries). The total sum of rent in society is:

$$R = \sum_j R_j. \tag{3.45}$$

Rental payments are actually made in monetary form. Since it is generally true that the prices of commodities are not the same as (proportional to) their values and, what is more, in the general case, fiat money not exchangeable for real money (gold) can be used, the actual mass of value arriving in the hands of landowners can be determined only as the value of the commodities they acquired for rental

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[1] "This is determination [of market price] by market-value as it asserts itself on the basis of capitalist production through competition; the latter creates a false social value... What society overpays for agricultural products in its capacity of consumer, what is a minus in the realisation of its labour-time in agricultural production, is now a plus for a portion of society, for the landlords" (Karl Marx, *Capital*, Vol. III, p. 661).

It is obvious that differential rent (together with absolute rent) is limited by the mass of the total social surplus-value and is actually always smaller. Hence, it follows that the diffusion of specific capitalist forms of farming implies a sufficiently high development level of capitalist production in general, i.e., a sufficient level of exploitation of the proletariat in society as a whole. After all, although differential rent is defined as $\sum_j (w_j^{\alpha_j} - w_j) Q_j$ (j are the above industries), the condition $wY^s > wY^R$ is obligatorily met, where Y^R are the commodities acquired at the expense of rental payments.

payments; according to the economic sense of the relations under discussion, these commodities are part of the social surplus-product. For this reason, the following determination of the actual volume of surplus-value contained in ground rent is used:

$$R = wY^R, \qquad (3.46)$$

where Y^R is the vector of the commodities purchased on ground rent.

Only now can the part of the surplus-value J that forms the net profits of producers be determined:

$$J = wY^s - w(Y^C + Y^D + Y^P + Y^U + Y^R), \qquad (3.47)$$

where Y^D, Y^P, Y^U are the vectors of the commodities purchased on the monetary profits of merchant capitalists, interest on credit, and taxes.

Accordingly, the final form of the simultaneous equations is constructed to describe the formation of the wholesale prices of production for industrial capitalists:

$$\bar{p}_j = \sum_i \bar{p}_i a_{ij} + \bar{p}_j^l l_j + \frac{C_j + V_j}{Q_j} r, \quad i, j = 1, \ldots, n \qquad (3.48)$$

$$r \sum_j (C_j + V_j) = \sum_i \bar{p}_i Y_i^J.$$

Here, Y^J is the vector of commodities bought out of profits. The level of average profit depends not only on the surplus-product Y^s and, accordingly, on the surplus-value L^s, but also on their distribution between the state, merchant and loaning capitalists, and landowners, which, in turn, provides an additional source of indeterminacy in the formation of the prices of production.

The laws governing the distribution of surplus-value between the various goals listed above require further formal studies. This is outside the bounds of an exposition of the main points of the theory of value.[1]

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[1] In passing, let us point out merely the following circumstances. As there exists a possibility of redistributing surplus-value among the various economic agents and, thus, the various price levels arise (wholesale with various gradations, and retail prices), and regular deviations of price from value take place, exchange ceases to be strictly equivalent (the merchant capitalist buys commodities for a price lower than value; the prices of production of some commodities are lower and of others higher than value, etc.). As for real

Involving nonreproducible resources in the system of equivalent exchange. Only now, after having discussed the problem of dividing surplus-value, it is possible finally to describe the principle behind the formation of the prices of natural (nonreproducible) resources. To do so means to show how these resources, having become commodities, are included in the system of equivalent exchange, although they have no value. Remember that the general quantitative determination of equivalent masses of different commodities appears as follows: quantities q_j and $q_{j'}$ of commodities j and j' arrive on the market as equivalent ones provided if,

$$p_j q_j = p_{j'} q_{j'}.$$

The sale of a natural resource, like that of any other commodity, is, by definition, its transfer to the buyer's ownership. The condition for it is, therefore, the receipt of equivalent property in exchange. In contrast to usual commodities that are not use-values to their producers, under capitalism natural resources serve their owners as a specific social use-value: they can be leased to entrepreneurs provided that the proprietor receives rent. In turn, the rental payment (if the fact is disregarded that, together with the plot of land, the capital invested in it is leased and interest is paid on it in form of rent) is, in fact, ground rent—differential and absolute. It is not important how accurately the sum total of rent corresponds to the real quantity of rent: the deviations are, in this case, similar to those of concrete prices around their normal level. The normal level of rent for a natural resource as such is the rent yielded as a component of the price of the output produced on the basis of that resource (output of agriculture or mining).

The sale of a natural resource is that of the possibility to receive income, i.e., rent. At the same time, money under capitalism possesses a similar ability to yield an income to its owners. Loan capital does the same, being again an expression of the specific social use-value of money, appearing

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money (gold and other precious metals), however, the possible violation of strict equivalence is fairly limited: an ingot of gold of a certain karat weighing 1 kilogram is equivalent only to an ingot of gold of the same karat weighing 1 kilogram. Here the incompatibility occurs in the requirements of real price-formation, which seems to explain why real money is ousted from the processes in which it serves the current exchange of commodities. The contradiction is the stronger, the greater is the share of surplus-value in the whole value of commodities.

as a result of its transformation into capital. Interest constitutes the price paid by the recipient of the loan to the owner of the money for the possibility of using, for a time, its ability to yield extra money, i.e., profit (i.e., the price of money as a special, capital, commodity). Then the ownership of money as such is realised precisely in the fact that it yields interest. It can be said that interest is rent yielded by ownership of money, and rental payment is rent yielded by ownership of land. Hence the principle of equivalent exchange of plots of land for money (i.e., the principle of the formation of prices for plots of land): the possibility of obtaining rental payment is exchanged for that of obtaining interest equal to it in monetary terms. The formula of the price of a plot of land j is ($j = n + 1, \ldots,$ $\ldots, T$, where T is the last number of commodities that are purely natural objects)[1]:

$$p_j = \frac{\theta_j}{\omega} \cdot 100, \qquad (3.49)$$

where θ_j is the rent yielded by plot j; ω is the average level of loan interest established for a given period of time. Price p_j is, in the given case, the size of capital equivalent to plot of land j under given social conditions. Formula (3.49) can easily be derived from the following relation of the equivalence of the sum total of loan interest and the sum total of rental payment:

$$\frac{p_j \omega}{100} = \theta_j.$$

Formula (3.49) already shows that the sale and purchase of land would inevitably lose its validity if the conditions determining them were themselves strictly determined. Indeed, if the sum total of rental payments and that of interest on capital are given and invariable, the exchange of land for capital is meaningless for both parties. The point is, however, precisely that formula (3.49) determines a certain normal price, whereas the parties are likely to expect a profit owing to the instability of both θ_j and ω. For this reason, the sale and purchase of land contain, from the very beginning, a speculative point. There are reasons for assuming that, as capitalism develops, it by itself affects the prices

[1] The division of the surface of the Earth into such specific commodities is, of course, arbitrary, but at each given point it is somehow justified.

of plots of land by changing them as compared with formula (3.49).

Anyhow, the prices of natural resources are based (as interest is based on the prices of money as capital) on the income anticipated from them, the latter being, as shown above, part of the surplus-value created by the working class.[1]

That is why the introduction of the sale and purchase of land and of money as capital into the system of equivalent exchange is based on the fact that this system has developed with respect to usual products. Accordingly, the theory first considers the prices of usual commodities and only then, on this basis, interest, rent, and the price of land.

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[1] Or, prior to capitalism, part of the surplus-product always created by labourers and transformed into commodities.

Chapter 4

STATISTICAL VERIFICATION OF THE LAW OF VALUE

Now we shall proceed to actual verification of the validity of the theory of value and surplus-value. This corresponds to the most profound principle of its construction. Like any really scientific theory, the Marxian political economy of capitalism is a generalisation of vast factual material. As Lenin put it, when describing Marx's economic theory, "testing by facts or by practice respectively, is to be found here in *each* step of the analysis".[1]

The law of value from which all the above theoretical conclusions are derived is directly, as such, the law of commodity prices. That is why its validity should be verified first of all using price statistics. Statistics of today being much more advanced than those of the 19th century on which Marx relied, modern mathematical models of the economy and mathematical-statistical methods of data analysis offer very ample new opportunities for actually verifying economic theories. In particular, it is possible to assess quantitatively the degree to which real prices correspond to value, and their fluctuations around value. Obviously a theory is the stronger, the greater the degree of accuracy with which it explains multiple facts, on which also its predictive power depend.

4.1. Requirements of the Law of Value Subjected to Statistical Verification

In order to make the above assessment, the theoretical statements should be translated into the language of statistics. Some repetitions are inevitable here, but will be kept

[1] V. I. Lenin, "Plan of Hegel's *Dialectics (Logic)*", *Collected Works*, Vol. 38, 1981, p. 318.

to the minimum and provide the reader with a concentrated exposition of the positions to be tested in a form corresponding to statistics specifics.

We have derived formulae to express the law of value in both its static and dynamic respects with reference to both total value and value added by living labour only. The method of their statistical approximation will be revealed as we turn to the corresponding sources of data. Let us begin with the most representative source—national accounts statistics. These allow us to verify the operation of the law for a sufficiently large number of successive years—although only in reference to net prices, i.e., to the excess of commodity prices over material input in money terms.

The initial stages of verification will apply to the action of the law in a static situation. The following mathematical expression of the law corresponds to this:

$$\frac{L_j}{N_j} \to const \text{ for all } j, \tag{4.1}$$

where j is the index assigned to a certain kind of commodity in some classification ($j = 1, \ldots, n$, where n is the total number of industries in the classification); N_j is the national income realised by industry j:

$$N_j = M_j - c_j, \quad j = 1, \ \ldots, \ n;$$

M_j is the sum total of the returns on the commodities of each industry j; c_j is the material input in industry j measured in money terms; L_j is the direct input of living labour in industry j (in working years). Expression (4.1) means that the amount of labour expended to obtain a unit of gross income in money form (i.e., say, per $1,000 of gross income calculated as the difference between the sum total of the returns on commodities and the sum total of the money spent on replacing the means of production expended), this amount of labour has to be approximately equal in all industries.

Now we shall clarify three essential points that must be taken into account in statistically verifying the statement expressed by formula (4.1).

First, capitalism in general and modern monopoly capitalism in particular see law-governed shifts of the centre around which prices fluctuate. Under premonopoly capitalism, the centre was the price of production only in exceptional cases coinciding with the value quantitatively. Under

monopoly capitalism, the law of the prices of production does not lose its validity, but its operation becomes more difficult owing to the shift from free to monopoly competition. Anyhow, a strict correspondence of prices to the requirement expressed by formula (4.1) is not expected in theory, but it is expected that the deviations from this requirement themselves might be explained, at least in part, as a natural consequence of modifications of value. In particular, it may be expected that the gross income of industries from sale will, per unit of living labour input, be higher in industries with a higher organic composition of capital. From general considerations, an evaluation can be made of the extent to which prices might be shifted with respect to the level directly corresponding to value. Let us merely point to simple consideration in accordance with which price deviations from value cannot, as a rule, be very great.

In the modern economy, the fund for replacing expended means of production is approximately 0.5 of the gross product in value terms, not less than a third of the national incomes of the developed capitalist nations consisting of the necessary product created by proletarians. Thus, surplus-value the redistribution of which leads to deviations of prices from value accounts for not more than two-thirds of the national income, or a third of GSP. A part of surplus-value has actually been withdrawn as rent, interest, etc. Then the share of the part to be redistributed is apparently no more than 25 per cent in the total commodity price. The redistribution is arranged, however, so that a part of surplus-value has, in any case, to be realised in the industry in which it was created. In other words, hardly more than 15 to 20 per cent of total commodity value is really redistributed; this is the estimate of the maximum amount to be redistributed. For this reason, deviations of prices from value should not, on average, exceed 10 to 15 per cent. The calculations given below corroborate these expectations.

Second, the prices at which producers sell their commodities under capitalism do not include the whole commodity value: some part of value is redistributed in favour of merchant's capital in the form of the difference between the final sale prices and producer's prices; another part becomes state income through indirect taxes included in commodity prices, and the like. As already shown, the first requirement of the law of value is fulfilled, since industry j as a whole is not unprofitable, i.e., prices correspond to the

condition

$$p_j > \sum_i p_i a_{ij} + p_j^l l_j \text{ for all } j,$$

where p_j (p_i) are prices of commodities j (i); i, $j = 1, \ldots, n$; p_j^l is the price of labour power in industry j.

Here we shall verify the law of value primarily on the basis of statistical evidence characterising the net receipts of producers from commodities, namely, on data of the national income received by the various industries. It is expected that the part of newly-created value that is realised in the producer's price (the wholesale price at which the industrialist sells commodities to the merchant capitalist) will turn out, in different industries, to be approximately proportional to the living labour spent in them, deviations from this proportionality being, at least in part, associated with the law-governed modification of value. As the producer's price does not include the whole of surplus-value, this additionally decreases the possibilities for prices to deviate from value. The share of producers' costs in their receipts from commodities is higher than the estimate given above (two-thirds of total value), the possibilities for a redistribution of surplus-value among them being correspondingly smaller. Then it might be all the more expected that the prices of different commodities (without indirect taxes) will be sufficiently close to those proportional to value.

Price deviations from value prove, of course, to be, on average, the higher, the narrower the groups of commodities to be analysed: as the commodity groups are disaggregated, the share of the chance component in the price inevitably increases (it is seen easily that, in contrast, as commodity groups are aggregated, the chance deviations of prices above and below the law-governed level would, at least partly, cancel each other out).

Third, labour input should be measured with regard to labour reduction. In this case

$$L_j = \tilde{L}_j \psi_j, \tag{4.2}$$

where ψ_j is the rate of reduction of labour in industry j to that considered as simple, unskilled labour.[1] In accordance

[1] Only the skill groups of labour power by industry have to be distinguished here. For this reason, set $\{\lambda\}$ of kinds of labour and set $\{j\}$ of industries are the same, and one may speak of rates of reduction of labour according to industries, ψ_j, being a particular case of rates ψ_λ, included in the general exposition of the theory.

with the theory of the subject under study (see section 2.5), the ratios of wages can be taken to represent the rates of reduction of labour:

$$\psi_j = p^l_j / p^l_{j'}, \quad j = 1, \ \ldots, \ n, \tag{4.3}$$

where j' is the industry considered in the given classification as that of simple labour. Statistically, this is the industry with the lowest average wage.[1] (Below we shall take agriculture to be j').

The proposed method for determining the rates of reduction ψ_j is based on the theoretical thesis that, given the unity of the market for labour power, the price of the latter in terms of simple labour is equal for all industries. Marx made the validity of this statement dependent on the extent to which capitalism could dominate all economic relations and thus achieve unity of the labour power market. There are particularly considerable reasons for believing that such a trend is developing in the most developed capitalist country, the USA, today. (Below it will be clarified that, even so, the differences in the labour power markets by industry are of a certain importance. The main analysis will, however, be made assuming that the labour power market is single and, for this reason, the rates of exploitation of the workers in different industries, are equal.) We shall certainly take into account that there exist some remnants of precapitalist forms of production, especially petty commodity one, in agriculture.

It is easy to see that the ratios of $L_j\psi_j$ do not change if they all are multiplied by the same number $p^l_{j'}$ (the average wage in industry j'). Then we obtain:

$$L_j\psi_j p^l_{j'} = L_j \frac{p^l_j p^l_{j'}}{p^l_{j'}} = L_j p^l_j, \quad j = 1, \ \ldots, \ n, \tag{4.4}$$

[1] In each industry, including that considered to be j', labour of different complexity is, of course, usually employed. That is why ψ_j themselves are not calculated precisely in the above way. This does not matter, however, for the calculations to follow. It can easily be seen that a more accurate determination of simple labour will change only $p^l_{j'}$ in (4.3). This, in turn, will change the absolute values of L_j, calculated with the aid of (4.2), but not their ratios. In this case, however, the dispersion and variation of these magnitudes will be unchanged, and it is precisely these indicators that have to be analysed.

which is nothing but the sum total of wages in industry j. Below we shall employ formula (4.1), taking simply the sum total of remuneration by industry as its numerator. This corresponds to the point of the theory that newly-created value is proportional to the wages of the workers who have created it. This is the simplest determination of the numerator of the formula: the statistics used to record precisely the sum total of remuneration (wages and salaries) by industry.

The calculations with formula (4.1), when its numerator is determined by formulae (4.2) and (4.4), are equivalent from the mathematical angle, but they differ substantially as follows. In the former case, the correspondence of the realised national income by industry to the *input* of reduced living labour has to be tested directly; in the latter—it is the correspondence of the realised national income by industry to the *remuneration* for living labour in them. Labour input and remuneration of labour are, of course, different things. The above equivalence of statistical verification methods is justified only because the reduction of labour is approximated by the indicators based on remuneration.

For the sake of completeness of the analysis, calculations will also be given where reduction of labour is not taken into consideration, i.e., it is assumed that

$$L_j = \tilde{L}_j, \quad j = 1, \ldots, n, \tag{4.5}$$

where $\tilde{L}_j$ is the number of full-time equivalent year-round workers in industry j. Then formula (4.1) assumes the form:

$$\frac{\tilde{L}_j}{N_j} \to const \text{ for all } j. \tag{4.6}$$

Below, when referring to formula (4.1), meant are the calculations taking account of the reduction of labour (using the indicators of remuneration), and mentioning (4.6)—those simply using data on the number of workers.

Further problems connected with methods for the statistical verification of the other formulae expressing the law of value will be considered below, where we shall employ these formulae.

Statistical verification will be performed mainly with data for the USA.

4.2. Calculations with National Accounts Statistics of the United States: Large Sectors of the Economy

We have processed US national accounts statistics for the realised national income, employment, and remuneration of employees by sector for 34 years (1948 to 1981).[1] The calculations will be given in detail for 1972, which we chose because a detailed comprehensive calculation may be made for it with the whole system of statistical sources, including the input-output table. It is important that this was business boom year, in which the structural shifts in the economy and therefore price deviations from value as well are felt most. Finally, the year 1972 was chosen in USA statistics as the base year for constructing price indices and, correspondingly, the indicators of so-called constant prices.

For the rest of the years of the period, we shall give chiefly the resulting characteristics of the price deviations from close observance of formula (4.1). Using the techniques for processing statistical data demonstrated for 1972, the reader may easily verify all the results we give below.

Basic data. Let us begin with the large sectors of the economy (see the list in Table 4.1). Besides those we take into account, US national accounts statistics also include a large sector called "Finance, insurance, and real estate", which is excluded from our calculations because it does not create any commodity output, whereas the law of value is that of commodity prices. Statistics also record the item "Government and government enterprises". More than 85 per cent of the national income shown by this item is accounted for by salaries of public employees (civil and military), which again does not correspond to the concept of commodity output. The revenues of government enterprises are given, but they are almost wholly used (over 90 per cent) to compensate the employees. Thus, the requirement of formula (4.1) is observed here with a high precision due to the fact that profit is hardly noticeable, but such a situation is of no importance for verifying the action of the law of value because it only expresses the specifics of a fairly limited sector of the economy (it accounted for mere 1.5

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[1] National accounts statistics are used as published in: *The National Income and Product Accounts of the United States, 1929-74. Statistical Tables. A Supplement to the Survey of Current Business*, U.S. Department of Commerce, 1977; *Survey of Current Business*, U.S. Department of Commerce, July 1977, July 1978, July 1982.

per cent of the total final product in 1972). Lastly two non-commodity branches of services, "Non-profit organisations" and "Private households", have been disregarded. The other service branches are analysed because the prices of their output are theoretically affected by the law of value. Wholesale and retail trade is included as a special sector in the calculation. It is well known that this sector partly performs the function of continuing the process of production and partly the pure functions of changing the forms of value. The available information does not allow one to be separated from the other.

In this work, the action of the law of value in US international economic relations is not considered, so only the indicators of the so-called domestic national income are considered. The sectoral data on realised national income are taken by us without any changes (see Table 6.3 of the system of national accounts). The main components of this magnitude are compensation of employees, profit-type returns, and net interest, i.e., interest paid minus that received. Profit-type return is defined as including incomes of individual private owners, rent-type returns, corporate profits and the like.[1] Taken as the sum total together with interest these kinds of return correspond to the concept of total profit, which is then subdivided into profit of enterprise, interest, and ground rent.[2] The incomes of the owners who do not exploit the labour of others are, however, also included in the incomes of individual private owners, which seems to play an essential role in agriculture. The formation of the indicators for this sector will be discussed in particular.

As can be seen directly from Table 6.5 of the system of national accounts, compensation of employees in the sector "Agriculture, forestry, and fisheries" amounted, in 1972, to $6,287 million and the realised national income (according to Table 6.3) to $30,649 million. The ratio of these magnitudes was thus 0.205, while for the whole U.S. economy the ratio was 0.759. The dramatic deviation in the sector under study seems to result, at least partly, from two circumstances. First, ground rent is realised in this sector ulti-

[1] Analysis of the national income components can be performed with the data of Tables 1.13 and 6.1 of the system of national accounts.

[2] Some inaccuracy appears due to the fact that *net* interest is included in realised national income; the magnitude with the interests paid included would correspond better to the concept of total profit.

mately as the redistribution of the surplus-value created in society as a whole in favour of landowners. For this reason, ground rent inevitably decreases the ratio of remuneration to realised national income in the given sector and increases it in the other sectors. Second, the above ratio is also reduced owing to the fact that profit-type returns here include the whole income of individual owners, the including earned incomes of those who do not exploit the labour of others, which are, by their social nature, closer to wages than to profit. No calculation that would eliminate the work of these factors is possible with national accounts statistics. We chose the following method for making the calculations for the USA: incomes of individual farmers were added to the remuneration of employees in agriculture.[1]

Data on compensation of employees for the other sectors were taken without any corrections being made (see Table 6.5 of the national accounts system). Data on numbers of employees by sector (in terms of conventional full-time equivalent employees, see Table 6.8 of the accounts system) were also accepted without any adjustments.

The concept of value as the law of prices applies to actual (current) prices, which is why we have taken these statistical data for analysis. Compensation of employees is taken at its full initial level, before direct taxes. Provided the law of value acts directly, profit should, in accordance with the above theory, be proportional precisely in relation to initial wages.

Estimate of real price deviations from value-level prices. Table 4.1 displays in detail the calculation procedure for evaluating deviations of prices from the conditions expressed by formulae (4.6) and (4.1). In evaluating the validity of formula (4.6) the data of columns 1 and 2 are used. Column 4 indicates the number of full-time employees by

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[1] Labour income in agriculture is, of course, somewhat exaggerated in such a calculation because there are capitalists among individual owners Yet it should be born in mind that the use of seasonal labour power (often foreigners and illegal residents of the USA) in agriculture is, in this sector, one of the obstacles to the establishment of the law-governed level of wages about which Marx wrote. Part of the value of the necessary product has thus been transformed in agriculture into profit through overexploitation of the workers. The two errors in the estimates of labour income in this sector have opposite signs and so offset each other, at least partly. The national accounts statistics do not permit more accurate calculations.

sector per one million dollars of the realised national income. In most sectors this level does not vary very considerably: from 69 to 94 man-years per one million dollars. Deviations are significant in the sectors "Agriculture, forestry and fisheries", "Communication" and "Utilities". The average labour input for 9 sectors in obtaining an income as high as a million dollars amounts to 0.066 thousand man-years (the average from 9 numbers of column 4 without regard for sectoral shares). This magnitude denoted by $\bar{x}$ is shown in column 4, row 11. The deviations from it can be calculated by sector. For example, the deviation for manufacturing is 0.073 — 0.066 = 0.007 thousand man-years per million dollars. If 0.066 is accepted as the average level for the economy as a whole, then manufacturing deviates from it by 0.007, i.e., by 10.6 per cent. Such deviations can be calculated for each sector.

To evaluate the average level of the deviations we use a well-known statistical indicator—the coefficient of variation. The dispersion of the magnitude under study has to be determined:

$$\sigma^2 = \sum_j (x_j - \bar{x})^2/n,$$

where j is the index of the sectors (here $j = 1, \ldots, 9$); x_j is the value of the indicator under study in sector j ($x_1 = 0.040$, $x_2 = 0.070$, etc.); n is the number of sectors (here $n = 9$). The mean square variance $\sigma = \sqrt{\sigma^2}$. This characterises the mean absolute level of the deviations of the magnitude under study from its mean value (see row 12). Finally, the coefficient of variation characterises the relative level of this mean deviation: $v = \sigma/\bar{x}$ (see row 13 of the Table).

In this case (column 4), the coefficient of variation equals 0.268. The number of year-round full-time workers per 1 million dollars of the national income thus deviates by sector from the total level for the economy as a whole on average by 26.8 per cent.

This calculation was performed without regard for sectoral shares in the economy. It can easily be seen that the sectors the indicators of which are closer to the average have a relatively greater share. For example, the share of manufacturing in the total realised national income is 37.7 per cent while the deviation for it, as shown above, is not 26.8 but 10.6 per cent. In contrast, the sectors with considerable deviations from the average (agriculture, communication, utilities)

Table 4.1

Evaluating Price Deviation from the Value-Level Pri-

	Realised national income, million dollars	Number of full-time equivalent employees, thousands	Compensation of employees, million dollars	Ratio: col. 2 to col. 1 (man-years per million dollars)	Ratio: col. 3 to col. 1
	1	2	3	4	5
1. Agriculture, forestry, and fisheries	30,649	1,229	24,290*	0.040	0.792
2. Mining	8,700	614	7,797	0.070	0.896
3. Contract construction	52,277	3,612	43,085	0.069	0.822
4. Manufacturing	251,811	18,548	203,304	0.073	0.807
5. Transportation	36,453	2,559	31,916	0.070	0.875
6. Communication	20,311	1,090	14,614	0.053	0.719
7. Utilities	17,589	693	9,336	0.039	0.530
8. Wholesale and retail trade	144,606	13,623	115,361	0.094	0.797
9. Services	106,349	9,073	76,836	0.085	0.722
10. Total	668,745	×	×	×	×
11. Arithmetic mean $\bar{x}$	×	×	×	0.066	0.773
12. Mean square deviation σ	×	×	×	0.018	0.103
13. Coefficient of variation v	×	×	×	0.268	0.133

Source: *The National Income and Product Accounts of the United States*, U.S. Department of Commerce, 1977, pp. 186, 206, 194, 36. Tables

* Including income of farmers: 18,003 millions of dollars.
Calculation of compensation of employees per employee in agriculture:
Compensation of wage employees – 6,287 millions of dollars
Number of wage employees – 1,229 thousand
Compensation per employee – 5,116 dollars/man-year

have small shares in the economy. The indicator of the average deviations should be expected as smaller if sectoral shares are taken into account.

The calculation of such a weighted coefficient of variation is shown in columns 6 and 7 of Table 4.1.[1] Share π_j of each sector j in the total realised national income is defined:

$$\pi_j = \frac{N_j}{N},$$

[1] For a precise mathematical exposition of the method see: Granio A. Korn, Theresa M. Korn, *Mathematical Handbook for Scientists and Engineers*, New York, 1968, pp. 595-96.

Share in realised national income (share of unity)	Weighted per unit employment (col. 4 multiplied by col. 6)	Weighted per unit compensation of employees (col. 5 multiplied by col. 6)	Per unit employment with regard to labour reduction (thousands of man-years per million dollars)	Weighted per unit employment with regard to labour reduction (col. 9 multiplied by col. 6)
6	7	8	9	10
0.0458	0.0018	0.0363	0.1549	0.0071
0.0130	0.0009	0.0117	0.1752	0.0023
0.0782	0.0054	0.0643	0.1611	0.0126
0.3765	0.0275	0.3039	0.1578	0.0594
0.0545	0.0038	0.0477	0.1712	0.0093
0.0304	0.0016	0.0218	0.1407	0.0042
0.0263	0.0010	0.0139	0.1038	0.0027
0.2162	0.0203	0.1723	0.1560	0.0337
0.1590	0.0135	0.1148	0.1412	0.0225
1.0000	×	×	×	×
×	0.0759	0.787	×	0.1539
×	0.0143	0.058	×	0.0114
×	0.188	0.074	×	0.074

1929-74. Statistical Tables. A Supplement to the Survey of Current Business. 6.3, 6.8, 6.5, 1.13.

where N_j is the national income in sector j, $N = \sum_j N_j$ is the national income within the system as a whole. The magnitudes of π_j are given in column 6.

Now let us calculate the average weighted quantity of labour input in obtaining 1 million dollars of national income:

$$M(x) = \sum_j x_j \pi_j.$$

Then the dispersion of x_j is

$$\sigma^2 = \sum_j (x_j - M(x))^2 \pi_j,$$

their mean square deviation

$$\sigma = \sqrt{\overline{\sigma^2}},$$

and the coefficient of variation

$$v = \sigma/(M(x)).$$

The calculation of these magnitudes is given in column 7. As can be seen, the coefficient of variation dropped to 0.188. The shares of the various sectors being taken into account, this is precisely the real indicator of the fluctuations of labour input (without regard for its reduction) in obtaining one million dollars of national income.[1] Already this indicator permits the assumption that the deviations of real prices from value were moderate in the US economy.

This outcome should, however, be considered only as the beginning of the analysis, because the differences in labour skills by sector were not taken into account. In order to do so we shall calculate with formula (4.1). Here it is assumed that the differences in labour skills correspond in general to those in average wages.[2]

This assumption is based not only on the theoretical grounds already discussed, but also on statistical observation. Table 4.2. shows data on wages and salaries per full-time equivalent employee. Nominal wages tend to grow in all sectors, but the rates of growth seem to be different. The differences in the rates seem to be due to gradual changes in the required skill of labour power by sectors and in part serve as a mechanism for bringing the wages in sectors to

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[1] In this place, the indicator of labour input in sector 1 is underestimated: the labour of farmers was not taken into account; the labour of wage-earning agricultural workers seems not to be taken completely into consideration because some of them are working in the USA illegally. Considering these circumstances, the indicator of labour intensity in sector 1 should prove closer to the average in the economy. Correspondingly, the coefficient of variation could also be reduced.

[2] The theory of value of labour power is also built upon the assumption that the intensity of labour may be taken as equal in different sectors of the economy. The real intensity of labour may, of course, be fundamentally different in different sectors, but these differences are also reflected in wages, again owing to the unity of the labour power market. That is why, strictly speaking, coefficients p_j^l/p_j^l, should be considered as a reflection not only of the differences in the skills (complexity) of labour among sectors, but also of those in the intensity of labour. This does not interfere with verification of the law of value: more intensive labour creates proportionally more value per unit of time and does not differ in this respect from more skilled labour.

Table 4.2

Wages and Salaries per Full-Time Equivalent Employee (in thousands of dollars)

	1948		1960		1972		1978		1981	
	Magnitude	Rank	Magnitude	Rank	Magnitude	Rank	Magnitude	Rank	Magnitude	Rank
1. Agriculture, forestry, and fisheries	1,595	1	2,003	1	4,718	1	7,649	1	9,886	1
2. Mining	3,396	8	5,724	6	10,790	7	19,016	9	26,089	9
3. Contract construction	3,126	6	5,756	7	10,758	6	15,397	5	20,374	5
4. Manufacturing	3,038	5	5,548	5	9,449	4	14,948	4	19,647	4
5. Transportation	3,428	9	5,836	8	10,953	8	17,431	6	22,370	6
6. Communication	2,869	4	5,531	4	10,540	5	18,338	8	23,610	7
7. Utilities	3,190	7	6,147	9	11,412	9	18,447	7	24,105	8
8. Wholesale and retail trade	2,843	3	4,478	3	7,659	3	11,211	2	14,318	2
9. Services	2,299	2	4,100	2	7,658	2	11,725	3	15,584	3

Source: *The National Income and Product Accounts*, Table 6.9, p. 210; *Survey of Current Business*, July 1982, Table 6.9B, p. 85.

a regular level (with regard to the skill of employees). Let us arrange the indicators of average wages in the various years from the lowest to the highest (the ranks are given in Table 4.2 next to the corresponding absolute quantities of average wages by sector for each year).

The order (rank) of the sectors with reference to the wage level has, on the whole, not changed substantially for a rather long period of more than 30 years. Only the wages in transportation, which were initially the highest, turned out, by the end of the period, to be at a medium level, while, on the contrary, those in communication rose substantially in the system of sectors with respect to this indicator. The share of these two sectors in the economy is not great. The rest of the sectors have virtually not changed their positions in the classification by average wage level

for these years. The conclusion seems to be justified that the differences in the wages express essential, stable specific features of the sectors determining the demands of the skills of the employees.

The calculation results of compensation of employees are given in column 5 of Table 4.1; there, also, the coefficient of variation of this indicator is calculated as 0.133 (the sectoral shares having been disregarded), i.e., as half the same indicator for labour intensity indicated above. The calculation outcome of CE per unit of the national income with regard to sectoral shares is shown in columns 6 and 8 of Table 4.1. The related coefficient of variation is 0.074. Thus the average deviation from precise correspondence to the law of value is, in the large sectors of the US economy, 7.4 per cent.

This result was obtained using the indicators of specific CE, which may be turned into those of *adjusted labour intensity* (i.e., labour intensity with regard to reduction of labour) being divided by the magnitude of compensation per employee in sector 1, which is considered as that of simple labour.[1] It has been demonstrated mathematically that such a recalculation does not change the coefficient of variation. This fact is illustrated by the two last columns of Table 4.1. Column 9 shows the magnitudes of adjusted labour intensity themselves, column 10—these magnitudes multiplied by π_j (from column 6) and the resulting $\bar{x}$, σ, and v. As is seen, this calculation also yields a coefficient of variation equal to 7.4 per cent. Below we shall show simply the calculation results of CE per unit of the national income.

Note that relatively great deviations may be observed for the sectors with a small share in the economy. The average weighted CE amounted to 78.7 cent per $1 of national income realised in the sectors[2] (row 11, column 8 of Table 4.1). More than 10 per cent deviations from this level up or down occurred in the following sectors: mining (+14 per

[1] The calculation of compensation per employee in agriculture is shown in the footnote to Table 4.1.

[2] In relation to the total amount of net output in the USA in 1972, this indicator is certainly substantially lower, being approximately 60 cents in the $1 (see the estimate of the net national product in Table 1.9 of the system of national accounts). Again let us emphasise that our calculation takes into account only the national income realised in the corresponding sectors.

cent), transportation (+11 per cent), utilities (—33 per cent) (compare the indicators by sector in column 5 with the above average magnitude). The total share of these sectors in the economy is 9.4 per cent (see their indicators π_j in column 6). The other sectors are situated within the range +4.4 to —9 per cent in relation to the average. For instance, the deviation for manufacturing is mere 2.5 per cent. Thus, the correspondence of prices to the law of value can be additionally corroborated statistically: the average level (78.7 cents in the $1) is made up not so much as a result of the mutual cancellation of the extremes as owing to the fact that the sectors with a considerable share in the economy are sufficiently similar to one another in relation to the indicator under study. In terms of mathematical statistics, we are dealing not with a U-type distribution, but with one with a clearly distinct central trend.

As yet, 4 estimates have been obtained of the extent to which the fulfilment of the law of value, taken without regard for its modifications, is observed.[1] The last estimate (7.4 per cent) is the lowest. It is also the most accurate, since the foregoing estimates do not take into account either the factor of the reduction of labour or the degree to which the various deviations of prices are spread, i.e., the shares of the sectors in the economy. As can be seen, the precision with which the law of value works is very high, prices being close to the level complying with the direct action of the law of value. Table 4.3 shows this not to be an expression of the specific features of just one year. In the other years of the period under review, the coefficients of variations were also fairly low for the large sectors of the economy. Our attention is drawn by the fact that they display a general trend towards a decrease from the beginning to the end of the period. In the economy of the USA, processes have been observed that regularly diminish the average level of price deviations from value. This fact is not subject to analysis in this book.

Let us, however, voice one supposition in this respect at this point. Marx, of course, when dealing with the problem of absolute rent, concluded the following: monopoly of private ownership of the land prevents a certain part of the surplus-value from being involved in the total process of the formation of the prices of production (and hence in the pro-

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[1] See in Appendix 1 for these estimates for all of the years of the period.

Table 4.3

Coefficients of Variation of Labour Intensity of Realised National Income with Regard to Sectoral Shares and Reduction of Labour According to Formula (4.1), (per cent)

Year	Coefficient of variation	Year	Coefficient of variation
1948	14.6	1965	10.0
1949	12.8	1966	10.0
1950	12.0	1967	10.1
1951	12.0	1968	9.9
1952	11.5	1969	9.7
1953	9.7	1970	9.4
1954	11.1	1971	8.7
1955	11.0	1972	7.4
1956	10.7	1973	7.1
1957	10.7	1974	6.3
1958	11.7	1975	7.4
1959	11.3	1976	7.1
1960	11.5	1977	7.3
1961	11.6	1978	7.2
1962	11.3	1979	6.4
1963	11.0	1980	8.3
1964	10.7	1981	8.6

cess of levelling the rate of profit between the sectors of the economy), retains this part of surplus-value in agriculture, with the result that the prices in this sector gravitate directly towards value and not to the price of production; it is the difference between the value of agricultural produce and its price of production that constitutes the source of absolute ground rent, i.e., the economic realisation of the monopoly of private ownership of land. Let us emphasise that this conclusion has been formulated for the real conditions under which the organic composition of capital was, in the agriculture of the developed capitalist nations, substantially lower than in manufacturing and other sectors where ground rent is not obtained (for this reason the value of agricultural produce was higher than its prices of production). Marx never generalised this thesis directly, nor stated that any monopoly preventing free competition would lead to prices in the sectors under study gravitating towards value. Nevertheless, such a generalised formulation of the problem itself is suggested by consideration of the above data.

The factors leading to the formation of prices of production as the centre of price deviations shifted in relation to value will be shown below to continue operating in part in the modern economy. At the same time, prices draw closer and closer to value rather than to the prices of production. It may be hypothetically assumed that, here, the form in which the law of value operates under highly developed monopoly capitalism manifests itself. This assumption has to be verified and discussed by economists.[1]

Stability of the average relation of CE to realised national income. The estimate of how essential the obtained result (the coefficient of variation in 1972 equal to 7.4 per cent) is for demonstrating the operation of the law of value should now be discussed in particular. The majority of readers will probably agree that this is a very small magnitude. The opinion of the majority (as, besides, of any minority as well) cannot, however, be a criterion of the quality of the outcome. In science, there are no absolute criteria of the quality of research, nor any absolute criteria of truth.[2] The importance of the result obtained at any rate increases if account of the fact is taken that the average level of the

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[1] Another factor seemingly able (at least in part, together with the strengthening of monopolisation) to explain the drawing of prices closer to value at the end of the considered period is the substantial decrease in the rate of growth of the US economy since the beginning of the 1970s. As is known from the theory and is perfectly corroborated by historical analysis, value directly regulates prices under the conditions of simple reproduction, whereas the conversion of value into the price of production is related to the period of law-governed extended reproduction, during the transition from manual to mechanised labour. This being so, the approach of capitalist reproduction to a type of simple one (a substantial decrease in the rates of economic growth) must evidently lead to prices again approaching to value directly.

Additional evidence of the weakening of economic growth in the US economy by the end of the period under review can be derived by the reader from the indicators of the growth rates by sector given below in Table 4.9 (for the purpose of comparison, it is necessary to reduce the indicators to the average annual level).

[2] Let us note in this connection that the criterion of practice is a component of the theory of knowledge, which maintains that any theoretical truth is a relative one, a certain approximation to absolute truth, never being complete. "...We must not forget that the criterion of practice can never, in the nature of things, either confirm or refute any human idea *completely*. This criterion too is sufficiently 'indefinite' not to allow human knowledge to become 'absolute', but at the same time it is sufficiently definite to wage a ruthless fight on all varieties of idealism and agnosticism" (V. I. Lenin, "Materialism and Empirio-Criticism", *Collected Works*, Vol. 14, 1977, pp. 142-43).

ratio of CE to the national income is a very stable magnitude over time (see Table 4.4). The calculation was performed for the totality of the above sectors covered by the concept of value as the law of prices.

The ratio under consideration tended gradually to increase up to the mid-1970s. Nevertheless, the range within which it fluctuates is, for 34 years, only 70.2 to 80.8 per cent. It is substantially narrower within each of the decades shown in Table 4.4: for the first decade 70.2 to 74.6 per cent,

Table 4.4

Compensation of Employees to Realised National Income Ratio in the US Economy as a Whole (per cent)

Year	Ratio	Year	Ratio
1948	71.3	1965	74.0
1949	71.3	1966	74.4
1950	70.2	1967	75.4
1951	70.8	1968	75.8
1952	72.7	1969	77.6
1953	74.1	1970	79.6
1954	74.5	1971	79.0
1955	72.5	1972	78.7
1956	74.0	1973	79.3
1957	74.6	1974	80.8
1958	75.6	1975	78.4
1959	74.0	1976	76.9
1960	75.8	1977	76.6
1961	75.6	1978	77.2
1962	75.6	1979	78.5
1963	75.3	1980	79.5
1964	74.6	1981	78.1

Source: *The National Accounts System*, Tables 6.3 and 6.5

for the second 74.0 to 75.8 per cent, for the third decade 75.8 to 80.8 per cent, finally for the last four years 77.2 to 79.5 per cent.

In turn, the fluctuations of the ratio have a distinctly expressed cyclic nature: in crisis years it is usually higher than in the directly preceding years or in those that follow immediately. The cyclic crises occurred in 1948-49, 1957-58,

1966-67, and 1973-75.[1] Table 4.4. shows that, for each of these years, the ratio of compensation to realised income was greater than for the respective precrisis year (we did not analyse the data for 1947). This ratio tended to go down in the years after the crisis.[2] The cyclic movement is added to the long-term trend towards an increase in the magnitude under consideration. Note that this trend was associated with the overall trend towards a decrease of economic growth rates, especially in the 1970s: the excess of the realised national income over CE is, for producers, the major source of funds for expanding production and at the same time, an incentive to capital to carry out such an expansion.

The high stability of the ratio itself and the sufficiently pronounced trend in its change testify that this ratio cannot be seen simply as a certain average resulting from the related sectoral ratios: it obviously obeys its own law. We shall deal with the nature of this law below, while analysing the data of I-O tables. At this point, we shall merely point out that the ratio of CE to the national income realised in the sectors of production ultimately depends on the total rate of surplus-value in a country and on the share of surplus-value withdrawn in favour of the state.

Since it is accepted that the overall ratio of employees' compensation to realised income for the economy as a whole has its own nature determining it, its magnitude by sector should be considered not so much as factors determining the average ratio (such a view seems natural, but in fact it expresses only the way the total magnitude is calculated and not its economic essence) as the phenomena determined by it. The range extending from —9 to +4.4 per cent in relation to the above average for 1972 happened to include the sectors producing as much as 90 per cent of the gross commodity output. The same narrow range as that within which the indicators for the sectors producing the overwhelming portion of output are found is also characteristic of the other years, especially of the end of the period. Yet the sectors discussed are very different: agriculture and construction, manufacturing and services, and the like. They differ not only technologically but also in a lot of

[1] See S. M. Menshikov, *Inflation and Crisis in Regulating the Economy*, Moscow, Mysl Publishers, 1979, p. 64 (in Russian).

[2] An exception is 1968. The 1966-67 crisis itself was, however, the least pronounced. In particular, industrial output even grew in these years by 1.2 per cent (see Menshikov, *op. cit.*, p. 30).

specific features of their economy (different ratios of fixed to circulating capital, and constant to variable capital, different production period, etc.). Nevertheless, the major indicator to characterise the distribution of their realised income between owners and employees proves very close in all the sectors. This is indirect proof (in reality a manifestation) of the fact that this distribution characterising the basic production relation of capitalism is regulated by a general economic law.

No statistics can, of course, provide direct, immediate proof of a theoretical statement relating to the inner laws of phenomena: the laws cannot be observed directly. Statistical corroboration of the laws means that observations do not contradict the expectations following from the laws, i.e., the logically obtained consequences from the laws discovered by science. In other words, the corroboration always remains relative. Even so, all sciences consider a theory, its productiveness, its practical importance to be corroborated if the expectations following from it can be confirmed by observation and if the observations can be explained as consequences from the laws discovered by science. Only given such corroboration may a theory be considered of any use for predicting phenomena not yet observed and being additionally verified in the course of obtaining such predictions and applying them in practice.[1]

Stability of the ratio of CE to realised income by large sectors of the economy and stability of its sectoral variations. In order not to overburden the text with information that cannot be analysed in detail, in this work we shall not give tables similar to Table 4.4 for each of the nine large sectors. Our calculations show, however, that not only the total average, but also the averages for each sector are highly stable and their trends are similar to the overall ones. We give only aggregate data for three large periods: 1948-57,

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[1] This work is not going to bring statistical verification of the law of value to the stage of forecasting, but we can indicate the way of forecasting that emerges from the above considerations: it seems reasonable not to forecast the ratios for individual sectors in order then to obtain the average level for them, but to independently forecast the average ratio and then to differentiate it for the sectors, with due regard for their specific characteristics. The average ratio of employees' remuneration to the national income realised in sectors is here determined mainly, first, by changes in the rate of surplus-value and, second, by the portion of it withdrawn in favour of the state by means of indirect taxes.

1958-73, 1974-81 (see Table 4.5). Each of them begins and ends in years that represent the culmination points of the precrisis booms (and the beginning of the crises), the first and last periods each covering one cycle and the second — two cycles. As already stated, however, the cyclical crisis of 1966-67 was only very slightly pronounced.

The ratios of CE to realised national income grew during the period under review in all sectors except agriculture, forestry, and fisheries, this being further confirmation of the fact that the processes in the economy as a whole determine those in the sectors.[1] The decrease in the ratio in agriculture seems to be due to the reduction of the incomes of farmers who do not exploit the labour of others.[2]

It is significant that, in the last period, the range from —5 to +3 per cent in relation to the average value of the indicator reviewed covers the sectors producing almost 85 per cent of total commodity outpur (mining, contract construction, manufacturing, wholesale and retail trade, services). In the first of the periods under consideration the range from —7 to +14 per cent in relation to the average included the sectors producing 80 per cent of total output (mining, manufacturing, contract construction, transportation, communication, wholesale and retail trade). The tendency for the various sectors to draw closer to one another in relation to the indicator under review is obvious.

There are, nevertheless, some differences. Our attention is drawn by the fact that these differences are very stable over time: the order of the sectors as regards the ratio does change, but only slowly and, as a rule, not particularly significantly. Table 4.5 shows the ranks characterising this order for the three periods. Only the rank of agriculture,

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[1] If the overall trend was only a resultant of the sectoral trends, these latter would be expected not to work in the same direction and to have opposite signs in various sectors.

[2] In the statistics used by us this segment of farmers is not specified, as we have already noted. On the whole, the incomes of the individual private owners in agriculture grew for the period under review from 17,505 million dollars in 1948 to 23,955 million dollars in 1981. Compensation of employees in agriculture increased over the same period from 3,319 million to 17,428 million dollars. Our conclusion that the share of farmers who do not exploit the labour of others went down significantly is based precisely on these data; moreover, the increase in the sums paid to wage-earning workers may be itself considered as evidence of the increase in the share of capitalist economies. The number of wage-earning workers (in full-time equivalent terms) was 2,072 thousand in 1948, and 1,550 thousand in 1981.

Table 4.5

Compensation of Employees to Realised National Income Ratios by Large US Sectors for Three Periods

	1948-1957		1958-1973		1974-1981	
	Ratio \$/\$	Rank	Ratio \$/\$	Rank	Ratio \$/\$	Rank
1. Agriculture, forestry, and fisheries	0.899	1	0.828	3	0.690	8
2. Mining	0.710	6	0.833	2	0.746	6
3. Contract construction	0.752	3	0.808	4	0.800	4
4. Manufacturing	0.743	4	0.796	5	0.806	2
5. Transportation	0.829	2	0.873	1	0.862	1
6. Communication	0.713	5	0.636	8	0.724	7
7. Utilities	0.509	9	0.490	9	0.516	9
8. Wholesale and retail trade	0.679	7	0.773	6	0.801	3
9. Services	0.595	8	0.671	7	0.769	5
$M(x)$	0.727	×	0.768	×	0.784	×
$\sigma(x)$	0.081	×	0.072	×	0.056	×
$v(x)$	0.111	×	0.094	×	0.071	×

forestry, and fisheries changed sharply: having had the highest ratio considered in the first decade, this sector moved to the last but one place by the end of the period. We have already mentioned the reasons for this. As regards the other sectors, i.e., disregarding agriculture, in the ordering, the ranks will be as follows:

	1948–1957	1958–1973	1974–1981
2. Mining	5	2	6
3. Contract construction	2	3	4
4. Manufacturing	3	4	2
5. Transportation	1	1	1
6. Communications	4	7	7
7. Utilities	8	8	8
8. Wholesale and retail trade	6	5	3
9. Services	7	6	5

Either a rather high stability of the ranks themselves or a regular transformation in them may be observed in the main; moreover, the trend is for the rank to change to a rather close one. The closeness of the ranks could be characterised with the aid of the Spearman correlation coefficient. Between the ranks of the third and first periods, this coefficient is equal to 0.5. With a very high probability, the conclusion follows that the ranks are interrelated. In other words, deviations by sector from the total level of the indicator reviewed are sufficiently steady over time. This stability should be considered as an external manifestation of certain stable features of the sectors themselves, as something law-governed. Let us proceed to discuss this problem on the basis of the properties of the action of the law of value known from the theory.

Deviations of prices from value: dependence on the growth rates of output by sector. The theory of value is not reduced to expressing the requirements of the law of value on the assumption that it works directly. What is more, the theory emphasises that this law, like the other laws of the capitalist economy, is realised through nonrealisation. The law of value works by means of the market "demand-supply-prices" mechanism. Thanks precisely to this it is, in particular, a regulator of the proportions of capitalist production.

Structural shifts in the economy arising as a result of technical progress, changes in nonproductive requirements, and the like, manifest themselves first of all as changes in the structure of demand, this increasing for some commodities quicker than for others. In considering sufficiently disaggregated groups of commodity it even turns out that the demand for some commodities gradually drops to nought, whereas for others it rises and expands quickly. This does not apply to the aggregation of commodities in the 9 major sectors of the economy: given the overall expansion of production, this tends to take place in each large sector even though with rates that are by no means equal. The sectors where the demand for output grows more rapidly, more often experience a situation where demand leaves supply behind. Yet such a situation makes prices, of course, deviate upward in relation to value (or to another centre of gravity of prices as modifications of value). The opposite situation (a relatively slow growth of demand) should, on the contrary, lead to deviations of prices downwards. Those deviations form one of the mechanisms for redistri-

buting resources in favour of the more rapidly growing sectors (a structural transformation of production). It may thus be expected that the above deviations of prices from value are connected with the differences in the growth rates of the sectors under review.[1]

This expectation is justified with a high degree of precision. Let us return, first of all, to the year 1972, the example of which we use regularly to illustrate the course of the calculations (see Table 4.6).

Table 4.6

Comparison of Prices Deviations from Value-Level Prices for 1972 and GNP Growth Rates for 1966—1973 by Nine Large Sectors of the US Economy

	Compensation of employees to realised national income ratio in 1972		GNP growth rate, 1973 to 1965		Calculated ratio of compensation of employees to realised national income in 1972, $/$
	$/$	Rank	Magnitude	Rank	
1. Agriculture, forestry, and fisheries	0.792	6	1.088	8	0.839
2. Mining	0.896	1	1.171	7	0.823
3. Contract construction	0.822	3	1.035	9	0.849
4. Manufacturing	0.807	4	1.331	6	0.792
5. Transportation	0.875	2	1.332	5	0.792
6. Communication	0.719	8	1.893	1	0.683
7. Utilities	0.530	9	1.546	2	0.750
8. Wholesale and retail trade	0.797	5	1.427	4	0.773
9. Services	0.722	7	1.450	3	0.769

Source for the data to calculate GNP growth rates: *The National Accounts System.* Table 6.2

The first column of Table 4.6 repeats the indicators expressing the ratio of CE to the realised national income

[1] The theory of value thus has consequences that cannot de derived from conceptions ignoring the law of value. Such conceptions consider the accelerated growth of demand for output of some sector usually as a factor giving rise to an increase in the relative prices of output, i. e., to a higher than the average price index. The statement resulting from the law of value by no means coincides with this: in this case, the price should be higher than that at the level of value, but its index can and must be lower than the total price index. We shall return once more to this problem when analysing price indices.

from Table 1. They have to be compared (see the third column) with the growth rates of output for the period 1966 to 1973 as a whole (i.e., with the ratios of the output volumes in 1973 to those in 1965); this period, as already mentioned, forms the U.S. reproduction cycle that included 1972.[1] To calculate the output indices, the data on final output (i.e., so-called value added) by sector in constant 1972 prices were employed. The indicators in the first column of Table 4.6 are expected to be the lower, the higher are those in the third column. This means that profitability (in relation to CE) is higher in the sectors that are growing more rapidly.

The first estimate of the correspondence of statistics to this expectation will be carried out by rank correlation. The ratios of CE to the national income are ranked in diminishing order (column 2 of Table 4.6). The first rank will thus belong to the sector with a relatively low level of profitability of variable capital, the ninth rank—to the sector with the highest of it. In other words, the sectors from which the price system withdraws a part of surplus-value are placed at the beginning of the series, those which this redistribution favours—at the end. The output of the latter sectors is expected to grow more rapidly than that of the former ones. The growth rates of output are ranked also in diminishing order (see column 4). The expected inverse dependence is quite distinctly pronounced: for example, rank 1 corresponds to rank 7, rank 3 to rank 9, rank 8 to rank 1, rank 9 to rank 2, and so on. The Spearman rank correlation coefficient is —0.7, which testifies to a high probability of an inverse statistical dependence of the two series of magnitudes.

It is fit to discover what share of the variation coefficient found in Table 1 can be explained by the dependence of the ratios under consideration on the sectoral rates. Having accepted the most simple (linear) hypothesis of the interrelationship between the magnitudes under study, we ob-

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[1] The ratios of CE to the realised national income by sector are rather stable within the cycles, whereas the growth rates of output are subject to short-term fluctuations determined by market conditions. The ratios are controlled by the overall class relations of the distribution of the national income and steady specific characteristics of the sectors, which cannot change as fast as the interplay of demand and supply. That is why the ratios for one year should, to be correct, be compared with the growth rates for the cycle as a whole, where transient fluctuations of the rates are eliminated.

tained the following regression equation:

$$(x_j - 0.787) = -0.1944\,(y_j - 1.357)$$

(the pair correlation coefficient $r = -0.52$).
Here j is the number of the sector ($j = 1, \ldots, 9$); x_j is employees' compensation per $1 of realised national income in sector j; 0.787 is the average weighted magnitude of this ratio; y_j is the growth rate of output in sector j; 1.357 is the average weighted growth rate of output in the system of sectors as a whole; —0.1944 is the coefficient of the regression equation. The indicators of CE per national income ratios calculated with this formula are shown in column 5 of Table 4.6.

Let us compare the calculated and actual figures of the ratio. In sectors 1, 3, 4, 8 and 9 the deviations of the calculated figures from the actual ones do not exceed 6.5 per cent (modulo). In 1972 these five sectors realised 88 per cent of total national income (from those parts realised directly in the commodity producing industries). In the rest of the sectors the deviations are greater (for sector 7 the deviation is up to +41 per cent), but their total share in the economy is 12 per cent (of sector 7—2.6 per cent). For this reason, the relatively large deviations of the calculated values from the actual ones for these sectors increase the average level of deviations only insignificantly. The variation of the ratio under consideration, as yet unexplained, fell, on the whole, from 7.4 to 6.3 per cent.

These 6.3 per cent of the variation involve the work of all the factors ignored so far, i.e., differences in the degree of monopolisation, differences between the sectors with regard to the organic composition of capital, market fluctuations of prices not cancelled out during the year under study, deficiencies of the data used, etc. Some of the factors express regular, stable specific features of the economy. Should they be taken into account, the explanation of the level of prices as derived from the properties of the law of value can be made more accurate.

There is some point in considering the dependence under study disregarding sectors 1 and 7 (Table 4.7). Sector 1 is disregarded owing to deficiencies in the data employed;[1] as for sector 7, a special explanation is needed.

[1] The fact that the calculated CE for agriculture almost coincided with the actual figure is no cause for self-congratulations. For an individual observation of a set, there is always a possibility of a *chance*

Table 4.7

Comparison of Prices Deviation from Value-Level Prices for 1972 and GNP Growth Rates for 1966-1973 by Seven Large Sectors of the US Economy

	Compensation of employees to realised national income ratio in 1972		Final product growth rate		Calculated ratio of compensation of employees to realised national income
	$/$	Rank	Magnitude	Rank	
2. Mining	0.896	1	1.171	6	0.827
3. Contract construction	0.822	3	1.035	7	0.850
4. Manufacturing	0.807	4	1.331	5	0.800
5. Transportation	0.875	2	1.332	4	0.799
6. Communication	0.719	7	1.893	1	0.703
8. Wholesale and retail trade	0.797	5	1.427	3	0.783
9. Services	0.722	6	1.450	2	0.779

With reference to developed free competition capitalism, the theory of value expects the prices to gravitate not so much towards the value of commodities as to its modification represented by the price of production. In other words, it is expected that the deviations of prices from value are due to the differences in the organic composition of capital. When analysing the above deviations of prices from value, we already noted that they are so small as to suggest that prices are directly regulated by the value of output under modern capitalism, at least as far as large sectors of the economy are concerned. The question may arise as to whether the indicators of the organic composition of capital are, in turn, so close for these sectors that the very problem of distinguishing between the two centres of price fluctuations no longer arises. Analysis of the statistics gives a negative answer to this question: the variation of the indicators characterising the organic composition of capital is very significant and, at any rate, by far exceeds that of the CE per national income ratio from which we judge the degree

coincidence. Essential considerations suggest that this sector should be characterised by a special ratio of remuneration to the realised national income in connection with rent.

of accuracy with which the law of value is fulfilled as expressed by formula (4.1).

It is impossible to calculate the organic composition by sector in full accord with its concept in value and even in price terms. Statistics do not involve indicators to be interpreted as circulating capital (a component of the constant capital advanced) and variable capital advanced. The only method available to us for getting an idea of the variation of the organic composition of capital, is to use the ratio of accumulated fixed capital to the number of employees. This ratio is, in its sense, similar to the concept of the technological structure of capital which, of course, forms the basis of the organic composition (see Table 4.8).

In this calculation the indicators of net (minus depreciation) fixed capital were used; precisely the latter seems to have to be taken as the fixed capital actually advanced and invested in production in a certain year. The volume of this capital was taken in 1972 prices. The indicators given in the basic statistics were aggregated for the composition of the nine sectors considered.[1] Note that the indicators for agriculture are overestimated for the reason already mentioned: the number of wage employees in this sector does not express the actual number of workers which includes farmers who do not exploit the labour of others (seasonal workers, especially those arriving in the USA illegally, are apparently not fully taken into account either).

Table 4.8, if compared with Table 4.1., shows that the volume of fixed capital per employee varies more than this of the realised national income per employee. While the range of the variations of the former is more than 46 times, that of the latter is less than 2.5 times.

The fact that prices are close to value cannot, therefore, be explained by the negligible variation of the organic composition of capital. Hence, additional evidence has been obtained to support the statement that there exists a powerful mechanism impeding the redistribution of value between sectors that would take place if the principle of prices of production dominated. Nevertheless, the deviations of prices from value are, to a certain extent, due to the variation of the organic composition of capital.

[1] The following correspondence of the large sectors and the sectors of the economy for which the initial data on fixed capital are given is accepted: sector 1 to sectors 1-4, 2 to 5-10, 3 to 11-12, 4 to 13-64, 5 to 65, 6 to 66-67, 7 to 68, 8 to 69, sector 9 to sectors 72-77.

Table 4.8

Calculation of Fixed Capital to Number of Employees Ratio for 1972 by Large Sectors of the US Economy

	Net fixed capital by the beginning of 1972, million dollars	Number of full-time equivalent employees, thousand	Col. 1 to col. 2 ratio	
			Magnitude	Rank
	1	2	3	4
1. Agriculture, forestry, and fisheries	91,009	1,229	74.1	3
2. Mining	82,433	614	134.3	2
3. Contract construction	18,407	3,612	5.1	9
4. Manufacturing	265,720	18,548	14.3	7
5. Transportation	133,304	2,559	52.1	5
6. Communication	75,042	1,090	68.9	4
7. Utilities	163,345	693	235.7	1
8. Wholesale and retail trade	112,860	13,623	8.3	8
9. Services	241,988 [1]	9,073	26.7	6
Total	1,184,108	51,041	23.2	×

[1] Together with capital of non-profit organisations disregarded in calculating the number of employees.

The source of the data to calculate fixed capital: *Capital Stock Estimates for Input-Output Industries: Methods and Data.* U. S. Department of Labor. Bulletin 2034. Washington, 1979, pp. 46–117.

Consider now the "Utilities" sector. Its ratio of fixed capital to the number of employees was approximately 10 times higher than that for the economy as a whole. This was not completely compensated for by the decrease in the ratio of CE to realised national income. Yet this can explain why the latter ratio proves, in this sector, to be regularly, throughout the whole period under review, the smallest: a certain redistribution of surplus-value clearly takes place in favour of this sector.

Both the facts that the high organic composition of capital was far from completely compensated for in the sector and that it was, nevertheless, in part compensated for, are characteristic. The specific feature of this sector seems to be satisfactorily explained by this. That is why the level of the ratio in it should be made to correspond rather to the level of the organic composition than to the growth rate of output. In other words, the sector should be disregarded in making a growth rates comparison.

In the other sectors, no significant correlation of the ratio reviewed with the organic composition of capital could be observed. The reader can see this by comparing the data in Tables 4.6 and 4.8.[1] Highly significant deviations of the organic composition of capital from its average level seem to be necessary under modern conditions in order for a substantial redistribution of value to occur in favour of the relevant sector.[2]

Thus, it is reasonable for the rest seven sectors to compare the CE to national income ratios with the growth rates of their output. Table 4.7 shows first of all that the inverse rank correlation is very high, its Spearman coefficient being —0.82.

The following regression equation was obtained: $(x_j - 0.794) = -0.1719\ (y_j - 1.365)$; $j = 2, 3, 4, 5, 6, 8, 9$; $r = -0.60$. The figures of the considered ratio calculated with this equation for seven large sectors are given in Table 4.7 (column 5). Note that the ratios themselves, sectors 1 and 7 being disregarded, are within the interval the upper limits of which exceed the lower by only 25 per cent; the weighted variation coefficient is 5.3 per cent. If the correlation dependence on the growth rates is taken into account, the residual variation coefficient decreases to 4.2 per cent. Thus, a very high accuracy of the explanation of the variation of the ratio is achieved.

Hence, even in an individual year where the market fluctuations are tangibly felt, the variation of the ratio may be, to a considerable extent, explained by comparing it with that of the growth rates of output for the cycle covering the given year. It should be expected that, if the ratios are taken not for a year but for a whole cycle, the impact of the above fluctuations will prove insignificant, and the regular specific features of the sectors will manifest themselves with increasing accuracy. Their correlation with the growth rates of output for an entire cycle is, there-

[1] The correlation coefficient of the ranks of CE per $1 of national income (Table 4.6, column 2) and of fixed capital per employee (Table 4.8, column 4) is equal to —0.05. The conclusion of no correlation between the indicators under review is justified with a high probability.

[2] Competition (and levelling of the rate of profit) among the large sectors of the economy seems to come up against very serious obstacles. It will be shown below that the correlation of CE per national income and the organic composition of capital is felt significantly within manufacturing.

Table 4.9

Comparison of Prices Deviation from Value-Level Prices and GNP Growth Rates by Nine Large Sectors of the US Economy for Three Periods

Equations of regression for:

1948-1957 $(x_j - 0.727) = -0.1753\,(y_j - 1.445)$; $r = -0.49$;

1958-1973 $(x_j - 0.768) = -0.1595\,(y_j - 1.880)$; $r = -0.72$;

1974-1981 $(x_j - 0.784) = -0.1197\,(y_j - 1.193)$; $r = -0.34$;

	1948-1957					1958-1973					1974-1981				
	Compensation of employees to realised national income ratio		GNP growth rate		Calculated ratio of compensation of employees to realised national income, $/$	Compensation of employees to realised national income ratio		GNP growth rate		Calculated ratio of compensation of employees to realised national income, $/$	Compensation of employees to realised national income ratio		GNP growth rate		Calculated ratio of compensation of employees to realised national income, $/$
	$/$	Rank	Magnitude	Rank		$/$	Rank	Magnitudes	Rank		$/$	Rank	Magnitude	Rank	
1. Agriculture, forestry, and fisheries	0.899	1	1.180	8	0.774	0.828	3	1.165	9	0.882	0.690	8	1.209	3	0.782
2. Mining	0.710	6	1.289	7	0.754	0.833	2	1.306	[illegible]	0.859	0.746	[illegible]	1.161	6	0.787
3. Contract construction	0.752	3	1.784	3	0.668	0.808	4	1.439	7	0.838	0.800	4	0.892	9	0.820
4. Manufacturing	0.743	4	1.460	4	0.724	0.796	[illegible]	1.865	5	0.770	0.806	2	1.148	7	0.789
5. Transportation	0.829	2	1.096	9	0.789	0.873	1	1.643	6	0.806	0.862	1	0.998	[illegible]	0.807
6. Communication	0.713	5	2.060	2	0.619	0.636	8	3.107	1	0.572	0.724	7	1.759	1	0.716
7. Utilities	0.509	9	2.365	1	0.566	0.490	[illegible]	2.439	2	0.679	0.516	9	1.177	[illegible]	0.786
8. Wholesale and retail trade	0.679	7	1.419	5	0.732	0.773	[illegible]	1.963	4	0.755	0.801	3	1.182	4	0.785
9. Services	0.595	8	1.360	6	0.742	0.671	7	2.074	[illegible]	0.737	0.769	5	1.375	2	0.762
Coefficient of variation:															
initial	0.111	×	×	×	×	0.094	×	×	×	×	0.071	×	×	×	×
residual	0.097	×	×	×	×	0.065	×	×	×	×	0.067	×	×	×	×

fore, of special interest. The results of such calculations are shown in Tables 4.9 and 4.10; in the former, for all the 9 sectors, in the latter, without sectors 1 and 7. For the reasons mentioned above, the second of the periods under consideration is considered as including two cycles: 1958 to 1966-67 and from these two years to 1973; the correlation calculation is thus given to further explain those figures presented in Table 4.5.

The growth rates were determined with the same data (Table 6.2, national accounts *system*) as for the calculations in Tables 4.6 and 4.7. Every time the year preceding the beginning of the period under consideration was taken as the base year for determining the growth rates; and the last year of the period as the final one. The growth rates were thus determined as the ratios of the indicators of final output in 1957 to 1947, 1973 to 1957, and 1981 to 1973.

Table 4.9 shows the results of the whole calculation. Columns 1 and 3 present all the basic data used for each period. The calculation given in Table 4.10 is based on the same information, the two mentioned sectors being disregarded, and this is not repeated: only calculated magnitudes, as well as the initial and residual variation coefficients, are shown.

If all the 9 sectors are involved in the calculation, the Spearman rank correlation coefficient is, for the first period, equal to —0.5, for the second period to —0.87 and for the third period to —0.55. Thus with considerable accuracy it may be stated that there exists an expected inverse dependence of the above ratios on the growth rates of output. Linear regression detects this dependence only slightly, however: the pair correlation coefficients are statistically insignificant (they are given in Table 4.9 after the regression equations); the residual coefficients of variation differ insignificantly from the initial ones (compare the last two rows of Table 4.9).

The calculation for 7 sectors results, first of all, in a sharp decrease in the initial variation coefficient, the latter, moreover, tending to decrease substantially from the beginning to the end of the period. This variation turns out to a great extent to be correlated with that of the growth rates of output. This dependence being taken into account, the residual coefficient is for the last period barely 2.1 per cent.[1] A very

[1] The residual variation coefficient is calculated in the following manner. The deviations of the calculated quantities of the function

high, may be even surprisingly high, accuracy of the explanation of the level of prices by the law of value is thus achieved. The law, as is now apparent, operates approximately as strongly as those of the natural sciences, at least at the level of large sectors of the economy.[1]

Table 4.10

Calculated Ratios of Compensation of Employees to Realised National Income by Seven Large Sectors of the US Economy for Three Periods (in dollars)

Equations of regression for:
1948-1957 $(x_j - 0.717) = -0{,}0189\ (y_j - 1.445)$; $r = -0.06$;
1958-1973 $(x_j - 0.773) = -0.1489\ (y_j - 1.902)$; $r = -0.73$;
1974-1981 $(x_j - 0.796) = -0.1216\ (y_j - 1.193)$; $r = -0.77$.

	1948-1957	1958-1973	1974-1981
2. Mining	0.721	0.861	0.800
3. Contract construction	0.711	0.842	0.833
4. Manufacturing	0.717	0.778	0.802
5. Transportation	0.724	0.811	0.820
6. Communication	0.706	0.593	0.727
8. Wholesale and retail trade	0.718	0.764	0.798
9. Services	0.719	0.747	0.774
Coefficient of variation			
initial	0.080	0.073	0.033
residual	0.080	0.049	0.021

(i.e., CE per national income ratio by sector) from the actual ones are determined; then their weighted residual variance, i.e., the sum total of the squares of the above deviations, weighted by the sectoral shares in the realised national income; finally, the residual mean square deviation. It is its relation to the average value of the magnitude under study (i.e., to the average CE per national income ratio) that is the residual variation coefficient.

[1] We also analysed the ratio of employees' compensation to realised national income for large sectors on the basis of the national accounts data for a number of other developed capitalist countries: Australia, Denmark, Finland, Great Britain, Japan, the Netherlands, New Zealand, Norway, and Sweden. A significant inverse correlation dependence between the variation of the ratios and that of the growth rates by sector was discovered in most of them. With regard to this dependence, the residual variation coefficients of the ratios considered are from 6.4 per cent (in Finland) to 16.2 per cent (in Denmark); the correlations were calculated disregarding agriculture where, in all the countries, the ratio is extremely low, apparently owing to ground rent. Statistical data do not confirm the existence of the dependence discussed for Great Britain and Japan.

It is noteworthy that both the initial and residual variation coefficients tend to diminish successively over time (see Table 4.10). This fact can be seen as further corroboration of the assumption that the closeness observed between prices and value level is due to monopolisation of the economy: the degree of monopolisation has increased in the postwar period, and, together with this, the deviations of prices from the value level have decreased fundamentally.

Note that the estimates obtained of the accuracy with which the law of value operates apply to newly-created value realised in sectors. Yet the value of commodities also includes the part transferred from the means of production used up. As regards the total price of commodities, it may be expected that its explanation as a result of the operation of the law of value will, on average, be even more accurate since the deviations of prices from value should be partly cancelled out in the process of the formation of the total value (total price) of commodities as the sum of the value transferred from the expended means of production (prices of the expended means of production) and newly-created value (realised national income). This will be further shown in the analysis of the statistical data of I-O tables.

Dynamics of labour intensity of output and dynamics of prices. So far we have analysed the correspondence of prices to value in the static situation: each time period was studied separately from the others and the correspondence of realised national income in sectors to labour input in them has been investigated. In conformity with the law of value, however, the changes in prices are expected to be determined by those in the value of corresponding commodities. Herein precisely lies the dynamic aspect of the operation of value as the law of prices.

To understand the dependence of prices on value in dynamics correctly and, therefore, to arrange statistical verification correctly, it is necessary to take the following circumstances into account.

Commodity prices depend not only on the value of commodities, but also on that of gold (given golden money circulation). If, say, the value of gold decreases more rapidly than that of some commodity, the price of the commodity is rising while its value is decreasing. If circulation is served by fiat money, a similar effect is caused by inflationary devaluation of paper money: commodity prices may grow despite a decrease in commodity values. The conclusion is

that the values of commodities affect not the absolute prices, but the ratios of the prices, i.e., relative prices in this sense of the word.[1]

Accordingly, the question of the dependence of changes in net receipts from commodities in sectors on living labour input in them is to be answered. In conformity with the law of value, the net receipts from commodities on the market should change in proportion to the changes in direct labour input in them, but by no means obligatorily in the same direction: net receipts may grow, whereas living labour input decreases, say, owing to an inflationary growth of prices. In this case, net receipts undoubtedly tend to increase in the sectors where living labour input increases. The sign of the changes in the net receipts may thus turn out to be invariable in relation to that of the changes in living labour input which will not contradict the law of value.

Similar reasoning is also valid in reference to the cyclical dynamics of the total level of prices. Let us explain this assuming the existence of gold money circulation. In periods of upswing, the level of prices tends to rise, an overall devaluation of money takes place (purchasing power decreases) irrespective of its value and the value of commodities as a result of the general excess of demand over supply. The opposite is characteristic of cyclical crises: the purchasing power of money tends to increase (the total level of prices to fall) simply owing to the excess of supply over demand, to overall overproduction.[2]

Hence the conclusion: it is rather the *relative prices and relative changes in prices* (*the ratios of the indices* of *prices*) than commodity prices as such, therefore not the changes in prices as such (i.e., not the indices of prices directly)

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[1] In some sense, price as such is relative: price is the ratio at which a commodity is exchanged for money. The ratios of prices defined in this way and the changes in these ratios may thus be considered.

[2] If a crisis of overproduction occurs where there is no stable gold standard for paper money exchange and an inflationary process is under way, no fall in the level of prices can be observed on the surface; prices may go up even more rapidly than in the previous upswing period. The matter is decided by the rate of inflation. In this case, however, the relative price of gold, i.e., real money, should rise during crisis years (that is, the index of the paper-money price of gold should be higher than the overall index of prices). In contrast, the index of the price of gold should lag behind the index of prices as a whole in boom years.

that must be derived from the law of value in accordance with its sense. This task is, in turn, broken down into two substantially different ones: to determine (1) the ratios of price indices from cycle to cycle, (2) their ratios within the cycle by phase. In this work we present only the results relating to the first aspect of the problem of price dynamics,[1] but this is the crucial aspect of verification of the law of value in dynamics: it is pertinent to study specific characteristics of the operation of the law of value by phase within the cycle if it has been demonstrated that the law affects long-term trends.

The stated possible specific features of the dynamics of relative prices as compared with those of value (for example, the chance that the signs of the increment in net prices of all commodity groups will prove common despite the differences in the signs of the increment in the labour intensity of commodities) constitute only one of the expressions of the overall fundamental characteristic of economic laws in general: although these laws, like any laws in general, are certain objectively existing constants, the magnitudes of the latter change historically, and moreover, quite rapidly. One should not see any purely logical contradiction in this statement. While changing historically, the constants are constants for a certain period of time; the magnitude of a constant tends to change historically, while its qualitative content remains intact.[2]

The historical changeability of the parameters of the law of value should be kept continuously in mind. In particular,

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[1] For this reason the changes in prices from the peak of one cycle to that of the next will be considered below.

[2] For example, in the expression of the law of value

$$\frac{\tilde{L}_j \Psi_j}{M_j - c_j} \to h \text{ for all } j = 1, \ldots, n$$

the magnitude of the invariant h changes historically owing to changes in the value of gold (h is the direct value of gold if gold coins are circulating, i.e., in this case $h = w_{gold}$, where w is the symbol of the magnitude of value); it is affected by the overall inflationary process (if paper money is not exchangeable for gold) and the like. Thus $h = h(t)$. For any time period t, however (certainly within the limits of the existence of commodity-money relations), the mathematical description of the law and the essential sense of parameter h remain unchanged: this is always a certain quantity of social labour required to receive a unit of money for commodities on the market, this amount not being dependent on the sectoral origin of commodities.

this is true of the sectoral skill differences, these differences changing in dependence on the speed and nature of the technical progress, which is inevitably different from sector to sector. The changes in the relative skill levels are reflected in changing wage ratios (see above, Table 4.2). These ratios also depend on a number of market factors. We do not, however, know any way to separate one from the other. For this reason, below, while evaluating the changes in the coefficients of reduction of labour ψ, we simply use the indices of the wage ratios in sectors $j = 1, \ldots, n$ to wage in one, fixed sector j'. Agriculture is taken as such a sector: in this sectoral classification, it regularly has the lowest level of wages per employee, which means that the labour spent there is closer to the concept of simple labour than that in any other sector.

Thus, in accordance with the law of value, i.e., as follows from formula (4.1), it is expected, first of all, that net receipts (total receipts less reimbursement of material input) per unit of commodity tend, for any time period, to be proportional to the commodities' direct labour intensity:

$$\frac{M_j - c_j}{Q_j} \to \frac{1}{h} \frac{\tilde{L}_j \psi_j}{Q_j}, \quad j = 1, \ldots, n \qquad (4.7)$$

where Q_j is the quantity of commodities of kind j sold (it is assumed to be equal to that of commodities produced over the same time period t); h is the constant from formula (4.1); $\frac{M_j - c_j}{Q_j}$ is a specific expression of the unit commodity price, being its price p_j with deduction of the sum total of prices of material input in its production.[1] Let us call it *the net price* of commodity j and denote it by π_j. Formula (4.7) maintains: the net price is proportional to the net (direct) labour intensity (with regard to reduction of labour). This formula is but a simple conversion of formula (4.1).

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[1] $M_j = p_j Q_j$; $c_j = Q_j \sum_i p_i a_{ij}$, where p_j, p_i are prices, a_{ij} are average coefficients of expenditure of means of production of kind i on producing unit commodity of kind j; $j, i = 1, \ldots, n$. Then $\frac{M_j - c_j}{Q_j} = p_j - \sum_i p_i a_{ij}$ are the net receipts for unit commodity j.

A formula for the net price index is easily obtained from (4.7):

$$I\frac{M_j-c_j}{Q_j}\equiv I\pi_j\to I\frac{1}{h}\cdot I\left(\frac{\tilde{L}_j\cdot\psi_j}{Q_j}\right)\equiv I\frac{1}{h}\cdot\frac{I\tilde{L}_j\cdot I\psi_j}{IQ_j}\equiv$$

$$\equiv I\frac{1}{h}\cdot I\tilde{l}_j I\psi_j\equiv I\frac{1}{h}\cdot I\tilde{l}_j\psi_j\equiv I\frac{1}{h}Il_j$$

$$\text{for all } j=1,\ \dots,\ n. \tag{4.8}$$

Here I is the symbol of the index, $\tilde{l}_j$ is the coefficient of the direct labour intensity of commodity j without regard for reduction of labour, $\tilde{l}_j=\frac{\tilde{L}_j}{Q_j}$. Note that, since h (t) itself is a constant for all j within a given time period t, $I\frac{1}{h}$ is also the invariant in relation to j. Simplifying somewhat, in formula (4.8), $I\frac{1}{h}$ may be assumed to be a reflection of the overall change in the purchasing power of money. In particular, if there is inflation, $I\frac{1}{h}>1$. Here we have a mathematical explanation of how the net prices of commodities π_j can go up while labour intensity, i.e. $\tilde{l}_j\psi_j$, falls. From now on $I\frac{1}{h}$ will be denoted by symbol β.

Formula (4.8) is merely an expression of the law of value with reference to the dynamics of prices. U.S. national accounts data allow the validity of this expression to be directly verified. Statistics (see Table 7.22) record price indices (implicit price deflators) with reference to the so-called value added. This quantity presents the full price reduced for reimbursement of direct material expenditure, while depreciation of fixed capital is included. It thus differs from π_j only by depreciation, the share of which in the total value added is, as a rule, small. At any rate, the price index for value added should, as a rule, be sufficiently close to $I\pi_j$. In the following we shall consider it simply to be $I\pi_j$.

To verify the operation of the law of value in dyn mics we shall employ, apart from formula (4.8), the following simplified formula, which does not take account of changes in the coefficients of the reduction of labour:

$$I\pi_j=\gamma I\tilde{l}_j,\ \ j=1,\ \dots,\ n \tag{4.9}$$

where γ is a constant similar to β. (4.9) could be easily derived from (4.6).

The calculations were carried out for the same three periods as above. Their course will be shown in detail using the example of the period 1973 to 1981. Let us begin by verifying formula (4.9). Table 4.11 presents all the data for calculating the indices of π_j and $\tilde{l}_j$.[1]

The last two columns of Table 4.11 present the ranks of $I\pi_j$ and $I\tilde{l}_j$ (by diminishing rank). The rank correlation coefficient $R = 0.87$. These ranks are closely interrelated, the sign of interrelation corresponding to that expected theoretically. The linear regression equation was also calculated:

$$(I\pi_j - 1.88) = 3.4339\,(I\tilde{l}_j - 0.97),\ r = 0.82,\ \text{residual}\ v = 0.16.$$

The variation of the price indices is thus almost half explained by the dependence of prices on the changes in labour intensity (initial $v = 0.28$). Nevertheless, the residual v should be acknowledged as substantial.

In agriculture and mining the price index should, in accordance with the theory of value, depend not only on the changes in labour intensity, but also on those in rent-formation. Note, in particular, that the price index is extremely high in mining ($I\pi_2 = 5.1$ while the average weighted $I\pi_j = 1.88$). There seem to be theoretical reasons for excluding these two sectors from the calculation. The share of the rest of the sectors in the economy will be 93.5 per cent.

These are the statistical characteristics for seven sectors:

	Price index	Simple labour intensity index
$M(x)$	1.83	0.96
σ	0.17	0.09
v	0.093	0.10

The average magnitudes of the indices, as well as the indicators of the variation of the index of labour intensity,

[1] In the official statistics (*Survey of Current Business*, July 1982, Table 7.22) the price index (implicit price deflator) is given for the "Services" sector only as a whole, so the services of non-profit organisations and households cannot be singled out. Accordingly, we give the rest of the information for the given sector as a whole, which distinguishes this estimate from those discussed above.

Note, also, that in this section the shares in the economy are determined in the same manner as above, by the sum of the realised national income for the period under review, in current prices.

Table 4.11

Estimation of Price Indices and Indices of Labour-Intensity Coefficients (for GNP) in Nine Large Sectors of the US Economy for 1973-1981

	Share in economy, per cent	Implicit price deflators for GNP (index numbers. 1972=100)		Price indices 1981/1973	Gross national product				Full-time equivalent employees			Labour-intensity indices 1981/1973	Ranks of indices 1981/1973	
					Billions of 1972 dollars		Index numbers 1981/1973	Ranks	Persons (thous.)		Index numbers 1981/1973		Prices	Labour-intensity
		1973	1981		1973	1981			1973	1981				
1. Agriculture, forestry, and fisheries	4.3	150.1	197.2	1.31	35.9	43.4	1.21	7	1,332	1,550	1.16	0.96	9	7
2. Mining	2.2	111.6	570·1	5.10	19.2	22.3	1.16	4	624	1,113	1.78	1.53	1	1
3. Contract construction	7.0	110.8	244.6	2.21	57.2	52.0	0.91	1	3,843	3,931	1.02	1.12	2	2
4. Manufacturing	35.4	102.8	179.3	1.74	313.0	359.2	1.15	3	19,566	19,666	1.01	0.88	7	8
5. Transportation	5.2	101.3	213.0	2.10	50.6	50.5	1.00	2	2,638	2,786	1.06	1.06	4	3
6. Communication	3.1	102.1	138.3	1.35	32.0	56.3	1.76	9	1,115	1,315	1.18	0.67	8	9
7. Electric, gas, and sanitary services	2.7	101.7	216.7	2.12	30.0	35.3	1.177	5	707	817	1.16	0.98	3	5
8. Wholesale and retail trade	21.0	105.5	188.7	1.79	212.0	250.5	1.182	6	14,288	17,344	1.21	1.03	6	4
9. Services	19.2	105.2	196.7	1.87	143.1	196.7	1.37	8	12,643	16,871	1.33	0.97	5	6
$M(x)$				1.88								0.97		
$\sigma(x)$				0.52								0.12		
$v(x)$				0.28								0.13		

hardly changed. Once the two sectors have been excluded, the initial variation coefficient of the price index fell sharply (from 0.276 to 0.093). Our attention is drawn by the fact that this coefficient proves to be virtually equal to that of the variation coefficient of the index of labour intensity. This corresponds to the expectations resulting from the law of value: if the law determines price indices directly, the variation in them should be approximately the same as that in the indices of labour intensity of output; such a conclusion could easily be obtained from formula (4.9), as well as from formula (4.8).

The Spearman rank correlation coefficient of the indices for the seven sectors under study is 0.82. The linear regression equation was calculated:

$$(I\pi_j - 1.83) = 1.4810\,(I\tilde{l}_j - 0.96);\ j = 3,\ \ldots,\ 9;$$
$$r = 0.82;\ \text{residual } v = 0.053.$$

The values of the price indices calculated from this equation are given in Table 4.13. There are sufficient reasons for maintaining that the variation of the price indices was quite well explained by that of the indices of the labour intensity of output.

Now let us consider the dependence of the price indices on those of labour intensity with regard to the reduction of labour, i.e., using formula (4.8). Table 4.12 presents their calculation.[1] First of all, it can be seen that the relations of the sectors with reference to the skills of the employees

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[1] A comparison of formulae (4.8) and (4.9) shows clearly that the index of labour intensity with regard to reduction of labour $Il_j = I\tilde{l}_j I\psi_j$. For this reason, Table 4.12 presents, first of all, the calculation result for $I\psi_j$. The index of the sum total of wages for each sector is divided by the index of the number of labourers; the resulting index of compensation per employee is then divided by the index of compensation per employee for sector 1, the result for each sector j being taken as $I\psi_j$ and shown in column 7. By multiplying $I\psi_j$ by $I\tilde{l}_j$ taken from Table 4.11 we obtain Il_j. These indices themselves are given in column 9 and their ranks for the sectors in column 10.

The calculation is arbitrary in that the level of complexity of the labour in sector 1 is assumed to be unchanged for the time period under study ($I\psi_j = 1$). It could easily be understood that, should we succeed in determining some change in the complexity of labour in this sector, all $I\psi_j$ would have been multiplied by one and the same magnitude, on which their variation and the results of the comparison of $I\pi_j$ and Il_j do not depend.

Table 4.12

Estimation of Indices of Reduced Labour-Intensity Coefficients (for GNP) in Nine Large Sectors of the US Economy for 1973-1981

	Share in economy, per cent	Compensation of employees: Billions of dollars, 1973	Compensation of employees: Billions of dollars, 1981	Compensation of employees: Index numbers $\frac{1981}{1973}$	Full-time equivalent employees indices, $\frac{1981}{1973}$	Index numbers of compensation per employee, $\frac{1981}{1973}$	Reduced index numbers of compensation per employee, $\frac{1981}{1973}$	Labour-intensity index numbers $\frac{1981}{1973}$	Reduced labour-intensity indices, 1981/1973: Magnitudes	Reduced labour-intensity indices, 1981/1973: Ranks
	1	2	3	4	5	6	7	8	[illegible]	10
1. Agriculture, forestry, and fisheries	4.3	7.4	17.4	2.35	1.16	2.03	1.00	0.96	0.96	6
2. Mining	2.2	8.7	34.1	3.92	1.78	2.20	1.08	1.53	1.66	1
3. Contract construction	7.0	48.4	91.5	1.89	1.02	1.85	0.91	1.12	1.02	2
4. Manufacturing	35.4	229.9	477.8	2.08	1.01	2.06	1.01	0.88	0.89	8
5. Transportation	5.2	36.6	74.6	2.04	1.06	1.92	0.94	1.06	1.01	3
6. Communication	3.1	16.3	40.2	2.47	1.18	2.09	1.03	0.67	0.69	9
7. Utilities	2.7	10.3	24.4	2.37	1.16	2.04	1.01	0.98	0.99	4
8. Wholesale and retail trade	21.0	128.3	283.1	2.21	1.21	1.82	0.90	1.03	0.92	7
9. Services	19.2	104.6	282.6	2.70	1.33	2.03	1.00	0.97	0.97	5
$M(x)$	×	×	×	×	×	×	×	0.97	0.94	×
$\sigma(x)$	×	×	×	×	×	×	×	0.12	0.12	×
$v(x)$	×	×	×	×		×	×	0.13	0.13	×

changed but slightly (to judge by the indices of CE per employee): the figures of column 7 considered to be $I\psi_j$ vary within the range from 0.90 to 1.08. For this reason the indices of adjusted labour intensity (column 9) are similar to those of simple labour intensity (column 8).[1] Accordingly, the estimates of correlation dependence of $I\pi_j$ on $I\tilde{l}_j$ and on Il_j are close to each other.

For 9 sectors, the rank correlation coefficient $I\pi_j$ with Il_j $R = 0.88$. For 7 sectors, i.e., disregarding sectors 1 and 2, where the price index should depend on rental relations, the coefficient $R = 0.96$. The following regression equation was obtained:

$$(I\pi_j - 1.83) = 2.5347\,(Il_j - 0.925), \quad j = 3, \ \ldots, \ 9; \quad r = 0.93,$$
$$\text{residual } v = 0.034.$$

The estimated $I\pi_j$ are, on the whole, rather similar to the actual ones (see Table 4.13).

Table 4.13

Comparison of Actual and Calculated Price Indices (for GNP) in Seven Large US sectors for 1973-1981

	Actual	Price indices calculated	
		in relation to $I\tilde{l}_j$	in relation to Il_j
3. Contract construction	2.21	2.07	2.07
4. Manufacturing	1.74	1.72	1.74
5. Transportation	2.10	1.99	2.05
6. Communication	1.35	1.40	1.23
7. Utilities	2.12	1.87	2.00
8. Wholesale and retail trade	1.79	1.94	1.82
9. Services	1.87	1.85	1.94

Initial statistical characteristics for $x = I\pi_j$:
$M(x) = 1.83$
$\sigma(x) = 0.17$
$v(x) = 0.093$

For the periods 1948 to 1957 and 1957 to 1973, we present in Tables 4.14 to 4.17 only the final results of the calcula-

[1] The ranks of the indices of simple and adjusted labour intensity hardly differ (compare the last columns of Tables 4.11 and 4.12).

Table 4.14

Comparison of Price Indices and Indices of Labour Intensity Coefficients (for GNP) in Large US Sectors for 1948-1957

	Share in economy, per cent	$I\pi_j$		$I\tilde{l}_j$		Il_j	
		Magnitudes	Ranks	Magnitudes	Ranks	Magnitudes	Ranks
1. Agriculture, forestry, and fisheries	8.0	0.74	9	0.76	6	0.76	9
2. Mining	2.5	1.18	6	0.68	7	0.93	6
3. Contract construction	6.6	1.21	5	0.77	5	1.11	3
4. Manufacturing	40.8	1.27	3	0.78	4	1.14	2
5. Transportation	6.6	1.28	2	0.83	3	1.10	4
6. Communication	2.0	1.23	4	0.66	8	0.92	7
7. Utilities	2.3	1.12	7	0.54	9	0.79	8
8. Wholesale and retail trade	20.8	1.10	8	0.84	2	1.04	5
9. Services	10.4	1.52	1	0.95	1	1.29	1
$M(x)$	×	1.21	×	0.80	×	1.08	×
$\sigma(x)$	×	0.18	×	0.07	×	0.13	×
$v(x)$	×	0.15	×	0.09	×	0.12	×

tions. The procedures for processing the data are similar to those employed for the period 1973 to 1981.[1]

The calculations for the three periods allow the following general conclusions to be drawn.

A close correlation has been regularly observed between the indices of prices and those of labour intensity (see Table 4.18). The sign of the correlation in all cases corresponds to the theoretical expectations. The closeness of the correlation in the second and third periods proved, on the whole, to be greater than in the first, the coefficients of correlation amounting here to very high levels; this shows that the changes in the direct labour intensity of output are the crucial factor on which the differences in the indices of net prices depend by large sectors.

Note that the actual differences in the price indices between the sectors tend to increase over time, the initial variation coefficients showing a steady tendency to in-

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1 We excluded from the "Services" sector the subsector "Private Households" for the period 1948 to 1957.

Table 4.15

Comparison of Actual and Calculated Price Indices in Seven Large US Sectors for 1948-1957

	Actual $I\pi_j$	Calculated $I\pi_j$ in relation to:	
		$I\tilde{l}_j$	Il_j
3. Contract construction	1.21	1.22	1.24
4. Manufacturing	1.27	1.23	1.28
5. Transportation	1.28	1.27	1.24
6. Communication	1.23	1.14	1.04
7. Utilities	1.12	1.04	0.90
8. Wholesale and retail trade	1.10	1.28	1.17
9. Services	1.52	1.36	1.44

Initial statistical characteristics for $x = I\pi_j$:

$M(x) = 1.25$
$\sigma(x) = 0.12$
$v(x) = 0.096$

Regression equation for the relationship to $I\tilde{l}_j$:

$(I\pi_j - 1.25) = 0.7794\,(I\tilde{l}_j - 0.81)$; $r = 0.48$;
residual $v = 0.085$.

Regression equation for the relationship to Il_j:

$(I\pi_j - 1.25) = 1.0908\,(Il_j - 1.11)$; $r = 0.85$;
residual $v = 0.051$.

Table 4.16

Comparison of Price Indices and Indices of Labour-Intensity Coefficients (for GNP) in Large US Sectors for 1957-1973

	Share in economy, per cent	$I\pi_j$		$I\tilde{l}_j$		Il_j	
		Magnitudes	Ranks	Magnitudes	Ranks	Magnitudes	Ranks
1. Agriculture, forestry, and fisheries	4.9	2.37	1	0.66	4	0.66	3
2. Mining	1.4	1.21	9	0.57	7	0.46	6
3. Contract construction	7.3	2.14	2	0.99	2	0.76	1
4. Manufacturing	39.0	1.31	6	0.63	5	0.45	7
5. Transportation	5.4	1.42	5	0.59	6	0.50	5
6. Communication	2.8	1.265	7	0.41	9	0.39	9
7. Utilities	2.7	1.257	8	0.49	8	0.40	8
8. Wholesale and retail trade	20.8	1.54	4	0.74	3	0.52	4
9. Services	15.6	1.92	3	1.14	1	0.73	2
$M(x)$	×	1.57	×	0.75	×	0.54	×
$\sigma(x)$	×	0.33	×	0.20	×	0.12	×
$v(x)$	×	0.21	×	0.27	×	0.22	×

Table 4.17

Comparison of Actual and Calculated Price Indices in Eight Large US Sectors for 1957-1973

	Actual $I\pi_j$	Calculated $I\pi_j$ in relation to:	
		$\widetilde{Il}_j$	Il_j
2. Mining	1.21	1.30	1.35
3. Contract construction	2.14	1.83	2.05
4. Manufacturing	1.31	1.37	1.33
5. Transportation	1.42	1.32	1.45
6. Communication	1.265	1.10	1.19
7. Utilities	1.257	1.20	1.21
8. Wholesale and retail trade	1.54	1.51	1.49
9. Services	1.92	2.01	1.98

Initial statistical characteristics for $x = I\pi_j$:

$M(x) = 1.53$
$\sigma(x) = 0.28$
$v(x) = 0.185$

Regression equation for the dependence on $\widetilde{Il}_j$:

$(I\pi_j - 1.53) = 1.2575\,(\widetilde{Il}_j - 0.75)$; $r = 0.92$.
residual $v = 0.073$.

Regression equation for the dependence [redacted] Il_j:

$(I\pi_j - 1.53) = 2.3148\,(Il_j - 0.535)$; $r = 0.99$;
residual $v = 0.032$.

crease. A still more important fact is that these differences are rather well explained by the law of value.

Owing to the exclusion of the rental sectors, the accuracy of the explanation rises fundamentally, with both the initial and residual variation coefficients of the net price indices going down. It is of interest, however, that for all the 9 sectors, the correlation coefficients of these indices with those of labour intensity are very high, particularly in the second and third periods.

It is significant that the accuracy of the explanation of the price indices by simple labour intensity tends to increase over time, with the residual variation coefficient diminishing from the first to the third period when considering the dependence of $I\pi_j$ on $\widetilde{Il}_j$. Both the process of the drawing of prices closer to value and the slackening of the

Table 4.18

Correlation Coefficients Between Price Indices and Indices of Labour-Intensity Coefficients (for GNP) in Large US Sectors

	For the relationship of $I\pi_j$ to $I\tilde{t}_j$			For the relationship of $I\pi_j$ to It_j		
	1948 – 1957	1957 – 1973	1973 – 1981	1948 – 1957	1957 – 1973	1973 – 1981
All 9 sectors						
Spearman rank correlation coefficients	0.43	0.80	0.87	0.79	0.83	0.87
Paired linear correlation coefficients	0.43	0.71	0.82	0.92	0.89	0.57
Variation coefficients for $I\pi_j$:						
initial	0.149	0.211	0.276	0.149	0.211	0.276
residual	0.134	0.149	0.159	0.058	0.080	0.226
*With exclusion of sector 1 and 2**						
Spearman rank correlation coefficients	0.43	0.86	0.82	0.73	0.92	0.78
Paired linear correlation coefficients	0.48	0.92	0.82	0.85	0.99	0.96
Variation coefficients for $I\pi_j$:						
initial	0.096	0.185	0.093	0.096	0.185	0.093
residual	0.085	0.073	0.053	0.051	0.032	0.034

* In calculation for 1957-1973 only sector 1 was excluded.

intensity of skill shifts are behind this.[1] Even so, the following conclusion should be drawn on the whole from the system of calculations: not only theoretically, but also statistically, formula (4.8) is more general and accurate, while formula (4.9) is a particular case of it that yields sufficiently accurate results only under specific conditions.

The following is confirmed for all three periods: the initial variation coefficients $I\pi_j$ are close to those of Il_j.

Variation coefficients (for 1948 to 1957 and 1973 to 1981 for sectors 3 to 9; for 1957 to 1973 for sectors 2 to 9)	1948 to 1957	1957 to 1973	1973 to 1981
$I\pi_j$ (initial)	0.096	0.185	0.093
Il_j	0.083	0.225	0.068

This fact demonstrates how strongly the price indices depend on changes in the values of output.

At the same time, the statistical data do not corroborate the theoretical conceptions that make price indices a direct function of the demand for output. We shall judge about the latter, as above, from the indices of net output (see Table 4.19). The price indices are ranked in diminishing order, as are those of output. In this case, the non-Marxian conception we are to test expects a positive rank correlation to be observed. First of all, the statistical data do not corroborate this correlation sign. Moreover, in the initial period, they show no correlation at all ($R = -0.05$). Subsequently, a correlation appears increasing in time, but with the opposite sign to that expected on the basis of the conceptions discussed. By the end of the period, it becomes perfectly clear that the indices of net prices and of the volume of GNP by sector are correlated, but that this correlation is opposite to that anticipated by non-Marxian conceptions of prices: the higher the output growth rates, i.e., the more rapidly the demand for corresponding products rises, the lower the rates of the rise in net prices tend to be. From 1974 to 1981, the rank correlation coefficient was -0.6.

[1] The following question must be studied: whether it was by chance or by necessity that, towards the end of the period, with strengthening monopolisation, a decrease in the rates of technical progress and of overall economic growth, etc., the ratios of the sectors by employee skills simultaneously became more steady. As a very tentative assumption here we may rather assert a relationship conforming to objective laws.

Comparison of Price Indices and Indices of GNP for Large Sectors of the US Economy

	1957 to 1948				1973 to 1957				1981 to 1973			
	$I\pi_j$		IP_j		$I\pi_j$		IP_j		$I\pi_j$		IP_j	
	Magnitude	Rank	Magnitude	Rank	Magnitude	Rank	Magnitude	Rank	Magnitude	Rank	Magnitude	Rank
1. Agriculture, forestry, and fisheries	0.74	9	1.10	[illegible]	2.37	1	1.17	9	1.31	9	1.21	3
2. Mining	1.18	[illegible]	1.24	7	1.21	9	1.31	8	5.10	1	1.16	[illegible]
3. Contract construction	1.21	5	1.55	3	2.14	2	1.44	7	2.21	2	0.91	9
4. Manufacturing	1.27	3	1.381	5	1.31	6	1.87	5	1.74	7	1.15	7
5. Transportation	1.28	2	1.14	8	1.42	5	1.64	6	2.10	4	1.00	8
6. Communication	1.23	4	1.81	2	1.265	7	3.11	1	1.35	8	1.76	1
7. Utilities	1.12	7	2.08	1	1.257	8	2.44	2	2.12	3	1.177	5
8. Wholesale and retail trade	1.10	8	1.385	4	1.54	4	1.96	3	1.79	6	1.182	4
9. Services	1.52	1	1.31	6	1.92	3	1.95	4	1.87	5	1.37	2
	$R = -0.05$				$R = -0.38$				$R = -0.60$			

On the basis of the theory of value, this phenomenon can be explained as conforming to the law. In sectors with high growth rates of output, there are favourable conditions for accelerated technological renewal of production where additional demand is to be met with the aid of new technologies. Technical progress leads to an accelerated decrease in the unit value of output in general and in direct labour intensity in particular. The decrease in relative value, i.e., that compared with the output of other sectors, leads to a fall in the relative price, i.e., in this case to the fact that the net price index in rapidly growing sectors is lower than in the economy on average.

At the same time, the net prices of rapidly growing output are at a level higher than value, as has already been demonstrated (see Table 4.10 and the relevant comments). So the corresponding sectors have an additional source for financing the accelerated development.

All these manifestations of the law of value were felt particularly in the third period. Again it turns out that the factors modifying the action of value as the law of prices have been relegated to the background in recent years and that this law worked directly to an ever greater degree.

Comparison of two conceptions relating to the problem of the ratios of the indices of prices to those of output is very characteristic. There is only one way available for choosing between the two competing theoretical concepts: they have to be brought to consequences relating to the surface of phenomena where they disagree with each other. Then these consequences are compared with the facts. This is the criterion of truth. Ultimately, precisely this criterion makes it possible to distinguish the true concept from false ones. Moreover, not simply some conclusion from a concept but the concept itself is refuted if it inevitably generates a false conclusion. This is precisely the case with the non-Marxian concepts of prices: in refuting the theory of value, they should inevitably come to the conclusion that an accelerated growth of the demand for certain commodities gives rise to an accelerated rise in their prices. Whoever accepts the opposite conclusion will agree, willy-nilly, that the ratios of the prices of commodities are regulated by those of their labour intensity.

The statistical data confirm precisely this. In order to emphasise this fact, let us note that the possible assumption that trends in the changes in net prices and the direct labour

intensity of output coincide only by chance is not correct. Such an assumption would have been admissible if the trends in the changes in relative prices were the same in all the periods, but this is not the case. The rank correlation coefficients between the net price[1] indices were, for the periods:

$$R\,(1948\text{-}57;\quad 1957\text{-}73) = -0.14;$$
$$R\,(1957\text{-}73;\quad 1973\text{-}81) = -0.37$$
$$R\,(1948\text{-}57;\quad 1973\text{-}81) = 0.12$$

(In the brackets, the periods for which the price indices have to be compared are placed next to the symbol of the rank correlation coefficient.) The coefficients, especially for the first and third cases, are close to zero. In other words, in the different periods the structure of prices tends to change in different directions. Yet every time, as already shown, the changes in relative prices correspond accurately to those in the relative labour intensity of commodities of different sectors. Such a coincidence of trends cannot be considered to occur by chance.

4.3. Calculations with US National Accounts Statistics: Manufacturing

The calculations made so far related only to the overall level of prices for 9 major sectors. To estimate the validity of the law of value it is, however, essential to know how accurately it operates not only at the level of large sectors taken as a whole, but also within each of them. If the indicators for some sector are taken in their total, the differences within it are cancelled out. It is essential to find out whether such differences could, in turn, be derived from the law of value. Below it is demonstrated that this is possible with a rather high degree of accuracy using, as an example, only one of the large sectors, manufacturing.

In the national accounts system, manufacturing is represented by 21 industries (10 industries producing non-durables and 11 industries producing durables). The more disaggregated the classification considered is, of course, the greater is the share of chance factors in price-formation. Let us first trace the variation of the ratio of compensation

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[1] The reader can easily check our calculation using the data of Table 4.19.

Table 4.20

Estimation of Prices Deviations from the Value-Level Prices for the Industries of Manufacturing of the USA for 1972

№ of the industry	Realised national income, millions of dollars	Compensation of employees, millions of dollars	Column 2 to column 1	Share in the total realised national income, share of unity	Weighted CE per national income ratio (3×2)
A	1	2	3	4	5
1	20,286	16,818	0.829	0.0806	0.0668
2	1,730	749	0.432	0.0069	0.0030
3	8,374	7,409	0.884	0.0333	0.0294
4	9,640	8,526	0.884	0.0383	0.0338
5	9,421	7,701	0.817	0.0374	0.0306
6	13,530	11,178	0.826	0.0537	0.0444
7	18,337	12,864	0.701	0.0728	0.0510
8	7,088	2,893	0.408	0.0281	0.0115
9	7,421	6,301	0.849	0.0295	0.0250
10	2,116	2,043	0.965	0.0084	0.0081
11	7,644	5,231	0.682	0.0304	0.0208
12	4,526	4,015	0.887	0.0180	0.0159
13	8,670	7,104	0.819	0.0344	0.0282
14	18,474	16,131	0.873	0.0734	0.0640
15	17,669	15,286	0.860	0.0706	0.0607
16	27,294	22,716	0.832	0.1084	0.0902
17	23,353	19,835	0.849	0.0927	0.0787
18	14,864	14,022	0.943	0.0590	0.0557
19	20,211	13,704	0.678	0.0803	0.0544
20	6,457	5,122	0.793	0.0256	0.0203
21	4,586	3,658	0.797	0.0182	0.0145
Total	251,811	203,304		1.0000	0.8071
Arithmetic mean			0.791		0.807
Mean square deviation		×	0.140	×	0.104
Coefficient of variation		×	0.177	×	0.129

Source: see Table 4.1

to the realised national income for 1972 (see Table 4.20).[1] Here we omit the calculation of simple labour intensity, i.e., without regard to the reduction of labour (the indicators of employment in annual full-time equivalent employee as presented in Table 4.1). We have made such estimates

[1] We do not give the names of the manufacturing industries because, in this work, we cannot analyse their specific characteristics. The reader will find a list of them in the sources of US national accounts statistics already mentioned.

and their results, in form of variation coefficients, are shown in Appendix 2 to this chapter.

The calculation shows the variance of the CE per realised national income ratios to be greater than in the large sectors of the economy: without allowing for the shares, the variation coefficient was 17.7 per cent, while the weighted coefficient was 12.9 per cent. The causes of this are obvious enough. In a more disaggregated classification we are dealing with industries in relation to each of which the actions of the factors giving rise to deviations of prices from value with opposite signs cancel out to a lesser degree than at the level of the largest sectors. It may not be expected that, in dealing with disaggregated industries, one might ultimately succeed in explaining variation with the same degree of accuracy as above: the share of particularly irregular, chance factors in forming real prices would inevitably increase. With regard to this consideration, it is even more essential that the level of prices (or rather the level of net prices) can be successfully explained by formula (4.1) with an average accuracy of 87 per cent.[1] The observation above is corroborated by the distribution of the magnitudes under study having a distinctly pronounced central tendency. Within the range of ± 5 per cent of the average the indicators are concentrated for the industries that, in 1972, produced 36 per cent of output (calculated in terms of the realised national income); within the range of ± 10 per cent—those of the industries accounting for 71 per cent of output. Yet there were several industries that differed sharply from the average level as regards the CE per national income ratio (industries 2, 8, 10, 18), which brings about a higher varia-

[1] Calculations similar to those shown in Table 4.20 were also made for the whole set of industries involved (according to the national accounts statistics) in the above major sectors of the US economy. These totalled 47 sectors: 2 in agriculture, forestry, and fisheries, 4 in mining, 21 in manufacturing, 7 in transport, 2 in communication, and 11 in services; in the rest sectors, industries are not indicated. The variation coefficients for such a set of industries is only slightly higher than for manufacturing alone, being (with regard to reduction of labour and the shares of industries) from 15 to 22 per cent in the various years. Thus, the variation of CE per realised national income ratio for this list of industries might be explained quite well directly from formula (4.1) with an accuracy of 78 to 85 per cent. With regard to the factors giving rise to deviations of prices from value, it seems possible to bring it to the level of approximately 90 per cent. How one such factor is taken into account is exemplified in the text by analysing the indicators for manufacturing.

Table 4.21

Comparison of Prices Deviations from Value-Level Prices and Fixed Capital to Number of Employees Ratios for 1972 by Industries of Manufacturing

Industries numbers	Fixed capital by the beginning of 1972, millions of dollars	Number of full-time equivalent employees, thousands of persons	Col. 1 to col. 2 ratio, thousands of dollars per employee	Compensation of employees per realised national income ratio, doll./doll.	Calculated ratio of compensation of employees per realised national income, doll./doll.	Deviation of calculated ratio from actual one
1	17,852	1,650	10,819	0.829	0.8196	—0.0093
2	686	74	9,270	0.432	0.8294	0.3974
3	5,854	969	6,041	0.884	0.8497	—0.0342
4	3,694	1,307	2,826	0.884	0.8699	—0.0140
5	13,182	674	19,557	0.817	0.7646	—0.0523
6	9,711	983	9,879	0.826	0.8255	—0.0004
7	26,999	993	27,189	0.701	0.7166	0.0156
8	11,864	183	64,830	0.408	0.4796	0.0716
9	6,347	612	10,370	0.849	0.8224	—0.0265
10	537	286	2,227	0.965	0.8737	—0.0912
11	5,377	603	8,917	0.682	0.8316	0.1496
12	1,754	488	3,594	0.887	0.8651	—0.0218
13	9,027	645	13,995	0.819	0.7996	—0.0193
14	28,460	1,214	23,443	0.873	0.7401	—0.1328
15	10,183	1,360	7,487	0.860	0.8406	—0.0193
16	15,690	1,864	8,417	0.832	0.8347	0.0027
17	12,759	1,810	7,049	0.849	0.8433	—0.0056
18	8,294	1,085	7,644	0.943	0.8396	—0.1033
19	9,131	877	10,411	0.678	0.8222	0.1442
20	3,325	455	7,307	0.793	0.8417	0.0487
21	1,686	416	4,052	0.797	0.8622	0.0652
	$M(x)$	×	12,816	0.807	×	×
	σ	×	11,044	0.104	×	×
	v	×	0.862	0.129	×	×

Equation of regression:
$(x_j - 0.807) = -0.00000631\ (y_j - 12816)$; $r = -0.67$.

tion coefficient than that resulting from a study of the large sectors of the economy. The reasons for such distinct differences are a subject for special study.[1]

Table 4.22 (columns 1 and 2) shows the final results of similar calculations for all the years of the period under

[1] Industry 2 representing the production of tobacco goods differs by the fairly well known specific characteristic of price formation yielding some kind of monopoly rent; this rent seems to partly remain in the hands of the entrepreneurs of this industry, which then explains the low indicator of the ratio under study here.

Table 4.22

Coefficients of Variation of Labour Intensity of Realised National Income by Industries of Manufacturing (with Regard to Labour Reduction)
(per cent)

Year	Without regard to sectoral shares	With regard to sectoral shares	With regard to the correlation dependence on capital composition	Coefficient of pair correlation
	1	2	3	4
1948	18.7	18.4	10.2	—0.83
1949	18.0	16.1	10.5	—0.76
1950	19.7	18.6	12.6	—0.73
1951	17.7	16.8	9.7	—0.81
1952	15.8	13.9	8.2	—0.81
1953	17.6	15.1	7.6	—0.86
1954	16.7	14.5	7.3	—0.86
1955	18.4	17.2	10.1	—0.81
1956	17.3	15.4	6.6	—0.90
1957	16.0	13.1	6.1	—0.89
1958	15.4	11.6	6.6	—0.82
1959	17.6	14.2	8.9	—0.78
1960	16.7	13.5	8.4	—0.78
1961	16.5	12.7	8.6	—0.74
1962	16.8	13.2	10.0	—0.65
1963	17.2	13.9	10.9	—0.62
1964	16.9	13.4	9.9	—0.68
1965	17.0	14.2	10.4	—0.68
1966	16.8	13.2	8.4	—0.77
1967	17.5	13.2	7.8	—0.81
1968	17.5	13.5	9.4	—0.72
1969	17.7	13.3	9.9	—0.66
1970	17.9	13.0	9.0	—0.72
1971	18.1	13.6	10.4	—0.65
1972	17.7	12.9	9.6	—0.67
1973	18.3	14.1	9.5	—0.74
1974	20.1	18.6	8.0	—0.90
1975	18.8	15.9	9.3	—0.81
1976	20.0	18.2	×	×
1977	19.3	16.4	×	×
1978	19.1	16.0	×	×
1979	20.6	18.6	×	×
1980	24.4	23.8	×	×
1981	21.9	20.9	×	×

review. As can be seen, up to the early 1970s inclusively, a trend could be observed towards a decrease in the variation of prices around value. Over approximately the following 10 years, however, the variation has again increased. The

reasons for this require special analysis. At any rate, the trend characteristic of manufacturing over the last decade differs from that for the economy as a whole. It may be assumed that the obstacles hindering the redistribution of surplus-value, due to the dominance of monopolies, prove less pronounced within the large sectors of the economy than between them. Nevertheless, the question has to be answered as to why the variation has reversed and began to increase again in recent years, bearing in mind in particular that the trend towards a decrease in growth rates was felt clearly in manufacturing as well.

The redistribution of surplus-value within manufacturing turns out to be markedly correlated to the differences between industries in the organic composition of capital. We shall interpret this quantity as before, by the ratio of the net fixed capital advanced to the number of employees.[1] The course of the correlation calculation is shown for 1972 (see Table 4.21).

As we repeated such calculations for a large number of years (1948 to 1975, for the others we could not obtain data on accumulated fixed capital), it is reasonable here to make use of the data on fixed capital not in 1972 prices, but in the relevant estimate historically taken shape by the beginning of each year and based on the original prices at which the elements of capital were acquired, as it was accumulated. Obviously, if we are dealing with the rate of profit, the latter is to be estimated not for a capital in terms of so-called constant prices in one and the same year, but by the real sum total of money capital accumulated during a set of successive years. Data on fixed capital by industry were taken from the same source as above.[2]

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[1] Let us repeat once more that the method is obviously not accurate enough because no allowance is made for advanced circulating capital. We do not possess the necessary accounts for doing so.

[2] The following correspondence of manufacturing industries to the sectors of the statistics of fixed capital has been accepted:

Industry	Sector	Industry	Sector	Industry	Sector
1	14	8	31	15	39-42
2	15	9	32	16	43-52
3	16,17	10	33,34	17	53-58
4	18,19	11	20,21	18	60,61,13
5	24,25	12	22,23	19	59
6	26	13	35,36	20	62,63
7	27-30	14	37,38	21	64

Taken into account the correlation under study, the residual variation coefficient in 1972 was 9.6 per cent. The specific characteristics of the ratios of employee remuneration to the national income for sectors 8, 10, 18 were largely explained: the residual deviation of the estimated ratio from the actual one for these industries was not much higher than the average (see column 6 of Table 4.21). At the same time, the error for sector 2 is very great, this seeming to provide additional proof of the statement that the specific characteristic of the sector under study is due to a specific monopoly effect.

The corresponding correlation calculations for 1948 to 1975 show that the residual variation coefficient is from 13 to 6 per cent (see Table 4.22, columns 3 and 4). It was especially low in the second half of the 1950s. The redistribution of surplus-value by means of price in favour of the sectors with a relatively high organic composition of capital seems to have been most extensive in that decade. At the same time, the calculations for the whole period show that this redistribution did not lead to a levelling of the profit rate in manufacturing industries.

Thus, while above, when examining the large sectors of the economy, we concluded that a substantial excess of the organic composition of capital is needed in order for surplus-value to be redistributed in terms of prices in favour of the relevant sector, here, as far as manufacturing industries are concerned, the conclusion is that such a redistribution does regularly occur, but does not result, however, in a levelling of the profit rate.

The variation in the CE to national income ratio is not fully explained by its dependence on the organic composition of capital. The deviations of the estimated figures from the actual ones (obtained if correlated with the indicators of the organic composition of capital) proved quite steady (in terms of sign and to a certain extent of order). This stability of errors testifies that there are steady factors of which the correlation calculation takes no account. The study needs to be continued. Here we shall merely indicate some directions.

In accordance with the presented feature of the mechanism by which the law of value operates, the CE to national income ratio may be expected to be lower in industries with a relatively high growth rate of output. There are reasons to believe that such a relation is actually observed in US

manufacturing: special calculations for 1958 to 1973 show that, if this factor is taken into consideration, the residual variation drops to an average of 8 per cent.

The question may also be raised as to whether or not there is any difference in the rate of surplus-value from industry to industry that might substantially account for the variation in the ratio under study.

The theory does not pose the unity of the labour power market as absolute, since differences in the skills of workers are not levelled overnight and therefore constitute an inevitable obstacle to the flow of labour power from one industry to another. These differences cannot isolate the labour power markets absolutely, yet they can engender a stable difference in the sectoral rates of exploitation. Let us note, in this connection, that the rate of exploitation should, in principle, be somewhat higher in industries where more skilled labour is employed.

The quantity of value created per unit time increases in direct proportion to the rise in skills, but this is not precisely true of the value of labour power itself. The reproduction of a skilled worker requires correspondingly higher outlays on his education and the satisfaction of his other cultural requirements; it also demands extra outlays on all his other needs (for food, clothing, housing, and the like) since higher skills are connected with a more developed overall stereotype of living standards: here the law of complexness of the development of requirements operates.

Even so, the growth of such outlays inevitably takes place much more slowly than that of expenditures on education (both general and special). This is confirmed by budget surveys; a growth of the total income is accompanied by a drop in the part of it used to satisfy the indicated requirements. In this case, there are reasons to believe that the value of skilled labour power grows somewhat more slowly than the value created by it. This, however, is tantamount to a higher rate of exploitation of such labour.

As the preliminary analysis already performed shows, these assumptions are confirmed statistically: as a rule, a lower ratio of CE to the realised national income is observed in industry with relatively high average wages.[1] The wages

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[1] An analysis of the correlation between the ratio of CE to realised national income and the average wage for manufacturing industries was carried out for 1972. Excluding industries 2, 8, 11, and 18, which are characterised by the particularly small (the first three of them)

are taken as the indicator of the level of skills. If this is so, the statistics demonstrate that more skilled labour yields more profit per 1 dollar of pay. The issue requires further study.

4.4. Calculations with US Input-Output Tables

So far, we have studied the expression of the law of value as applied to the gross money income of sectors, i.e., the sum total of the revenue from their commodities, minus the material costs. Such, precisely, are formulae (4.1) and (4.6). The merit of this approach is that it allows the mass statistical data of national accounts for many years running to be used in the study. Its shortcoming is the impossibility of comparing the total value of commodities with the sum total of money receipts from their sale. Above it was assumed that, in an analysis of total value, the indicators characterising price deviations from it would diminish. Such an analysis is possible relying on the statistical data provided by I-O tables.

Statical analysis. As applied to the total value of commodities, the law of value may be expressed as follows:

$$\frac{w_j Q_j}{M_j} \to h \text{ for all } j = 1, \ldots, n, \tag{4.10}$$

where w_j is the total value of a unit commodity of kind j; Q_j is the quantity of corresponding commodities sold over the time period for which the value of commodities is determined; M_j is the sum total of the prices of these commodities; h is a constant.

The magnitudes of value are determined, as in Chapter 2, from the simultaneous equations:

$$w_j = \sum_i w_i a_{ij} + l_j; \quad i, j = 1, \ldots, n,$$

where a_{ij} are the coefficients of direct material inputs (including that for replacing of fixed capital withdrawn per unit output); l_j are the coefficients of direct labour inputs.

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or particularly large (the last one) ratios, the residual coefficient of variation was 5.7 per cent. The specifics of industry 2 have already been mentioned; the low ratio in industry 8 is well explained by the extremely high ratio of fixed capital to the numbers employed (see Table 4.21). The specifics of industries 11 and 18 are to be studied further.

Below the coefficients a_{ij} and l_j, based on the data of the I-0 table in money terms (the US 1972 input-output table in the corresponding prices) will be used. For this reason, the magnitudes of total labour inputs w_j calculated using these coefficients have, from the very outset, the following dimensions: labour inputs per 1 dollar of receipts from commodities in current prices. As applied to such information, the law of value, if it operates directly, appears as follows:

$$w_j \to h \text{ for all } j = 1, \ldots, n. \qquad (4.11)$$

Expression (4.11) means: in accordance with the law of value, the total amount of labour necessary in order to receive a fixed sum total of money on the market (for instance, 1 billion dollars) tends to be equal for all industries. Moreover, it is implied, as before, that, considering the mechanism by which modifying factors, too, operate, a certain variation of the magnitudes w_j may be derived from the law of value.

Labour inputs should, of course, be measured with regard to their reduction. When using the information of the I-0 tables this means that l_j are determined as the compensation of employee per 1 dollar of output; correspondingly, w_j are seen as the coefficients of the full pay intensiveness of 1 dollar of output.[1]

Statistical I-0 tables lack the part of the coefficients of direct material inputs that describes the replacement of fixed capital expended. This disadvantage is also inherent in the US I-0 table for 1972. Hence a certain inaccuracy in the calculation of the coefficients w_j. No account was taken of the flows of the I-0 table first quadrant while calculating coefficients a_{ij} (the basis for the calculation of w_j). These flows do not exceed 0.5 million dollars (the precise size of such flows is not indicated in the published table either).

The initial I-0 table contains 79 industries for which the product flows in the first quadrant are shown. We have excluded two of them: industry 70 "Finances and Insurance" and industry 71 "Real Estate and Rent", for the reasons already discussed. For comparison with the indicators obtained from processing national accounts, we present the ratios of CE to realised value added and the indicators of total

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[1] Also of a certain interest is the determination of the simple total labour intensiveness of output (in terms of labour time per one dollar). We do not present these indicators in the current work.

Table 4.28

Direct Labour Intensity of Value Added and Total Labour Intensity of Output by Input-Output Industries (with Regard to Labour Reduction) for 1972

($/$)

Industries numbers	Direct labour intensity	Total labour intensity	Industries numbers	Direct labour intensity	Total labour intensity
A	1	2	A	1	2
1	0.166	0.297	42	0.663	0.631
2	0.108	0.228	43	0.688	0.654
3	0.301	0.394	44	0.639	0.629
4	0.660	0.597	45	0.689	0.655
5	0.315	0.395	46	0.727	0.674
6	0.434	0.467	47	0.781	0.721
7	0.618	0.604	48	0.703	0.663
8	0.146	0.217	49	0.751	0.689
9	0.488	0.506	50	0.739	0.685
10	0.447	0.473	51	0.696	0.669
11	0.746	0.654	52	0.624	0.620
12	0.826	0.721	53	0.768	0.692
13	0.865	0.775	54	0.615	0.614
14	0.487	0.437	55	0.642	0.623
15	0.292	0.366	56	0.824	0.757
16	0.805	0.638	57	0.825	0.734
17	0.966	0.666	58	0.694	0.643
18	0.494	0.539	59	0.569	0.598
19	0.115	0.737	60	0.936	0.802
20	0.598	0.563	61	0.860	0.708
21	0.829	0.665	62	0.728	0.677
22	0.774	0.664	63	0.536	0.553
23	0.795	0.687	64	0.638	0.610
24	0.606	0.579	65	0.622	0.604
25	0.769	0.651	66	0.452	0.475
26	0.721	0.663	67	0.585	0.571
27	0.493	0.507	68	0.266	0.332
28	0.629	0.570	69	0.515	0.521
29	0.446	0.510	70	0.538	0.546
30	0.614	0.572	71	0.565	0.568
31	0.399	0.339	72	0.665	0.562
32	0.635	0.604	73	0.498	0.541
33	0.343	0.386	74	0.558	0.548
34	0.819	0.662	75	0.687	0.656
35	0.658	0.620	76	1.027	0.915
36	0.633	0.594	77	0.442	0.510
37	0.692	0.628			
38	0.596	0.563			
39	0.672	0.627			
40	0.715	0.651			
41	0.749	0.677			

labour intensity (with regard to reduction of labour) (see Table 4.23)[1].

The weighted variation coefficient of magnitude w_j for the 77 industries turned out to be 22 per cent. Let us exclude industries 1 and 2 relating to agriculture. In them, the ratio of CE to realised value added is 0.166 and 0.108 respectively, whereas the average for the 77 industries is 0.627. The set of the factors already mentioned that diminish this ratio are obviously operating here: ground rent and the exclusion of the earnings of farmers who do not exploit the labour of others from the sum total of pay. Let us also exclude industry 68 "Utilities" (with a ratio of 0.266) for the above reasons. The three industries with a 7.5 per cent share of the gross product thus being excluded, the weighted variation coefficient is already 17.3 per cent.

Let us focus on the calculation results obtained.

The weighted variation coefficient of magnitudes w_j hardly differs from that obtained by simple averaging of these magnitudes, as is demonstrated by the following data:

	With simple averaging	With regard to the shares of the industries
Arithmetic mean of magnitudes w_j	0.589	0.558
Mean square deviation	0.125	0.123
Variation coefficient	0.213	0.220

This is to be expected because, if the economy is presented in sufficiently disaggregated form, the shares of the industries covered by averaging do not differ as sharply as when major industries are considered, and a mutual cancelling out of the differences in the shares of the industries with high and low (compared with the average) magnitudes under study is inevitable. This is demonstrated by calculations for 21 manufacturing industries. This specific of the calculations manifests itsefs most fully when 77 industries are studied.

The variation coefficient of magnitudes w_j, expressing total labour inputs per 1 dollar of realised commercial out-

[1] Realised value added is defined as the difference between the sum total of proceeds from the commodities of each industry (the total for the corresponding column of the I-O table) and the sum total of material inputs in money terms (i.e., the sum of the flows into the given industry from all the 77 industries); it is greater than the realised national income since these flows do not take account of expenditure on replacing of fixed capital expended and also due to the exclusion of a number of minor flows.

put is much lower than that of the magnitudes expressing the ratio of CE to realised value added, for which it is 0.304. This confirms the above statement that the real deviations of total prices from total values are even smaller than was found from analysing the indicators characterising such deviations with reference only to newly created value realised in the industries.

Finally, the calculations with the I-O table additionally corroborate the thesis that the average, generalised expression of the amount of labour inputs per 1 dollar of realised value is not simply a function of the corresponding sectoral indicators: it is decisively determined by the rate of surplus-value and then varies somewhat in the industries under the impact of various factors modifying value and forming the mechanism by which the law of value operates. This statement assumes that the rate of surplus-value can be determined closely enough using the sum total of the prices of commodities consumed by industrial workers (as vital means of subsistence) and of surplus-product. We do not possess full enough information to determine the rate of surplus-value accurately, but, as a first approximation, we may use the estimate of the share of the consumption fund in the final product. If this share calculated in terms of prices and the share calculated in estimates proportional to the values of output approximately coincide, the conclusion should be drawn that the former estimate is an external manifestation of the latter, corresponding to the law of phenomena.

According to the US I-O table for 1972 the two estimates are as follows:

	($ billions)	
	in current prices	in prices proportional to value
Final product	934.2	540.5
Consumer goods	579.8	311.5
The share of the consumption fund in the final product, per cent	62	58

When rounded, the two estimates coincide, the share of the consumption fund amounting to approximately 60 per cent. There are reasons to believe that the former is nothing but a manifestation of the latter, and quite an accurate one in quantitative terms.[1]

[1] To determine the rate of surplus-value, it is necessary, in addition, to estimate the share of the consumption of the industrial workers

By analogy with the studies set out in the previous paragraphs, the factors giving rise to deviations of price from value should be studied. The total capital intensities of output by the industries of the I-O table (according to fixed capital advanced) have been estimated, and a correlation analysis has been carried out of the dependence of magnitudes w_j on the ratio between the total capital intensity and total labour inputs. This dependence was expected to explain in part the variation of magnitudes w_j as a consequence of the redistribution of surplus-value in favour of the industries with a higher organic composition of capital. The expectations came true: the variation coefficient for the 77 industries went down from 22 to 19 per cent (if the abovementioned three industries are excluded, from 17.3 to 14 per cent). Should account be taken of other factors already mentioned (differences in the growth rates of the industries, a certain variation in the rate of surplus-value depending on the skills of the workers, and the like), the variation coefficient will be brought down to roughly 10 per cent. At this stage of the study, too, however, it may be concluded that the statistics correspond fully to the assumptions arising from the specifics of the law of value.

4.5. Summary

Statistical verification allows the following conclusions to be drawn concerning the quantitative fulfilment of the law of value and of other related economic laws:

— the prices are determined to a decisive extent by the law of value as such; the formulae expressing its characteristics, assuming its direct action, make it possible to explain the realised national income for major industries with an accuracy of up to 90 per cent or more (especially for recent years), and for disaggregated industries with an accuracy as high as 80 per cent or more;

— the dynamics of net relative prices are mostly explained by those of direct labour intensiveness (with regard to reduction of labour), and the dependence of the changes in the total relative prices of commodities on their total labour intensiveness can be traced;

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in the total consumption fund, by component, as well as to exclude from the final product the part intended for replacing fixed capital expended.

— factors inherent in the law of value give rise to stable deviations of prices from value (the market "demand—supply—prices" mechanism, the redistribution of surplus-value in favour of industries with a high organic composition of capital, and the like); consideration of these factors already makes it possible to raise the accuracy of the explanation to 95 per cent or more for aggregated industries and to 85 to 90 per cent or more for disaggregated ones.

The statistical observations were analysed assuming the value newly created by production workers to be proportional to their wages. The only exception was the calculation already mentioned of the dependence of the ratio of pay to realised national income (for manufacturing) on the average wages of workers, taken as indicators of their skills. Such a dependence was actually observed. In other words, the rate of surplus-value seems to tend to rise as the skills of the industrial workers increase. The same calculation, however, showed that this rise in the exploitation rate in statics is not great (i.e., if the exploitation rate of workers in the various industries is studied for the given time period). So the principle usually accepted in the theory concerning the equality of the rate of surplus-value in different industries remains the sufficiently precise first expression of the law operating in this respect. This was the assumption accepted in the basic analysis ensuring sufficiently great precision in the explanation of the observed facts.[1]

The Marxist-Leninist theory considers all the laws of the capitalist economy without exception as very approximate trends, on average. Let us recall that Engels stressed this circumstance in particular with reference to the law of value, to the law of the overall rate of profit, etc. in his well-known

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[1] Let us note that the statistics were processed directly to verify the operation of the law of value as expressed by formulae (4.1), (4.6), (4.8), (4.9), (4.11). The same information can be processed by a similar transformation—as the ratio of surplus-value realised in the industries (excess of the realised national income over pay) to the workers' wages. Given such a conversion of the data, the variation coefficients themselves tend to grow significantly, since only that part of the receipts directly affected by the factors making prices deviate from value is now presented in the numerator of the ratio. Since, however, consideration of these factors inherent in the law itself explains the level of net prices, it also explains the variation in such a ratio. It may easily be demonstrated that the variation tends to go down rapidly if the dependence on the differences in the organic composition of capital, the growth rates of output, and the like between industries are taken into account.

letter to Schmidt of 12 March 1895.[1] If the mass of facts observed can be presented as a consequence of theoretical laws with an accuracy of 90 to 95 per cent or even more the conclusion follows that the laws do, in fact, operate. The US statistics show that the law of value holds.

Yet since the law of value is valid, the whole conglomerate of bourgeois theoretical conceptions refuting the concept of value and surplus-value cannot be considered correct.

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[1] See: "Engels to Conrad Schmidt in Zurich, March 12, 1895", in: Marx, Engels, *Selected Correspondence*, p. 456-60.

Chapter 5

THE THEORY OF THE SOCIAL UTILITY OF THE PRODUCTS OF LABOUR

The law regulating the proportions of the exchange of commodities (determining their exchange value) is directly formulated abstracting from the use-value of commodities. This fact was also reflected in the formulae providing a mathematical expression of the law of value. It by no means follows from this, however, that the Marxian commodity theory is reduced to that of value and ignores use-value.

5.1. A Methodological Approach to Studying the Utility of Products in Economic Theory

The utility of products as a research topic in economic theory. In accordance with the definition already presented in Chapter 1 (section "Utility of Products"), the use-value (utility) of a thing consists in its ability to meet the requirements of society (both productive and personal). Things possess such an ability thanks to their natural (mechanical, physical, chemical, biological, geometrical) properties, which cannot, of course, be the subject of economic science. Nevertheless, the various natural properties do not, in themselves, make a thing useful even though they do form the basis of its utility. Some properties may become useful, but social development is required to discover this. It was Marx who distinguished between the utility of products and the basis intrinsic in the natural properties of things conditioning this utility: "A commodity is, in the first place, an object outside us, a thing that by its properties satisfies human wants of some sort or another...

"It [every useful thing] is an assemblage of many properties, and may therefore be of use in various ways. To discover the various uses of things is the work of history...

"The utility of a thing makes it a use-value. But this utility is not a thing of air. Being limited by the physical properties of the commodity, it has no existence apart from that commodity. A commodity, such as iron, corn, or a diamond, is therefore, so far as it is a material thing, a use-value, something useful."[1]

The commodity is, according to Marx, use-value not simply because it possesses certain physical properties, but because these properties can meet certain human needs. In other words, a thing becomes a use-value thanks to its correspondence to historically determined social needs. So utility is socio-economic in nature. As such, it constitutes the subject-matter of economic science.

Economic theory is called on to study the objective foundations of the social evaluation of utility, to determine the criteria for such an evaluation, to point out the ways to compare the utilities of the various objects. The importance of such a study consists in the fact that, without it, the scale of production and the uses of the objects cannot be determined correctly. Hardly anyone would deny that the laws determining the scale of the social production of various products are social laws. The proportions of the development of production are, of course, to a certai extent technological ones. Nevertheless, the products' structure is primarily a socio-economic problem.

Although the opinion that the classics of Marxism excluded use-value from the subject-matter of economic theory has been refuted repeatedly in recent years, it still persists. Since this opinion hampers the development of economic science, we consider it necessary, without fearing accusations of quotation-mongering, to present the main statements made by Marx and Engels on the given issue. This is particularly necessary since, when formulating their attitude to the issue of utility, they also determined the foundations of its scientific analysis.

Marx, of course, drew a precise distinction between political economy and commercial knowledge of commodities, which studies specific useful qualities of commodities. The conclusion is often drawn from this that Marx completely excluded the use-value from the subject-matter of political economy. It should be noted that this conclusion by no means follows even from those theses in Marx's works crit-

[1] Karl Marx, *Capital*, Vol. I, pp. 43-44.

icising the confusion of political economy and commercial knowledge of commodities. Let us quote the statements in which this criticism is expressed most sharply.

"Whatever its social form may be, wealth always consists of use-values, which in the first instance are not affected by this form. From the taste of wheat it is not possible to tell who produced it, a Russian serf, a French peasant or an English capitalist. Although use-values serve social needs and therefore exist within the social framework, they do not express the social relations of production. For instance, let us take as a use-value a commodity such as a diamond. We cannot tell by looking at it that the diamond is a commodity. Where it serves as an aesthetic or mechanical use-value, on the neck of a courtesan or in the hand of a glass-cutter, it is a diamond and not a commodity. To be a use-value is evidently a necessary prerequisite of the commodity, but it is immaterial to the use-value whether it is a commodity. Use-value as such, since it is independent of the determinate economic form, lies outside the sphere of investigation of political economy.* It belongs in this sphere only when it is itself a determinate form. Use-value is the immediate physical entity in which a definite economic relationship—*exchange-value* is expressed."[1]

It would be emphasised that Marx criticised the inclusion of use-value in the subject-matter of political economy in those sections of his works which were devoted to clarifying the essence of the exchange-value of commodities, i.e., the determination of value. His criticism was directed at those bourgeois economists who tried to define the exchange-value of commodities by their use-value. Marx wrote: "In the same way the exchange-values of commodities must be capable of being expressed in terms of something common to them all, of which thing they represent a greater or less quantity.

"This common 'something' cannot be either a geometrical, a chemical, or any other natural property of commodities. Such properties claim our attention only in so far as they

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* That is why German compilers write *con amore* about use-values, calling them "goods". See, for example, the section on "goods" in L. Stein, *System der Staatswissenschaft*, Bd. I. Useful information on "goods" may be found in "manuals dealing with merchandise". (Footnote by Marx.)

[1] Karl Marx, *A Contribution to the Critique of Political Economy*, Progress Publishers, Moscow, 1977, pp. 27-28.

affect the utility of those commodities, make them use-values. But the exchange of commodities is evidently an act characterised by a total abstraction from use-value."[1]

A number of important conclusions may be drawn from these statements by Marx. First, he refuted attempts to include natural properties of commodities as such (use-value) in the subject-matter of political economy. Second, he rejected, in particular, attempts to consider use-value as the essence of the exchange-value of commodities and, for this reason, initially disregarded use-value in studying value. Third, he pointed out that use-value, since it reflects the definite form of production, has to be studied by economic theory. Such an understanding of the above statements by Marx is corroborated by his own comments on them.

Thus, in presenting, in a letter to Engels on April 2, 1858, a brief plan of his work "A Contribution to the Critique of Political Economy", Marx pointed out: "I. *Value* is reduced entirely to quantity of labour. Time as the measure of labour. Use-value—whether considered subjectively as usefullness of labour, or objectively as utility of the product—appears here simply as the material presupposition of value, which for the time being drops completely out of the economic determination of the form. Value as such has no other 'substance' than labour itself."[2]

Thus, use-value can only temporarily be removed from the study in analysing value as such.

Marx's *Capital* provides a methodological basis for a correct understanding of the economic significance of use-value. Marx revealed the meaning of the thesis to the effect that use-value is part of the set of problems covered by economic theory when it itself becomes the definiteness of the social form. The ratio of labour inputs in production of the output to its use-value (in other words, in what use-values social labour is embodied) cannot be understood outside a definite form of social production.

The statements by the classics of Marxism reveal not only their attitude to the issue of the economic significance of utility, but also theoretical views of the actual content of the problem of utility. While criticising bourgeois theories of utility, Marxism at the same time scientifically reveals the actual ratio of labour inputs in producing the

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[1] Karl Marx, *Capital*, Vol. I, p. 45.

[2] "Marx to Engels in Manchester, April 2, 1858", Marx, Engels, *Selected Correspondence*, p. 98.

various goods to their comparative utility. Marxism has elaborated a theory of the commodity, combining the theory of value and that of utility. Strictly speaking, Marx's theory of value would not itself have been quite scientific if it had lost touch with the theory of utility, since the commodity is an inseparable unity of use-value and value. Let us consider the methodological fundamentals of the approach taken by Marx and Engels to the given problem.

The objective character of utility. Marx and Engels considered the utility of products to be an objective phenomenon determined not by the subjective, voluntaristic evaluations of individuals, but by the objective processes of the development of society. Their approach to utility is opposed both to the vulgar-materialistic view of utility, which reduces it to a simple totality of natural properties of things, and to the approach of the subjectivists, who considered the estimates of utility primarily as ones depending on the psychology, tastes, and inclinations of individuals.

As already stated, there exists a difference between the objective natural properties of things and their objective importance for the existence and development of society. To point out this difference is of fundamental methodological importance. First, unless this fact is acknowledged, the historical changeability of utility cannot be understood; the utility of objects with equal natural characteristics can change under changing social requirements and also given the discovery of new, formerly unknown properties capable of satisfying new requirements.[1] Second, without acknowledgement of this fact, one cannot speak of comparing the utility of products differing in their natural characteristics.[2] The difference in natural characteristics and utility must, however, be understood correctly.

Acknowledgement of the objective utility of things, not coinciding with their natural properties (though related to them) and being social in nature, constitutes a fundamental difference between Marxism and both vulgar-materialistic

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[1] The utility of products also changes in connection with changes in their natural properties, but it must be borne in mind that it can also change irrespective of this.

[2] It is no mere chance that economists who deny the possibility of comparing the utilities of different products usually ignore the difference between the utilities of products and their basis -the natural properties of these products.

views on the issue and subjectivist views, which fit in well with the vulgar-materialistic approach. Moreover, without denying the subjective dimension of the utility of products, Marxism does not, either, confuse the objective importance of products for the existence and development of society with the subjective utility evaluations by individual consumers.

Marx spoke of objective utility as that of products and of subjective utility as that of the labour creating these products. At the same time, he pointed out that there also exists a subjective aspect to the estimations of product utility. "The product supplied is not useful in itself. It is the consumer who determines its utility. And even when its quality of being useful is admitted it does not exclusively represent utility."[1] Marx also noted that buyers act, of course, according to their own opinion. All these are obvious truths, expressing incontestable, even though the most superficial, facts. In this connection, it should be noted, in general, that people only undertake all actions after corresponding stimuli have been comprehended and have, therefore, obtained a subjective expression. All this does not, however, provide any grounds for interpreting utility subjectively.

Concepts to the effect that to evaluate the utility of different objects is a subjective matter depending only on individual tastes and inclinations, and hence does not obey objective laws, are a consequence of a completely unscientific understanding of the subjective as such, and Marxists should by no means accept this extremely superficial point of view. Different people do, of course, give different estimates of the utility of various objects and there are probably no two families that would spend their incomes in absolutely the same way. The subjective is not, however, something quite independent, being ultimately a reflection and expression of the objective world. Behind the subjective actions of humans lie objective social laws. When a seller and a buyer negotiate on a private market, it looks as if the price of the commodity results from a confrontation between their subjective evaluations. Yet, scientific political economy reveals, behind this externality, the objective basis, the

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[1] Karl Marx, "The Poverty of Philosophy", in: Karl Marx, Frederick Engels, *Collected Works*, Vol. 6, Progress Publishers, Moscow, 1984, p. 118.

law of prices, i.e., value. Similarly, subjective evaluations of utility made by different consumers are based on certain social conditions that determine them and on their objective basis—the real social utility of products. Subjective evaluations taken in their sum are but a reflection of this; they do not emerge spontaneously, but as an expression of the views determined by all aspects of social life, above all the level of its economic development, the social position of the members of society, the level of their incomes, the size of family, intellectual level, and so on.

It is useful to remember that Marx scoffed at Proudhon's idea that the opinions of consumers concerning products were a manifestation of their "free will", i.e., arbitrary power. "The consumer is no freer than the producer. His estimation depends on his means and his needs. Both of these are determined by his social position, which itself depends on the whole social organisation. True, the worker who buys potatoes and the kept woman who buys lace both follow their respective estimations. But the difference in their estimations is explained by the difference in the positions which they occupy in society, and which themselves are the product of social organisation."[1]

Marx came back repeatedly to this idea, thereby laying the scientific foundations for analysing social needs and the utility of products.[2]

It is important to note that acknowledgement of the objective character of utility is related to acknowledgement of its social character. Marxism considers the utility of products from the standpoint of society as a whole, of social groups and classes, this distinguishing it fundamentally from the subjective theory that considers utility primarily from the standpoint of a particular individual. Characteristic of the subjective school is an individualistic approach to utility, of Marxism—a social one.

Subjectivists do not, of course, entirely refute the approach to utility and the related categories (for example, the category of demand) from the standpoint of society. They cannot do so, because the goal of their theory is to explain prices, and the prices at which commodities are sold are, in principle, equal for all consumers. For this reason, sub-

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[1] Karl Marx, "The Poverty of Philosophy", in: Karl Marx, Frederick Engels, *Collected Works*, Vol. 6, pp. 118-19.

[2] See: Karl Marx, *Capital*, Vol. III, pp. 181-82, 194-95.

jectivists acknowledge not only the subjective, but also an objective value of things. Yet the understanding of this objective value is entirely nonscientific.

Eugen Böhm-Bawerk confused the objective utility of things with their natural characteristics. He also acknowledged the existence of an objective exchange-value of the commodity. Yet he considered this objective exchange-value not as a manifestation of the value of the commodity, but as an expression of utility, which itself contradicts the real foundations of equivalent exchange. At the same time, this "objecive" exchange-value was not considered either to be an expression of the objective social utility of commodities, having been derived by Böhm-Bawerk from subjective evaluations of utility.

V. Pareto emphasised that he was studying not the psychological reactions of individual consumers, but objective aspects of the behaviour of the mass of consumers under different conditions, thereby trying to convince the reader of his scientific objectivity. Yet he maintained the same individualistic positions. The point is that Pareto refused to analyse causal links in the economy and thus rejected a causal study of the behaviour of consumers. He suggested that the facts of such behaviour should be simply recorded, in order to derive empirical patterns from them.[1] The reason for such an approach is that Pareto considered human requirements simply as an expression of their tastes, proceeding from the idea of utility as satisfaction experienced by an individual. He believed, however, that this satisfaction was hardly measurable as such. Thus, Pareto's "objective approach" (in reality, pure empiricism) is based on the same individualistic misconception of utility.

John Hicks went still further to suggest that only marginal rates of substitution of commodities should be used, even purely verbal indications of their connection with utility being cast aside. Superficial empiricism is thus taken to the extreme, the idea of causality and, together with it, any attempt at a scientific approach to the phenomenon, is done away with entirely.

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[1] While criticising Pareto's empiricism, it should not be concluded that analysis of mass consumer behaviour would be pointless. On the contrary, such analysis is of particular importance. It can reveal how the phenomena appear on the surface and thereby helps in posing the correct questions concerning the reasons for these phenomena.

There is nothing surprising in this development of the bourgeois doctrines of utility. Their ideas of utility being completely subjective in nature, an expression of the consumers' arbitrary will in fact bars the way to studying the laws regulating the development of social needs and the utility of products. Rejection of the very concept of utility is the logically inevitable final step in doctrines originating in a subjective interpretation of this phenomenon. In its rejection of scientific analysis of the phenomena of commodity production, bourgeois economics could not stop half-way, i.e., ignore value and substitute utility for it, it would also have been forced to deny utility itself. That is why there is no greater misconception than to say that vulgar economics elaborated the issues of utility.[1]

The laws governing the shaping of the social utility of products are, of course, very complicated, utility being dependent on a tremendous number of specific factors and changing over history. Nevertheless, it is governed by the objective laws. It would be wrong to simplify these laws and try to determine utility simply by analysing the natural characteristics of things. The laws governing the formation of utility are social laws, and a researcher has to take them into account in all their true complexity. Yet, given this complexity, the laws are quite cognisable.

The opposition and unity of the use-value and value of products. The Marxist methodology for studying the economic significance of utility is focused on revealing the actual interrelation between use-value and value, on criticising every attempt to confuse them. This is where the main demarcation line might be drawn between the scientific theory of utility and the vulgar theories on this subject. Marxism differs from vulgar political economy not in denying the economic significance of the utility of things, but in criticising attempts to determine value by utility.

The idea of the opposition of value to utility and of their unity was repeatedly emphasised by Marx. A classic example of its consistent elaboration is provided by *Capital*, where the whole system of the political economy of capitalism was derived from the unity of these two opposing aspects of the commodity as a cell of bourgeois production.

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[1] Here we are not considering a number of valuable empirical studies on consumption development carried out within the framework of bourgeois doctrines; rather we are dealing with the theoretical comprehension of these studies.

Vulgar schools in bourgeois political economy try to demonstrate that the law of the exchange of commodities does not imply their equivalence in value, but in utility. Price is seen directly as an expression and a measure of utility. Hence the rejection of the theory of value and surplus-value, which expresses the objective fact that the proletariat is exploited by capital. At the same time, capitalism is shown to be a "natural" formation, complying with "human nature" and geared to satisfy the needs of the members of society. Thus, the confusion of use-value and value serves the purposes of overt bourgeois apologetics.

The value of commodities reflects the socially necessary labour expenditure on their production and is realised by exchange. The utility of commodities is, in its very essence something opposed to expenditure, namely, its outcome, to be realised in consumption. The fact that the labour spent on producing use-values represents their value reflects only certain relations characteristic of commodity production, i.e., the division of labour among the various owners. In fact, utility reflects all the dimensions of society's life, since these determine the level of development, the pattern, and the degree of satisfaction of needs. For this reason alone, utility can in no way determine the value of commodities, value nor can determine their utility. This is felt not only in their qualitative difference, but also in the quantitative expression of value. The value of commodities changes along with the productivity of labour, even though a commodity's ability to satisfy social needs may remain unchanged.

Commodity production is based on a deep social division of labour where the product has no utility for its producer, i.e., can not satisfy the producer's needs. A commodity is a use-value not for its producer, but for others. Thus, the exchange of commodities is by no means an exchange of equivalents as regards utility: everyone engaged in exchange receives instead of a commodity of no use-value to him, a commodity useful for him. If the exchange of commodities were reduced to that of equivalents in terms of utility, it would be pointless and would simply not exist.

Marx wrote: "If therefore, as regards the use-values exchanged, both buyer and seller may possibly gain something, this is not the case as regards the exchange-values."[1] He criticised Condillac, an 18th-century economist, who believed

[1] Karl Marx, *Capital*, Vol. I, p. 156.

that the value of a thing consists solely in its relation to our wants and thus profit would be created in circulation where each of contracting parties obtained some gain by exchanging a useless thing for another, useful one. "We see in this passage, how Condillac not only confuses use-value with exchange-value, but in a really childish manner assumes, that in a society, in which the production of commodities is well developed, each producer produces his own means of subsistence, and throws into circulation only the excess over his own requirements."[1] The same methodological error, where a specific historical approach to the phenomena is lacking, and developed commodity production and exchange are confused with an exchange of surpluses, is also characteristic of vulgar doctrines of utility, which, of course, is not attributable to "childish manner".

While being opposites, value and use-value are closely related to each other, so that without either one of them the commodity itself would not exist. In particular, a product has no value if it lacks social utility. Value is always that of some socially useful product; this is true both of the essence of value and of its magnitude.

The theory of Marx and Engels relating to the opposition and unity of use-value and value, if taken directly in the form in which it is presented, concerns only commodity production, since value is a characteristic not of every product, but only of ones produced for sale, of commodities. Behind the relation of value and use-value, however, a more general relationship may be found between labour expenditure on production and the useful product of this labour, that is a relationship characteristic in some form of all production.

Marx repeatedly pointed out that the regulation of the proportions of capitalist production by the law of value is merely a specific form of the regulation of production by the working time at the disposal of society. Since this regulation is characteristic of all production, "Indeed, *no form* of society can prevent the working time at the disposal of society from regulating production one way or another".[2] For this reason social expenditure on production is always expenditure of labour-time: "*Labour-time*, even if exchange-va-

[1] *Ibid.*, p. 157. The words by Condillac are taken from a quotation by Marx given on the same page.

[2] "Marx to Engels in Manchester, January 8, 1868", Marx, Engels, *Selected Correspondence*, p. 187.

lue is eliminated, always remains the creative substance of wealth and the measure of the *cost* of its production".[1] Similarly, the products of labour, while always being those of social production and satisfying not only their producers' own requirements, are always social use-values.

The objective foundations of utility in general have to be sought in the utility of consumer goods, otherwise no correct understanding is possible of the utility of means of production. To study the utility of consumer goods is the most complex aspect of the issue, inasmuch as the structure of their production is affected by all the processes of social life—both economic and noneconomic, the structure of the output of means of production being, in fact, almost entirely determined by economic and technological laws.[2] Moreover, it is precisely the problem of consumer goods utility that has been confused most by the vulgar schools of bourgeois political economy, but because we are dealing here with the most difficult aspect of the issue, an analysis of it reveals the most fundamental characteristics of the whole.

5.2. The Basic Concepts of the Theory of the Utility of Consumer Goods

Needs, effective demand, welfare, consumption. A scientific basis for a comprehensive analysis of personal consumption and for forecasting its future development is provided by the Marxian theory of the interrelation between production and consumption—a section of overall economic theory. It includes a conceptual apparatus and an advanced materialistic research methodology and reveals general laws of the evolution of needs and consumption.

The basic concepts defined and employed in the considered section of the Marxian general economic theory are those of needs, effective demand, welfare, and consumption.

Needs (wants) are defined as the material, intellectual, and social living conditions the creation of which becomes necessary as society develops and which society, its individual groups, and members aim to attain. Marx distinguished

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[1] Karl Marx, *Theories of Surplus-Value*, Part III, Progress Publishers, Moscow, 1975, p. 257.

[2] The structure of the production of means of production is, of course, to some extent affected by the necessity, when using them, to meet the demands of the hygiene and physiology of labour, and the like, which are no longer determined by economic laws alone.

between physical, intellectual, and social needs.[1] He also divided needs into natural and social ones.[2]

The essence of the concept of needs is shown in the following words by Marx: "...Consumption creates the need for *new* production, and therefore provides the conceptual, intrinsically actuating reason for production, which is the pre-condition for production. Consumption furnishes the impulse to produce, and also provides the object which acts as the determining purpose of production. If it is evident that externally production supplies the object of consumption, it is equally evident that consumption *posits* the object of production as a *concept*, an internal image, a need, a motive, a purpose. Consumption furnishes the object of production in a form that is still subjective. There is no production without a need, but consumption re-creates the need".[3]

Acknowledgement of the existence of a set of various needs is combined in Marxist theory with the assertion that they exist in complexes. "While the extent of these wants differs quantitatively, still there exists an inner relation which settles their proportions into a regular system."[4] The complexes are formed primarily by needs each of which cannot be satisfied—completely or partly—unless the others are. For example, one cannot cook and eat without certain dishes and cooking utensils. This sort of complementarity of needs is known as physical. There also exists social complementarity of them, meaning that the development of a need is correlated with (is engendered by) the rise in the overall level of needs. For example, a society of culturally developed people requires not only a greater quantity of and more intellectual benefits than one at a lower level of culture, but also higher quality goods to meet the needs for food, clothing, housing, utensils, etc. The social complementarity of needs and consumption means that there is a certain minimum of satisfaction of each need for each level of welfare; moreover, the minimum tends to rise (both quantitatively and qualitatively) as welfare does.

While forming a system, needs are not, at the same time, reduced to some "general need": we are dealing with a sys-

[1] See: Karl Marx, *Capital*, Vol. I, pp. 223-24.
[2] See: Karl Marx, *A Contribution to the Critique of Political Economy*, p. 134.
[3] *Ibid.*, p. 196-97.
[4] Karl Marx, *Capital*, Vol. I, p. 336.

tem of diverse needs. Each of the numerous needs has a specific unit of measure. For example, the need for energy required for the functioning of the human body is measured in calories; the requirements for the various substances necessary to build the tissues of the body, in substance weight units; the need for leisure, in hours, and so on. For some needs, no such simple measure can be given and they have to be expressed in units of the products that satisfy them. For example, the need for footwear is expressed in pairs of footwear, that for radios, in the number of sets, and the like.

Needs should be distinguished from the population's effective demand, the difference between them resulting from the following circumstances.

First, not every need is directly aimed at some production output (material or intellectual); some are met by things provided by nature; others, social ones, concern certain conditions of life even if engendered by the development of production, but not representing components of social product (the need for labour, the receipt of a job in an appropriate trade, a desired post, for working conditions, contacts with other people, for diffusion in society of one's knowledge and experience, and the like). Effective demand always applies to some specific product.

Second, some needs, even if commodity-money relations exist in society, are satisfied free of charge.[1]

The effective demand of the consumer himself is only for goods transmitted to the consumer along goods circulation channels.

Third, since the full satisfaction of needs cannot be achieved, effective demand is always smaller than the full demand for the corresponding goods and paid services.

Finally, the demand is also smaller than needs, because the latter's satisfaction is achieved with the help of consumer goods already accumulated.

Two stages should be distinguished in the formation of effective demand: the initial demand which, in more or less pure shape, reflects which commodities the people would like to buy, and the final demand, which is expressed by actual purchases of goods and services. The latter is a

[1] This term is, of course, arbitrary. The production of corresponding goods requires labour. Their producers receive money for the output, but not from the final consumer.

modified expression of the former and depends not only on the wishes of the population, but also on the supply of the respective goods and services.

The social and physical conditions of human existence and development affect the human welfare level. These conditions include the social position of a given social group (or individual), the forms and size of income, the quantity and quality of material consumer goods obtained (food, clothing, housing, household goods, etc.), the quantity and quality of services obtained, work, leisure, and everyday living conditions, the conditions for preserving health and longevity, and opportunities for developing and applying one's abilities. All the aspects of social life, and all the processes of its development affect the well-being of the members of society, this being decisively influenced by the development of social production. The concept of well-being is a generalised, integrated expression of the satisfaction of requirements, which complies with the concept of the integrated character of needs themselves.

Sometimes the concepts of welfare and consumption are identified with each other. In a certain sense, these are, indeed, concepts of the same order, because both of them express the real, actual satisfaction of needs. The concept of consumption is, however, narrower, being the expression of the satisfaction of needs for material and intellectual output as such. Since the processes of developing and satisfying social needs are not analysed in this work (only their impact on the needs for products is considered), we are dealing strictly with the level and structure of consumption-consumer purchases, and the like.

The complementary character of needs leads to consumption as such also developing as the use of a complex of physically and socially mutually complementary goods. Even an individual kind of consumer goods usually serves to meet a complex set of needs (for instance, clothing serves to meet a lot of physiological, aesthetical, and other needs). This applies particularly to the set of various consumer goods. As long as the total level of consumption grows, consumption of definite kinds of good tends to increase and, moreover, new products begin to be consumed, the increase in the consumption of different products varies. At the same time, there is a continuous process of renewal of the set of products satisfying even one and the same need, obsolete ones being replaced by new, better ones. Thus, the propor-

tions of needs and those of consumption to an even greater degree undergo continuous changes throughout history. Their development, on the whole, obeys the law of the rising needs stated by Marx and Lenin.

The higher the level and rate of development of society, the greater, also, the speed at which the well-being of its members grows, the stronger the historical character of consumption proportions develops. They are less and less determined by natural needs and more and more by those associated with the development of culture, the individual, and the like.

The whole process of the rise in the level of consumption is ultimately determined by the development of production. The natural needs (for food, clothing fuel, housing, etc.) in their basic form are, of course, determined by the necessary conditions for the normal vital activities of the human body. From the very beginning of the existence of humanity, however, and with time more and more, even these needs are determined by the development of production: to the extent that production develops, they change quantitatively and qualitatively, i.e., express themselves in a striving to obtain goods of increasing sophistication. In their modern form, these needs are largely the outcome of material and intellectual culture. This is particularly true of intellectual needs.[1]

The subjective form of needs and chance factor in consumer behaviour. While asserting the objective character of the rise and development of needs, it would obviously be entirely wrong to go to extremes and deny the subjective differences between the desires and tastes of society's members. Such differences, being random for society as a whole and its large groups, do exist for individuals. Needs have a subjective form of manifestation, being expressed in the aspirations, desires, and tastes of the members of society. These differences, however, only bring a diversity to the actual manifestation of needs within the framework of their total level brought about by objective processes in social life. It is precisely here that the demarcation line lies between the subjectivist methodology and historical materialism in interpreting the trends in mass consumer behaviour.

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[1] See: Karl Marx, *Capital*, Vol. I, p. 223.

Only superficially does the subjectivist look as if the social trend consists simply of the sum total of individual trends. In fact, before being composed of them, it substantially affects individual goals, is divided into them and specified in them, thereby assuming the form of its own manifestation. That is why the development of consumption must be cognised not as a chance result of arbitrary actions by individuals, but as an inevitable result of the objective conditions of social development that have, on the whole, determined the goals and actions of individuals. Not only the patterns of consumption development, but also all the patterns in general of the development of society, are realised through subjective actions (irrespective of whether the laws of social life are cognised by people), from which it by no means follows that there are no objective social laws fully independent of people's will. The latter point of view would render any scientific approach to society, in particular to consumption, impossible.

At the same time, a denial of the chance factor in phenomena in general, and in mass consumer behaviour in particular, corresponds not to a dialectical but to a metaphysical concept of the operation of objective laws. "...Law, every law, is narrow, incomplete, approximate." "Appearance is *richer* than law" but law is more profound than phenomenon, "law is essential appearance".[1] Engels put forward the following criticism of "...determinism ...which tries to dispose of chance by denying it altogether": "...Chance is not here explained by necessity, rather necessity is degraded to the production of what is merely accidental".[2]

To deny the chance component in the indicators characterising consumption and consumer purchases by the population leads to a direct identification of empirical patterns with profound laws. Given such an approach the very problem of discovering the internal laws determining specific features of the surface of phenomena disappears. In contrast, to acknowledge that data on consumption and purchases express both the laws and chance characteristics of specific circumstances results in the problem of distinguishing between the two, of clarifying the inner laws of phenomena.

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[1] V. I. Lenin, "Conspectus of Hegel's Book *The Science of Logic*", *Collected Works*, Vol. 38, 1981, pp. 151-52.

[2] Frederick Engels, *Dialectics of Nature*, Progress Publishers, Moscow, 1974, pp. 218, 219.

The qualitative analysis, contained in Marxist economic theory, of the problem of the development of needs and consumption, provides the basic prerequisites for proceeding to a formal description of the problem and to derivation, on this basis, of the law governing changes in the consumption structure, which explains the observed quantitative patterns. The conclusions from this analysis may be presented in the form of concepts that are expressible quantitatively.

The concepts of total needs, the normal level of their satisfaction, and long-term needs. It follows from the dependence of personal needs on the development of production that, at each given moment, they are limited quantitatively: only those needs really exist that were engendered by the previous, always limited, development of production. A further development of production surely leads to an expansion of and qualitative changes in requirements. Hence, inevitably, the conclusion arises of the historical nature of needs, of their unlimited development.

Proceeding from the statement concerning the limited nature of needs at every given moment, use can be made of the concept of total needs. The historical changeability of their magnitude has to be kept in mind.

The historical development of requirements and of consumption leads to the historically achieved level of satisfaction of various needs eventually being established as traditional and habitual and, hence, coming to be the norm, or standard, of consumption.

The existence of such a standard under capitalism was revealed by Marx when he analysed the value of labour-power. The value of labour power "is in every country determined by a *traditional standard of life.* It is not mere physical life, but it is the satisfaction of certain wants springing from the social conditions in which people are placed and reared up".[1]

Under capitalism, the welfare of most members of society, i.e., the working people, can rise only as a result of their struggle against the propertied classes; in these circumstances the rise in welfare achieved does not constitute just a starting point for further growth, but also a subject of

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[1] Karl Marx, "Wages, Price and Profit", in: Karl Marx and Frederick Engels, *Selected Works* in three volumes, Vol. Two, Progress Publishers, Moscow, 1976, pp. 71-72.

class struggle; moreover, under some conditions, when the economic situation takes a turn for the worse, capital usually manages to reduce the real incomes of the working people. It in no way follows from this, however, that the concept of the historical standard of satisfaction of needs of the working class loses validity. On the contrary, it serves as a scientific substantiation of the struggle waged by the proletariat against attempts by capital to ignore this standard and reduce the workers' living conditions to the minimum of subsistence. The successes scored in the economic struggle of the workers demonstrate that the need to ensure traditional living standards paves its way despite the resistance put up by capital; moreover, under certain conditions, the workers succeed not only in maintaining, but also in raising this level. The concept of the traditional living standards was introduced by Marx to reveal the nature of the law of value of labour-power. This standard is thus seen as a law, a necessity determining, on the whole, the extent of and proportions in which the needs of most members of society are satisfied under capitalism.[1] Like the other laws of the capitalist economy, it is realised only through nonrealisation, through continuous deviations depending on the economic situation.

The standard of consumption actually established in society does not ensure full satisfaction of total needs. There exists a certain difference between total needs and the normal level of their satisfaction, i.e., an unsatisfied part of needs. As long as such a difference has a substantial positive quantity for basic needs, a further rise in the level of consumption is aimed at satisfying needs formed previously, but not satisfied. This part of total needs (i.e., unsatisfied ones) will be called perspective needs, which emphasises that, in a long term, should the total level of consumption rise, precisely these needs will be satisfied, this being an important factor in determining the changes in the structure of consumption and consumer purchases. Perspective needs are thus not those that will arise in the future; they are a component of total needs already formed. The economic struggle

[1] With reference to these needs Marx repeatedly used the concept of necessity: necessary wants, necessities of life (see, in particular, *Capital*, Vol. I., Ch. VI; Vol. II, Ch. XX). Note the difference between this concept and that of necessaries for satisfying only the purely physical minimum of the requirements of proletarians.

of the proletariat under capitalism pursues the satisfaction of such needs as one of its major aims.

Both normal and perspective needs tend to change continuously. Normal needs rise rapidly, in other words, perspective needs tend to change into normal ones. At the same time, the quantitative and qualitative rise in total wants leads to the development of perspective needs.

5.3. The Objective Consumption Function

The generalising function of the increase in consumption. Since the level of development and satisfaction of particular needs can be expressed quantitatively, a theoretical function may be formed to express, in a generalised form, the increase in the satisfaction of the whole aggregate of material and intellectual needs.

From the qualitative point of view, this function is merely a formal presentation of the concept of consumption as satisfaction of a variety of needs that are not isolated from each other but form a certain system, a presentation of the concept of the level of consumption in general already introduced above. At this point, however, some specification is made following from the statement concerning the existence of a normal level of satisfaction of needs: the extent to which the system of total needs is satisfied at the normal level is not quantified, the generalising consumption function being seen as that of the rise in the consumption level over and above the normal one. There is no need here to discuss whether the total level of satisfaction of needs can be measured in relation to the state corresponding to the historically established traditional standard of consumption (note that this is, at any rate, possible for each particular need: the ratio of the normal level of satisfaction to the total level of each need). All statistical patterns to be analysed here express, however, specific characteristics of the increase in consumption and consumer purchases. To analyse these patterns, a generalising function of the increase in consumption is enough.

Having noted this, in order to simplify the presentation, we shall speak of the consumption function, meaning the generalising function of the increase in consumption.

Let us assume n kinds of needs have been formed by a certain moment, they are denoted by index j ($j = 1, \ldots, n$). Denote by N_j^{max} the total volume of need j, N_j^{min} — the normal level of its satisfaction (each need being expressed

by a specific unit of measure).[1] The situation to be considered is where $N_j^{max} > N_j^{min}$ for all $j = 1, \ldots, n$. The volume of perspective need $N_j^{persp} = N_j^{max} - N_j^{min}$. The set of vectors $\{N: N^{min} \leqslant N \leqslant N^{max}\}$ is that of all admissible levels of satisfaction of perspective needs.[2] This set determines the domain within which a choice can be, and in reality is, made of the structure of consumption, given an increase in its total. Let us call this domain that of the choice of the structure of consumption.

Suppose m kinds of good are used in consumption by society's members. The volume of their purchases (current receipts) is expressed by vector $x = (x_1, \ldots, x_m)$[3]; their quantity used in consumption—by vector $v = (v_1, \ldots, v_m)$. The level at which each need j is satisfied is a function of the quantity of the various consumer goods: $N_j = N_j(v)$.[4]

[1] It should be emphasised that, in each case and strictly speaking, vectors N^{min}, N^{max} change over time, so: $N^{min}(t)$, $N^{max}(t)$. Their dependence on t is ignored merely for the sake of simplicity.

[2] Satisfaction of needs at below the normal level means violation of the economic necessity discussed above. Satisfaction of a given need to above saturation point, has a negative effect on the increase in welfare as a whole and is, at any rate, pointless wastefulness. For this reason it, too, is disregarded from the very beginning in our analysis of the problem of establishing the structure of real consumption, because it does not correspond to its laws and to consumer behaviour (even though such things do occur for individual consumers and even groups of them).

Note that, in comparing the vectors, we employ the inequality signs in the following way; $a > b$ means that vector a is greater than vector b in all its components; $a \geqslant b$ means that vector a involves two, and only two, sets of components: one the components of which are strictly greater than the respective components of vector b and another the components of which are equal to the respective components of vector b (each of the sets may be empty or nonempty, but at least one of them is not empty every time); $a \geqq b$ means that either $a > b$ or $a = b$ for all the components. The latter case differs from the foregoing in not involving vectors $\bar{a}$ such that both sets mentioned are nonempty. For example $\{N: N \geqq N^{min}\}$ means that, among the set of vectors N, there is no $\bar{N}$, i.e., an N such that, for some components, they are strictly equal and for others they are strictly greater than N^{min}.

[3] Since the problems involved in changes in the stocks of consumer goods on the way from production to consumption are everywhere ignored, vector x may also be interpreted as the volume of the current output of consumer goods.

[4] Thus, index j denotes the needs not for specific kinds of product but for large groups of substitutable ones; one and the same level of need satisfaction may be achieved using very different collections of corresponding products.

As can be seen from the very notion of the general level of consumption, changes in it depend on those in the satisfaction of each need. Indeed, it is obvious that, for example, other things being equal, if the satisfaction of any specific need is growing, there is an overall rise in welfare. In order to introduce the consumption function, no more is required than to proceed from the fact that the rise in the total level of consumption over and above the normal minimum depends on how perspective needs are satisfied.

The total increase in consumption is thus a function of the level at which perspective needs are satisfied: $r = r(N)$ with $N^{min} \leqslant N \leqslant N^{max}$. Function $r(N)$ is introduced here as an axiom, as a mathematical presentation of the concept of the increase in the total level at which needs are satisfied. What it amounts to is that, with a rise in the satisfaction of various needs over an above the normal minimum (but not higher than the maximum), a total rise in the level of consumption takes place, this being the higher, the greater the extent to which particular perspective needs are satisfied.[1] It is also accepted that, with the increase in satisfaction of particular needs and of the whole set of them, the overall increase in the level of consumption tends to be continuous.

The following statement is thus accepted: if some basket of goods v^2 ensures that all needs, without exception, could be satisfied at a level not lower than that of basket v^1 and certain individual needs (at least one) at a higher level, then basket v^2 results in a greater increase in total consumption:

$$r[N(v^2)] > r[N(v^1)] \text{ if } N(v^2) \geqslant N(v^1), \qquad (5.1)$$

with at least one component $N_j(v^2) > N_j(v^1)$. Add that, if $N(v^1) = N(v^2)$, i.e., all needs in the two vectors v compared are satisfied equally, then $r[N(v^1)] = r[N(v^2)]$.

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[1] The question inevitably arises here as to whether there is any point in using function $r(N)$ in the complex case of comparing vectors N, the components of which differ from one another in opposite directions, i.e., for example, vectors one of which is greater than the other in component j, but smaller in component j'. As yet, function r has been interpreted without regard to this case. The answer to this question is given below. To put it briefly, this simply means that, should the substantiation given here and the interpretation of function $r(N)$ be accepted for the simple case, and should certain specific features of the range of perspective needs be additionally taken into account, this function will rank any vectors within the above domain.

For the time being, no quantitative sense is attributed to the difference in magnitudes r: to regulate all the feasible vectors of the satisfaction of perspective needs, no more is required than to know for which of them the value of r is greater, smaller, or equal; it is not necessary to point out exactly the extent to which this is the case.

Proceeding from rather obvious premises, it may be demonstrated that, among the various structures of consumption making up the domain of choice, there exist sets of structures each of which contains vectors N ensuring an equal overall increase in welfare (i.e., those structures for which $r(N) = const$). Such vectors will be called equivalent ones with regard to the increase in welfare. They form continuous hypersurfaces of indifference within the domain of choice.

Initially let us exclude from consideration all vectors $\overline{N}$, i.e., those strictly greater than N^{min} for some components and equal to N^{min} for certain other components (at least one). This substantially simplifies the further reasoning and, at the same time, corresponds to the fact that, if there is an increase in consumption, the complex nature of needs makes this increase occur simultaneously for all needs. Vector N^{min}, as such, remains within our consideration.

The demonstration of the existence of equivalent vectors in the domain of choice is based on certain assumptions derived from a qualitative analysis of the processes involved in the increase in welfare.

First, the actual existence is accepted of a nonempty set of vectors of the satisfaction of perspective needs making up the domain of choice, limited both from above and below.

Second, it is accepted that the levels at which each particular need is satisfied continuously change. This assertion is valid inasmuch as we are dealing here not with the needs of an individual or a particular family (needs for many goods, in particular durables, are, for them, essentially discrete), but with those of society.

Third, it is acknowledged that there exists function $r(N)$ for which (5.1) is valid, the function continuously increasing in all its arguments.

Fourth, it is accepted that the increase in welfare, while passing from any vector N to any other vector N', does not depend on the course taken. Indeed, the domain of choice involves some vectors, achievable in the future, of the levels of satisfaction of perspective needs. The vectors are to be

estimated and compared at a certain moment of time, since the domain of choice is itself determined for just such a moment; such an estimate of each vector N, i.e., the value of function $r(N)$ for it, is naturally unique.[1]

In this work it will suffice to demonstrate that there exist vectors equivalent (indifferent) with regard to the increase in welfare for the case when the vectors compared differ from one another in the magnitudes of no more than two components, the rest of their components being equal. These latter components may be ignored at this point, and only the two needs (j and j') that are satisfied to different extents considered.

By $\hat{N}$ we shall denote all vectors in the domain of choice, with the exception of N^{min} and N^{max}. The following is true of any such vector $\hat{N}$: $N^{min} < \hat{N} \leqslant N^{max}$. Then, in accordance with (5.1), we have for any $\hat{N}$:

$$r(N^{min}) < r(\hat{N}) < r(N^{max}). \quad (5.2)$$

Inasmuch as the domain of choice involves all N: $N^{min} \leqq N \leqslant N^{max}$ (all $\overline{N}$ being, as already indicated, disregarded) and each need satisfaction changes continuously, there exists a set of various continuous sequences of vectors forming the transition from N^{min} to N^{max} such that each subsequent vector of a given sequence is greater than the previous one (if any subsequent vector is denoted by N' and the previous one by N, $N' > N$). In any such sequence, the value of function $r(N)$ tends to grow continuously in accordance with the third premise. At the same time, it remains within $r(N^{min}) = r^{min}$ and $r(N^{max}) = r^{max}$. Hence, in any such sequence, the value of the function tends to grow continuously from r^{min} to r^{max}. It follows from this

[1] Estimates of the social significance of the various sets of goods tend to change over time and it may be assumed that these changes depend on how the structure of consumption changes concretely if welfare increases, i.e., on the course taken. In particular, it may be assumed that the change in the level and structure of total needs depends on the transition course; however, since the level of various needs changes, the estimate of any given set of goods inevitably changes as well. Certain patterns in the changes of the estimates can probably be discovered and predicted, but this is a considerably more complicated task than the one we are discussing. A start has to be made with the more simple task, that is, disregard changes in the estimates.

that, in any such sequence, function $r(N)$ runs through one and the same continuous series of values.

The sequences are formed, however, so as to have common vectors only at their extremes N^{min} and N^{max}, the other vectors of the various sequences being different from one another. It follows from this that function $r(N)$ for different sequences takes on equal values for different vectors $\hat{N}$. This means that there exist sets of various vectors for which $r(\hat{N}) = const$, i.e., equivalent vectors of the levels of satisfaction of different needs.

The demonstration of the existence of vectors equivalent as regards the increase in levels at which perspective needs are satisfied has a clean economic sense. If, by some moment in time, certain needs and a certain standard level of their satisfaction are established, any increase in their satisfaction over and above the minimum (but not above saturation point) will mean a certain approximation to complete satisfaction. In other words, any increase in the satisfaction of needs over and above the minimum represents some extent to which the aggregate of perspective needs are satisfied. The level of consumption can, however, be increased in different ways, i.e., the same level at which the aggregate of perspective needs is satisfied can be achieved differently.

It is quite obvious, and in this work that is not particularly mathematical in character any demonstration of this may be disregarded, that there exist equivalent vectors differing in the magnitudes of more than two of their components, as well as that the set of various vectors for which $r(N) = const$ is continuous. This set forms, within the domain of choice, smooth hypersurfaces with the same increase in welfare characteristics.

Different vectors N for which $r(N) = const$ are distinct at least in the magnitude of two components: one vector is obligatorily smaller than the other as regards the satisfaction of some need j but, in contrast, it is greater as regards the satisfaction of need j'. Indeed, these vectors are not equal, yet, at the same time, they are equivalent; that is, the positive and negative increments in the levels at which the various needs are satisfied offset one another:

$$\text{given } r(N) = const, \ \sum_j \frac{\partial r}{\partial N_j} dN_j = 0. \tag{5.3}$$

The ratio of these mutually offsetting increments $(dN_j)/(dN_{j'})$ is called the rate of substitution.[1] The specific form of function r is determined by the extent to which these rates change, i.e., the change in the rate at which a decrease in the satisfaction of each need can be offset by an increase in that of others.

Hereinafter, the presentation of the rate of substitution will be interpreted in the following way: among the increments offsetting each other, negative ones are given in the numerator and positive ones in the denominator. Let us note certain characteristics of the rates of substitution.

The rates of substitution are nonpositive, since the negative increment in the satisfaction of one need is offset by the positive increment in that of another:

$$\frac{dN_j}{dN_{j'}} \leqslant 0. \tag{5.4}$$

As the satisfaction of need j' approaches full saturation, the rate of substitution tends to zero (i.e., the absolute magnitude of that decrease in satisfaction of need j that can be offset by a certain increase in satisfaction of need j'):

$$\frac{dN_j}{dN_{j'}} \to 0, \text{ given } N_{j'} \to N_{j'}^{max}. \tag{5.5}$$

Indeed, if, given some vector v, need j' is satisfied at the maximum level, no increment in consumption above this limit would result in a greater satisfaction of need j' or a rise in total welfare; therefore no such increase in consumption could offset even the most insignificant decrease in the satisfaction of another need j. It is natural to assume that the rate of substitution gradually approaches this level.[2]

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[1] Strictly speaking, in the presentation of the rate of substitution one should indicate constantly $\sum_j \frac{\partial r}{\partial N_j} dN_j = 0$, but we omit this in order to simplify the text.

[2] It cannot be equal to zero within the domain of choice, otherwise, if a certain need is satisfied to a greater extent, the other being satisfied at a constant level, the value of function r would not change, which contradicts (5.1). The equality of the rate of substitution to zero at the upper boundaries of the domain of choice is but another expression of the fact that the domain of choice is a limited range of definition of the consumption function.

Inasmuch as the rate of substitution within the domain of choice is non-zero, and equal to zero at its upper boundaries, this means it tends to zero. We assume this change in the rate of substitution to be monotonous and continuous.

Should these statements be accepted, and they are merely an expression of the characteristics of the domain of choice resulting from its theoretical definition, the hypersurfaces for which $r(N) = const$ will be:

(A) orthogonal to the upper boundaries of the domain of choice at the points common with these boundaries;

(B) convex towards the origin.

These distinctive features should be taken into account in determining the specific form of function r.

The hypothesis relating to the form of the consumption function. Not one function, but a whole class of functions $r(N)$ in the domain of choice correspond to the conditions (A) and (B) formulated above. In other words, these conditions are not sufficient for a single-valued determination of which vectors are equivalent to one another. Several functions may be indicated, each of which will fulfil this demand. This is not surprising. Our knowledge of the objective laws governing the development and satisfaction of needs is still in its infancy; there are, as yet, no adequate data for further specifying the patterns of the changes in the rates of substitution. For this reason, there is no other alternative but to suggest some hypothesis relating to a formula of the rates of substitution that would comply with the regularites already discovered, to build the consumption function on its basis, and to try to discover which fields of its application would yield substantiation of the accepted hypothesis. Such a verification will reveal further laws allowing the rates of substitution and the consumption function to be determined more accurately. This method is a common one.

Let us employ the vector of the extent to which perspective needs are satisfied $z = (z_1, \ldots, z_j, \ldots, z_n)$, the components of vector z being measured in fractions of a unit:

$$z_j = \frac{N_j - N_j^{min}}{N_j^{max} - N_j^{min}} \quad (N_j^{min} \leq N_j \leqslant N_j^{max}, \quad N_j^{min} < N_j^{max}).$$

The following formula is proposed for the rate of substitution:

$$\frac{dz_j}{dz_{j'}} = -\frac{z_j(1 - z_{j'})}{z_{j'}(1 - z_j)} \tag{5.6}$$

which corresponds to the specific characteristics (A) and (B) of function $r(N)$ inasmuch as condition (5.5) is ful-

filled: given $N_{j'} \to N_{j'}^{max}$, the rate of substitution tends to zero because the magnitude of $z_{j'}$ tends to unity. Formula (5.6) also meets an additional assumption:

(C) the rate of substitution, $dz_j/dz_{j'}$, can depend only on the extent to which perspective needs j and j' are satiated in some vector z and on the extent to which they are not satiated in this vector.

The assumption is quite natural since it seems impossible to find, besides indicators of the extent to which each of vectors v ensures satiation of perspective needs or, on the other hand, leaves them unsatiated, any other characteristics of vectors v essential for determining the increase in consumption in the domain of choice. Formula (5.6) presents a very simple form of the dependence of the rate of substitution on these characteristics: a direct dependence on the ratio of the degrees of satiation of perspective needs j and j', an inverse dependence on the ratio of the degrees of their non-satiation.

By integrating differential equation (5.6) and generalising the outcome for the case $n \geqslant 2$, we conclude that function $r(N)$, defined as follows, meets formula (5.6)[1]:

$$\frac{\prod\limits_j z_j}{e^{\sum\limits_j z_j}} = r(N). \tag{5.7}$$

Thus the specific form of function $r(N)$ has been determined. Its arguments are magnitudes z_j, so far determined [see (5.6)] as changing within the limits: $0 \leqslant z_j \leqslant 1$. It may be demonstrated, however, that, with exception of vector $z = 0$, where all the components of this vector are equal to zero (i.e., points N^{min}), magnitudes $z_j = 0$ should be excluded from the domain of the determination of function (5.7).

Indeed, if at least one of the components of vector z (for instance, some magnitude z_j) is equal to zero, then, in accordance with (5.7), $r(N) = 0$ irrespective of the magnitude of the other components of vector z, so irrespective of any increase or decrease in any of them either. This, however, contradicts (5.1), i.e., the condition being a logical expression of the concept of the consumption function. Thus, if

[1] The conclusion was derived by I. A. Itskovich (see I. A. Itskovich, "On an Analysis of the Objective Welfare Function", in: *Problems of National Economic Optimum*. Issue 2, Nauka Publishers, Novosibirsk, 1969, p. 198 (in Russian).

the zero values are admitted for particular z_j, formula (5.7) fails to correspond to the necessary characteristics of function r, i.e., the function of which this formula is merely a specific form. This means that, for all cases with the exception of $z = 0$, zero values of z_j are excluded from the domain of the definition of function (5.7). As for the specific form of function r, we have arrived at the conclusion introduced above as a premise: that all the vectors $\overline{N}$ should be excluded.

Note that, when the function is built up in practice, it may turn out that certain needs should be aggregated for some reasons. Magnitude z_j for such an aggregate expresses the average degree of satisfaction of several needs. Were no aggregation to be made, satisfaction of each of these needs would be expressed by a particular magnitude z_j; each of them would be involved as a multiplier in the numerator of formula (5.7) and as an item of the exponent in the denominator of the formula. It follows that, with regard to the aggregated character of the magnitudes of needs in specific calculations, the indifference surfaces should be expressed as follows:

$$\frac{\prod\limits_j z_j^{k_j}}{e^{\sum\limits_j z_j k_j}} = r(N). \tag{5.8}$$

Here k_j is the indicator to eliminate the effect of aggregating on determining $r(N)$.[1]

The specific nature of magnitudes z_j, consisting in the fact that all of them express the degree of satisfaction of individual perspective needs, as well as the fact that $r(N)$ serves as a generalising function of these degrees, allows the consumption function to be shaped to express the degree of satiation of aggregate perspective needs within the limits of 0 to 1. This is achieved under the following natural assumption:

(D) if the degrees of satisfaction of all perspective needs in some vector z are equal ($z_1 = z_2 = \ldots = z_n = t$), the total degree of satisfaction of all these needs is also equal to t. Given this assumption, from function r a monotonically increasing function $D = D[r(N)]$ can be derived to estab-

[1] It may be demonstrated that the attribute by which indicators k_j may be calculated is expenditure on full satisfaction of needs (k_j are the greater, the more complete satiation of need j costs). Below we shall dwell on this specifically.

lish equivalence of the same vectors N as function $r(N)$. Let us derive function D.

If $z^{(t)}$ denotes vector z, each component of which is equal to t, function (5.8) assumes the following form:

$$t^{\sum_j k_j} \Big/ e^{t \sum_j k_j} = r(z).$$

$$\frac{1}{\sum_j k_j}$$

Hence $t = e^t [r(z)]$. But in accordance with the demand formulated $D(z^{(t)}) = t$. For this reason

$$D = e^D [r(z)]^{\frac{1}{\sum_j k_j}}.$$

Each total level of satisfaction of long-term needs is achieved not only by some vector $z^{(t)}$, but also by the set of other vectors z forming, together with this $z^{(t)}$, a hypersurface that is specifically characterised, according to (5.8), by the constant nature of magnitude r. Consequently, each such hypersurface is also characterised by some constant magnitude $D(r)$. Thus we finally get:

$$D = e^D \left(\frac{\prod_j z_j^{k_j}}{e^{\sum_j z_j k_j}} \right)^{\frac{1}{\sum_j k_j}}. \tag{5.9}$$

As can be seen from the presentation, function D belongs to the class of *implicit* functions.

The formula of the consumption structure. Let us proceed from the assumption that the mass of consumers aspire in their behaviour (i.e., in shaping the structure of consumption and consumer purchases) to the highest degree of satisfaction of all perspective needs. Since function D measures the overall level of satiation of perspective needs, this tendency of the mass consumer may be described as a desire to maximise magnitude $D(z)$. In view of the limited nature of resources consumers cannot achieve full satiation of needs, i.e., $D = 1$, and they determine the pattern of their consumption so that the maximum D might be achieved given the resources at their disposal. In other words, we assume $D \to$ $\to$ *max*.

The crucial resource limiting the production possibilities at each given moment is the labour time available in society which, given a definite productivity of labour, ultimately determines the volume of output. Superficially, the limited nature of resources is seen as that of the money incomes of the consumers, which ultimately determines the overall volume of their purchases at given prices (the budget frontier).

For vector N, which maximises the value of D, resources being given, the following is true:

$$\frac{D_j}{h_j} = const \quad (j = 1, \ldots, n),$$

therefore

$$\frac{D_j}{D_{j'}} = \frac{h_j}{h_{j'}} \text{ for any } j, j'. \tag{5.10}$$

Here $D_j \equiv \frac{\partial D}{\partial z_j}$ is a partial derivative of function D in relation to the satisfaction of perspective need j; h_j is a similar partial derivative of the function expressing the expenditure of a limiting resource.[1]

The sense of (5.10) consists in the following: if (5.10) is not met in some vector N lying on the budget frontier, the value of function D may be raised without changing the total amount of the limiting resource. Indeed, in this case for some j and j', $D_j/h_j \neq D_{j'}/h_{j'}$. It follows from this, however, that a portion of the resource employed having been removed to satisfy that need (j or j') where the increment (partial derivative) of function D per unit expenditure is higher, an overall increase in D may be achieved (even though the degree to which that need is satisfied will decrease where the above ratio is lower).

The partial derivative of function D, as this function is defined in (5.9), is expressed by the formula:

$$\frac{\partial D}{\partial z_j} \equiv D_j = \frac{D(1 - z_j)\,k_j}{z_j(1 - D)\sum_j k_j}, \tag{5.11}$$

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[1] The vector N^0 in which function D achieves the possible maximum should be viewed as the optimal structure of consumption, and the appropriate value of D as the optimal one (D^0). The same applies to vector N^0, its components N_j^0, and so on. However, in order not to complicate the presentation, we shall employ simply D, N, N_j, etc.

where z_j and k_j are as explained above. In turn, the ratio of partial derivatives

$$\frac{h_j}{h_{j'}} = \frac{l_j (N_j^{max} - N_j^{min})}{l_{j'} (N_{j'}^{max} - N_{j'}^{min})}, \tag{5.12}$$

where l_j is the total expenditure of social labour (or the price) per unit use-value intended to satisfy need j'.[1]

Generally speaking, l_j tend to change if the structure of the satisfaction of needs N changes, but we shall usually ignore this dependence.

Comparing formulae (5.12) and (5.11) above all allows the significance of indicator k_j to be determined, its purpose being to eliminate the effect of aggregation on the calculation with function D. This effect is due to the fact that ratio $h_j/h_{j'}$ depends on aggregation resulting from the classification of needs employed and being expressed by determining magnitudes N_j^{max}. Indeed, it follows from (5.12):

$$\frac{h_j}{h_{j'}} = \frac{l_j N_j^{max} q_j}{l_{j'} N_{j'}^{max} q_{j'}}, \tag{5.13}$$

where $q_j = 1 - \frac{N_{j'}^{min}}{N_{j'}^{max}}$, $q_{j'} = 1 - \frac{N_{j'}^{min}}{N_{j'}^{max}}$ are the degrees to which needs j and j' are not satisfied at point N^{min} (dimensionless magnitudes expressed by fractions of a unit). While l_j and, q_j, and, accordingly, $l_{j'}$ and $q_{j'}$, can be independent of aggregation, even though they are not necessarily so, this is obviously not the case as regards N_j^{max} and $N_{j'}^{max}$. The more general and broad the indicator of the aggregate j separation, the greater the magnitude N_j^{max}, even if all the components of the aggregate have equal l_j and equal q_j[2]. Now let us note that, in accordance

[1] At this point magnitudes h_j ensure the commensurability of expenditure on satisfying perspective needs j and not per unit output satisfying these needs, which is necessary as the arguments of function D are represented by the degrees of satisfaction perspective needs (magnitudes z_j), not simply by the quantities of goods consumed.

[2] We disregard here the problem of the N_j^{max} unit of measurement. Let us note that, in the given case, it cannot be a specific unit of some consumer good, as virtually any need is satisfied using a number of kinds of good. Far from always can a certain conventional unit for the measurement of the given need exist into which the units of measurement of specific goods might be converted. For this reason, it seems justified that, in principle, the needs to be reflected by func-

with (5.10), if k_j are disregarded, the ratios of magnitudes z_j and $z_{j'}$ prove, at the optimum point, to be dependent on aggregation, which obviously should not take place. It is precisely magnitudes k_j that eliminate this effect of aggregation on determining the structure of the optimum satisfaction of perspective needs. In order to remove this effect of aggregation it is necessary to make the needs themselves commensurable in some manner. The measuring of magnitudes N_j^{max} seems to require that the expenditures on achieving them should be used; at any rate, there is no way to make them commensurable with one another except to use magnitudes $l_j N_j^{max}$, which we shall denote by L_j^{max}. It follows from this that

$$k_j = \frac{L_j^{max}}{L_{j'}^{max}} \quad (j = 1, \ldots, n), \tag{5.14}$$

where j' is one of the needs j, k_j being merely the ratio of total labour expenditure on satisfying any need j and some need j' (evidently $k_{j'} = 1$). Perhaps magnitudes k_j might answer the question as to how many such needs as some j' are aggregated to make j (as regards expenditure on full saturation). The choice of a need j' to measure the others is not, from this point of view, essential: in the calculations with function D, only the ratios of different k_j are essential, but these, as it follows from (5.14), do not depend on the choice of j' (i.e., magnitude $L_{j'}^{max}$). All $k_j > 0$ as $l_j > 0$, $N_j^{max} > 0$.

From (5.14) it follows that: $L_j^{max} = k_j L_{j'}^{max}$; we shall also employ symbol $A_j = \frac{N_j^{max} - N_j^{min}}{N_j^{max}}$. Hence, $A_j = q_j z_j$, where $z_j = \frac{N_j - N_j^{min}}{N_j^{max} - N_j^{min}}$. Note that $N_j^{min} =$

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tion D can be measured only by the amounts of total expenditure of social labour on their full satisfaction (this does not apply to needs that are completely satisfied without production; as regards these, however, $N_j^{min} = N_j^{max}$ by definition and they are withdrawn from the process of optimising consumption; for any j for which $N_j^{max} > N_j^{min}$, society organises the production of appropriate goods). If N_j^{max} is measured in labour units, then, certainly, l_j in formula (5.12) is of purely formal significance, as it is itself measured in the same units ($l_j = 1$ in this case); it is meaningful if it represents a monetary unit.

$= (1 - q_j)N_j^{max}$, $N_j^{max} > N_j^{min}$ for all j. Only such vectors N that $N^{min} < N < N^{max}$ are studied further. From this it follows that $0 < z_j < 1$; $0 < A_j < q_j$ for all j. By comparing (5.10), (5.11), (5.12) and (5.14) we obtain for an optimal N:

$$\frac{D_j}{D_{j'}} = \frac{(1-z_j)\, z_{j'}}{(1-z_{j'})\, z_j} \cdot \frac{k_j}{k_{j'}} = \frac{h_j}{h_{j'}} =$$

$$= \frac{l_j\,(N_j^{max} - N_j^{min})}{l_{j'}\,(N_{j'}^{max} - N_{j'}^{min})} = \frac{L_j^{\max} q_j}{L_{j'}^{\max} q_{j'}} = \frac{k_j q_j}{q_{j'}}.$$

But $k_{j'} = 1$. So

$$\frac{(1-z_j)\, z_{j'}}{(1-z_{j'})\, z_j} \cdot k_j = \frac{q_j}{q_{j'}} \cdot k_j;\quad \frac{(1-z_j)\, z_{j'}}{(1-z_{j'})\, z_j} = \frac{q_j}{q_{j'}}.$$

Hence

$$\frac{1-z_j}{z_j} = \frac{q_j\,(1-z_{j'})}{q_{j'} z_{j'}};\quad \frac{1}{z_j} = \frac{q_j\,(1-z_{j'})}{A_{j'}} + 1;$$

$$\frac{q_j}{A_j} = \frac{q_j\,(1-z_{j'})}{A_{j'}} + 1;\quad \frac{1}{A_j} = \frac{1-z_{j'}}{A_{j'}} + \frac{1}{q_j} =$$

$$= \frac{q_j - z_{j'} q_j + A_{j'}}{A_{j'} q_j} = \frac{q_j - z_{j'} q_j + z_{j'} q_{j'}}{A_{j'} q_j};$$

$$A_j = \frac{N_j - N_j^{min}}{N_j^{max}} = \frac{A_{j'} q_j}{q_j - z_{j'}\,(q_j - q_{j'})}.$$

Finally,

$$N_j = \frac{A_{j'} q_j}{q_j - z_{j'}\,(q_j - q_{j'})} \cdot N_j^{max} + N_j^{min}. \tag{5.15}$$

Hereinafter we shall test formula (5.15) using statistics on consumer purchases of large aggregates of goods, each aggregate meeting a particular need (for food, clothing and footwear, housing, etc.). For such data it may be assumed that: $N_j \equiv N_j(v) = v_j$ for all j, where j is the index of the above aggregates ($j = 1, \ldots, n$); v_j is as explained above; accordingly, $N_j^{min} = v_j^{min}$, $N_j^{max} = v_j^{max}$. Then formula (5.15) will take the following form:

$$v_j = \frac{A_{j'} q_j}{q_j - z_{j'}\,(q_j - q_{j'})}\; v_j^{max} + v_j^{min}. \tag{5.16}$$

These formulae express the ratio of the components of optimum vector N, i.e., the structure of consumption in the case when function D reaches its possible maximum. Indeed, N_j ($j = 1, \ldots, n$) are the components N. It would be enough to give some values N_j for one of the needs, i.e. $N_{j'}$, in order, once N_j^{max} and N_j^{min} are known, to obtain the values of the rest of N_j.[1] Thus, all other N_j, i.e., the whole system of the components of vector N, all of their ratios, the structure of the vector, are determined with regard to the ratio to $N_{j'}$.

The question still remains open, of course, as to how magnitude $N_{j'}$ can be found for which the other N_j have to be discovered. The point is, however, that, according to (5.15), N_j can be found for any magnitude of $N_{j'}$ (within the range from $N_{j'}^{min}$ to $N_{j'}^{max}$). In other words, this formula determines the continuous change in the optimum structure of consumption, corresponding to the uninterrupted increase in the satisfaction of some need j', this being a specific *optimum path (trajectory) of the rise in consumption.* The point on this path that is attainable depends on the resources to be employed to satisfy personal needs.

The above provides grounds for calling (5.15) and, accordingly, (5.16) the formula to express the optimum ratio of the satisfaction of various needs or, more briefly, the formula of the consumption structure.

The fundamental characteristics of the formula of the consumption structure. This formula methodologically differs fundamentally from those widely current in the literature for determining the amounts of purchases (consumption), those obtained by means of mathematical statistics techniques in processing actual data. Usually, a specific formula is sought for each type of need, this being nothing but a formalised presentation of an appropriate empirical regularity. Not only the specific values of the parameters, but also the entire shape of these formulae are different. In contrast, formula (5.15) has an overall form common to all needs (each of them has, of course, its own specific basic parameters N_j^{max}, N_j^{min}). It thus expresses *a general law of the optimum change in the structure of consumption*, with the regularities for individual needs being merely particular manifestations

[1] Given N_j^{max} and N_j^{min}, we determine q_j for all j. Having given $N_{j'}$, we determine $A_{j'}$, $z_{j'}$. Then N_j is determined for any j with the aid of (5.15).

of it. The meaning of this law is clear from that of the factors representing the arguments in (5.15): the changes in the structure of consumption, given an increase in its total level, are a compound function of the N_j^{min} to N_j^{max} ratios for all j, i.e., ultimately of the degrees of satisfaction of all needs established by a given moment of time. (5.15) expresses, of course, only a theoretical hypothesis of the general law of the changes in the structure of consumption, since it has been derived from the hypothesis of a specific form of function D.

The theory of consumption usually emphasises that the choice made by the mass consumer (vector v) is the sum total of those made by individual consumers (households): $v_j = \sum_i v_j^i$ for all j; here i is the household's index. This is indisputable, but from this it is often also concluded that the theory of choice should be constructed with reference to the individual consumer. In contrast and in accordance with the above arguments we consider it impossible, in principle, to construct a theory to apply at the level of the individual consumer. The utility function (5.9) and the optimum trajectory (5.16) derived from it are constructed directly for mass consumer behaviour (i.e., for rather large social groups and the population as a whole). Yet, in reality, even economists who attempt to construct individual utility functions process statistical data on mass consumer behaviour by employing them.

Below, the following two basic specific features of the optimum trajectory (5.16) will be tested (with the aid of a variety of statistical techniques and a retrospective forecast).

First, the independence of structural shifts in consumption (at the level of large aggregates of goods corresponding to different needs) on those in relative prices. Formula (5.16) does not include prices or their indices as arguments in determining v_j corresponding to some $v_{j'}$ (q_j and $q_{j'}$ being given). The formula does not imply that prices fail to affect the formation of the vector of total needs v^{max} and thereby the magnitudes of q_j and $q_{j'}$. Yet it is quite obvious that total needs tend to change slowly and under the impact not only of prices, but also of all the processes involved in society's development. For a period of 5 to 10 years or even more, v^{max} may be taken as constant, the error being not particularly large. In this case, the changes in relative prices are

expected, over 5 to 10 years, virtually not to affect the changes in the structure of consumption.[1]

Second, formula (5.16) is a curvilinear trajectory of the increase in the consumption of goods j, given an increase in the consumption of goods j' (q_j and $q_{j'}$ being given). Only in the special case (if $q_j = q_{j'}$) is (5.16) a linear equation:

$$v_j = v_j^{min} + A_{j'} v_j^{max}.$$

The case of $q_j = q_{j'}$ is merely an exception, however. Total needs v_j^{max} and the traditional levels at which they are satisfied v_j^{min}, inevitably develop non-proportionally. So the degrees q_j of non-satisfaction of particular needs are usually not equal for different j.

Let need j' be chosen such that $q_j > q_{j'}$. Then formula (5.16) expresses a convex trajectory of increase in consumption. Indeed, the first derivative:

$$\frac{dv_j}{dv_{j'}} = \frac{v_j^{max} - v_j^{min}}{v_{j'}^{max} - v_{j'}^{min}} \cdot \frac{q_j q_{j'}}{[q_j - z_{j'}(q_j - q_{j'})]^2} > 0;$$

and the second derivative:

$$\frac{d^2 v_j}{dv_{j'}^2} = \frac{v_j^{max} - v_j^{min}}{v_{j'}^{max} - v_{j'}^{min}} \cdot \frac{2q_j q_{j'}(q_j - q_{j'})}{[q_j - z_{j'}(q_j - q_{j'})]^3} > 0.$$

If the derivatives are strictly positive, given $q_j > q_{j'}$, the trajectory (5.16) is convex.

5.4. Statistical Verification of the Objective Consumption Function

The objective consumption function and the law of the consumption structure following from it were statistically verified using data for more than 20 countries (socialist, developed capitalist, and developing)[2]. The law expressed in formula (5.16) is proved to operate with a high degree of accuracy in actual mass consumer behaviour. In this work

[1] Prices are also expected to be an essential factor of the consumption structure within large aggregates of goods.

[2] See K. K. Valtukh, *The Objective Consumption Function: An Analysis and Applications*, Novosibirsk, Nauka Publishers, 1980 (in Russian); see also the author's articles in the journals *Quality and Quantity*, No. 9, 1975 (Amsterdam); *IHS-Journal*, No. 5, 1981 (Wien); *Empirical Economics*, No. 6, 1981 (Vienna), etc.

we shall demonstrate this using statistical data on private consumer expenditures of the French population for 1959 to 1971. French mass consumer behaviour will be shown to correspond well to the specific features of the theoretical utility function proposed above. This means, at one and the same time, first, a statistical corroboration of the function and, second, explanation of mass consumer behaviour with the aid of this function.

Statistical data on private consumer expenditures are used as elaborated in F. Pascaud's study "La consommation des ménages de 1959 a 1972" (No. 134 des Collections de l'I.N.S.E.E., série M, No. 35, juin 1974).

We shall focus on the latter characteristic of the consumption growth trajectory.[1]

The first statistical test: increase in ratios $\frac{\Delta x_j}{\Delta x_{j'}}$. Pascaud's study includes thoroughly developed statistical data on consumer purchases, classified in aggregates of goods meeting particular needs. This aggregation principle allows the statistical data to be used directly to test the proposed theory. It is also very important that Pascaud distinguishes clearly nondurables, semidurables, and durables within the large aggregates.

For all the calculations we fix item j'. It would be convenient if this were to meet the following requirement:

$$q_{j'} = \min_j q_j.$$

In processing the statistical data we do not usually know vectors v^{min}, v^{max}, so we cannot, strictly speaking, judge precisely about relations $\frac{v_{j'}^{min}}{v_{j'}^{max}}$ and $\frac{v_j^{min}}{v_j^{max}}$, $q_{j'}$ and q_j. Apparently, however, the need for foodstuffs in all developed countries might, as a rule, be considered as more satiated than the people's other needs in these countries (this does not, of course, mean that the need will be fully satiated); anyway,

[1] F. Pascaud's study contains numerous results showing that the statistical data do not corroborate the idea of structural shifts in consumption being dependent on changes in relative prices. In particular, the reader may easily convince himself that the Spearman rank correlation coefficient is exactly equal to 0 between the growth rates of consumer purchases for 8 basic aggregates for 1959 to 1971 (see Table 5.1) and those of the prices (see Pascaud, *op. cit.*, pp. 115-22). Thus, the former of the indicated characteristics of the trajectory is corroborated by this information.

exceptions to this rule are expected to be rare.[1] So calculations where the consumption of foodstuffs is taken as item j' may be considered justified[2]. Alcoholic beverages will be excluded from foodstuffs ($j' = 0$) and are considered as a special consumption item ($j = 1$).

As has been shown, in the optimal curve $\frac{d^2 v_j}{dv_{j'}^2} > 0$. A statistical corroboration of this property, where discrete survey data are used, means that the following law of structural shifts is valid: relations

$$\frac{v_j(t)-v_j(t-1)}{v_{j'}(t)-v_{j'}(t-1)}=\frac{\Delta v_j^t}{\Delta v_{j'}^t}$$

grow with a growth of v_j (here t is the survey index). To interpret this: the structural shifts in this sense become increasingly marked. In other words, given sufficiently long intervals of real trajectories, not only an increase in relations $v_j^t/v_{j'}^t$, but also in relations $\Delta v_j^t/\Delta v_{j'}^t$, is expected. Thus, not Engel-type laws (stable trends in structural shifts), but stronger and more subtle laws are examined: a strengthening of structural shifts over sufficiently long intervals (in this case the Engel-type laws are even more valid). The interval within which relation $\frac{\Delta v_j^t}{\Delta v_{j'}^t}$ increases may, of course, prove shorter than that within which $\frac{v_j^t}{v_{j'}^t}$ increases, owing to changes in people's needs and tastes. Moreover, also within the interval where an increase in $\Delta v_j^t/\Delta v_{j'}^t$ takes place, it may prove less stable than in $v_j^t/v_{j'}^t$.

For the reader's convenience, the complete data on private consumer expenditures on large aggregates of goods and services in constant 1963 prices are given in Table 5.1. They will be used in both this and the next sections. We are at once, however, confronted with the following shortcoming of the statistical data: they do not cover the direct values of consumption v_j^t, but those of current incoming goods x_j^t; $x_j^t \approx v_j^t$ is valid, however, only for nondurables

[1] Below we shall find only one exception: alcoholic beverages.

[2] Any other item used as j' would not affect the conclusions of this work, but it would complicate the analysis techniques and the drawing of conclusions.

Table 5.1

Private Consumption Expenditures

	1959	1960	1961	1962	1963
0. Food excluding alcoholic beverages	62.05	63.81	65.60	68.30	70.68
1. Alcoholic beverages	10.33	10.51	10.63	11.03	11.60
2. Clothing and footwear	21.45	22.63	24.24	26.53	29.42
3. Housing	34.47	36.45	38.82	42.38	46.66
4. Hygiene and health	16.60	17.77	19.62	21.53	23.54
5. Transport and communication	15.30	16.53	18.29	20.56	22.66
6. Culture and leisure	14.84	16.21	17.39	18.89	20.62
7. Hotels, restaurants, cafés and other	18.22	19.00	20.04	21.28	22.54
8. Sum total	193.30	202.90	214.60	230.50	247.70

and services. As far as durables and semidurables are concerned, $v_j^t \gg x_j^t$ since v_j^t includes goods previously accumulated. The following formula is approximately valid:

$$v_j^T = \sum_{t=T-l_j+1}^{T} x_j^t,$$

where l_j is the life-time of the j-kind goods in consumption; $l_j = 1$ is assumed for nondurables and services, $l_j > 1$ for the rest.

Since the data include for all kinds of goods only values x_j^t, they give a false impression about the dynamics of the consumption of goods for which $l_j > 1$. Let us start, however, with the direct statistical data.

Table 5.2 shows variables Δx_j, $\Delta x_j/\Delta x_{j'}$. The interval from 1959 to 1971 is divided into two equal parts: 1959-1965, 1965-1971.

As can be seen, relations $\Delta x_j/\Delta x_{j'}$ are greater in the second part than in the first for all items but two: "Alcoholic Beverages" and "Clothing and footwear". Let us dwell on these findings in more detail.

The first exception ("Alcoholic beverages") merely confirms the rule. An increase in relations $\Delta v_j/\Delta v_{j'}$ is expected if consumption item j meets a less satiated need than j'. In this case, however, there are reasons to assume the demand for alcoholic beverages to be more satiated than that for

^t_j) (thousand mill. 1963 francs)

1964	1965	1966	1967	1968	1969	1970	1971	Ix_j (1971/1959)
73.25	75.55	77.83	80.68	82.80	85.64	87.77	90.50	1.46
12.02	12.20	12.34	12.61	12.55	12.76	12.80	13.16	1.27
30.51	30.65	31.50	32.21	33.56	35.89	37.27	38.74	1.81
49.33	51.63	53.84	57.40	61.72	66.51	69.74	75.57	2.19
25.80	27.82	30.30	33.41	34.38	37.81	41.57	45.51	2.74
24.66	26.15	28.25	29.87	31.29	34.05	35.81	39.31	2.57
21.93	23.18	24.66	26.43	28.32	30.41	32.89	35.68	2.40
23.50	24.70	26.22	27.30	27.79	29.39	30.41	32.03	1.76
261.00	271.90	284.90	299.90	312.40	332.50	348.50	370.50	1.92

Table 5.2

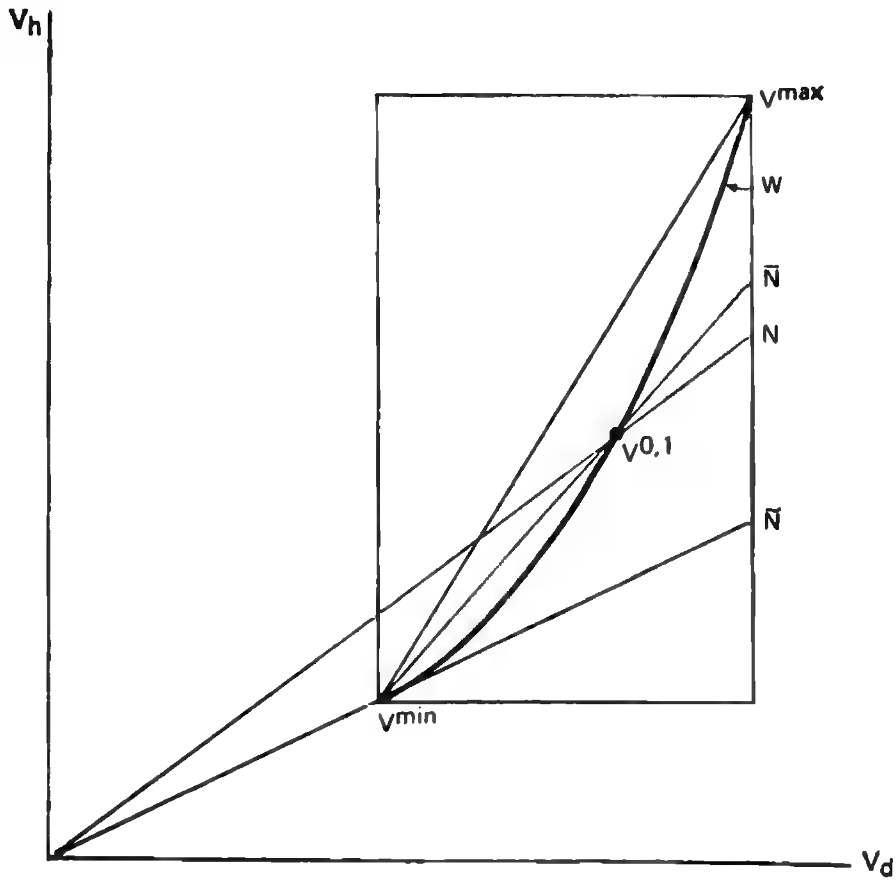
Increments in PCE (Δx_j) and Their Ratios ($\Delta x_j / \Delta x_{j'}$) (thousand mill. 1963 francs)

	Δx_j		$\Delta x_j / \Delta x_{j'}$	
	1960-1965	1966-1971	1960-1965	1966-1971
0. Food excluding alcoholic beverages	13.5	14.95	×	×
1. Alcoholic beverages	1.87	0.96	0.14	0.06
2. Clothing and footwear	9.20	8.09	0.68	0.54
3. Housing	17.16	23.94	1.27	1.60
4. Hygiene and health	11.22	17.69	0.83	1.18
5. Transport and communication	10.85	13.16	0.80	0.88
6. Culture and leisure	8.34	12.50	0.62	0.84
7. Hotels, restaurants, cafés and other	6.48	7.33	0.48	0.49
8. Sum total	78.6	98.6	5.82	6.60

foodstuffs as such. Let us explain how this conclusion was drawn.

In accordance with the properties of trajectory (5.16), the satisfaction of a more satiated need would be expected to grow slower than a less satiated one. While omitting a

detailed mathematical presentation of this fact, we shall explain it with the help of a diagram.



$$\tilde{N}=\left\{v\colon \frac{v_j}{v_j^{min}}=const \text{ for all } j\right\}$$

$$N=\left\{v\colon \frac{v_j}{v_j^{0,1}}=const \text{ for all } j\right\}$$

$$\bar{N}=\left\{v\colon \frac{v_j-v_j^{min}}{v_j^{0,1}-v_j^{min}}=const \text{ for all } j\right\}$$

The optimal growth curve (5.16) is denoted by W. It is strictly above ray $\tilde{N}$, which connects the beginning of the coordinates and point v^{min}. It follows that

$$v_h^0/v_h^{min}>v_d^0/v_d^{min}\ (j=h,\ d)$$

for any point v^0 belonging to curve W. This means that the consumption growth rate of goods h in relation to the level v_h^{min} is higher than that of goods d in relation to v_d^{min}.

The same is also true, however, of the comparison between any two points $v^{0,1}$, $v^{0,2}$ on curve W. Consider two points such that $v^{0,2} > v^{0,1}$ (in other words, point $v^{0,2}$ is any point of the segment of curve W that ascends from $v^{0,1}$). Then

$$\frac{v_h^{0,2}}{v_h^{0,1}} > \frac{v_d^{0,2}}{v_d^{0,1}}$$

(the pointed segment of curve W lies above ray N).[1]

As can easily be seen, the diagram complies with condition $q_d < q_h$.

If any q_d and q_h are compared, the lower consumption growth rate should always correspond to the need with the higher degree of satiation (i.e., a lower degree of non-satiation—q_j). That is why the consumption growth rates may be regarded within the framework of this theory as indicators of the degrees of satiation (non-satiation) of the needs under consideration. The consumption growth rates will be denoted by Iv_j, those of purchases by Ix_j. Then the above conclusion means:

$$Iv_h > Iv_d \Leftrightarrow q_h > q_d.$$

Variables q_h, q_d cannot be found from Iv_h, Iv_d, but it may be discovered which of them is greater, i.e., they may be ranked.

It can be seen from the above that, strictly speaking, not the property of function (5.16)

$$q_h > q_d \Rightarrow \Delta v_h / \Delta v_d \text{ increases}$$

that is to be checked, but the following one: that

$$Iv_h > Iv_d \Rightarrow \Delta v_h / \Delta v_d \text{ increases.}^2$$

If items 0 and 1 are compared, $d = 1$ ("Alcoholic Bever-

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[1] Moreover, this part of W lies above ray $\overline{N}$:

$$\frac{v_h^{0,2} - v_h^{min}}{v_h^{0,1} - v_h^{min}} > \frac{v_d^{0,2} - v_d^{min}}{v_d^{0,1} - v_d^{min}}$$

[2] It should not be assumed that this expression is a mere tautology. Consider the growth rates and increments of some variables a and b. From the argument: $Ia > Ib$ it does not necessarily follow that: $\Delta a/\Delta b$ increases. For example, in our computations for the item "Clothing and footwear" we have: Ix_2 (1971/1959) $= 1.81 > Ix_0$ (1971/1959) $= 1.46$ (see Table 5.1), but $\Delta x_2/\Delta x_0$ does not grow; it decreases (see Table 5.2).

ages") and $h = 0$ ("Food"). Note that both items cover non-durables only, for which $Ix_j \approx Iv_j$, but Ix_0 (1971/1959) > > Ix_1 (1971/1959):

$$Ix_0\,(1971/1959) = \frac{x_0\,(1971)}{x_0\,(1959)} = 1.46$$

$$Ix_1\,(1971/1959) = \frac{x_1\,(1971)}{x_1\,(1959)} = 1.27$$

Accordingly, Iv_0 (1971/1959) > Iv_1 (1971/1959). We conclude that $q_0 > q_1$.

In this case, concavity rather than convexity of curve (5.16) is expected:

$$\frac{d^2v_1}{dv_0^2} < 0.$$

Ratio $\Delta v_1/\Delta v_0$ should not grow, but decrease. This is the theoretical expectation, and it is corroborated by the data in Table 5.2.

For the other items Ix_j(1971/1959) > Ix_0(1971/1959) ($j \neq 0,1$). As for the items not including durables and semi-durables or those including both as only a small share, the conclusion Iv_j (1971/1959) > Iv_0 (1971/1959) is justified. This concerns the items "Housing", "Hygiene and health", "Hotels, restaurants, cafés, and other". In all of these cases, the increase in relations $\Delta x_j/\Delta x_0$ (see Table 5.2) may be regarded as the increase in relations $\Delta v_j/\Delta v_0$. The theoretical expectations are confirmed statistically.

An increase in relations $\Delta x_j/\Delta x_0$ may be also observed for the items "Transport and communication", and "Culture and leisure". These have a relatively large share of durables and semidurables. That is why the indicators for them in Table 5.2 cannot be regarded as direct statistical proof of the theoretical expectations. At the same time, the statistical data do not contradict these expectations.

Below, the statistical data on these two items will be shown, in fact, to confirm the expected curvature of the trajectories.

The decrease in ratio $\Delta x_2/\Delta x_0$ (the item "Clothing and footwear") took place under the condition that Ix_2 (1971/1959) > Ix_0 (1971/1959) (see Table 5.1). A more detailed consideration of the consumption growth processes for the item will also be presented below.

Thus, the first statistical test proved quite successful. Even if the statistics of x_j^t instead of v_j^t are used, the expected

sign of the changes in the ratios $\Delta x_j/\Delta x_0$ is confirmed in 6 cases of 7. It should be stressed, however, that the regular trend in the changes in $\Delta x_j/\Delta x_0$ manifests itself only over rather long time intervals, while within these intervals, if the data are examined from year to year, the trend looks very unstable. At the same time, the trends of ratios x_j/x_0 manifest themselves rather steadily, from year to year virtually without exception.

This is apparently why trends in the changes in ratios x_j/x_0 only are noted in the literature, while trends $\Delta x_j/\Delta x_0$ remain unnoticed.

Second statistical test: evaluations of the trajectories' degrees of curvature. The first test made it possible to check statistically the theoretical expectations concerning the existence of curvature of real consumer purchase growth trajectories and the sign of this curvature. Yet a more subtle property of curve (5.16) is the following: it differentiates the *degree* of curvature of trajectories depending on the difference between $q_{j'}$ and q_j. As already stated, given $q_{j'} = q_j$, this trajectory becomes a straight line. As might easily be seen, this line connects points v^{min} and v^{max}. Its general formula is:

$$v_j = v_j^{min} + z_{j'} (v_j^{max} - v_j^{min}). \tag{5.17}$$

If $q_{j'} \neq q_j$, curve (5.16) has no common points with the straight line connecting points v^{min} and v^{max} (mind that curve (5.16) was obtained for $\{v\colon v^{min} < v < v^{max}\}$ and does not, therefore, include points v^{min} and v^{max}). The deviation of curve (5.16) from the straight line being the greater, the greater is the value of the difference $(q_j - q_{j'})$. If this difference is negative, the corresponding deviation of curve (5.16) from the straight line is also assumed to be negative; the deviation is positive if $(q_j - q_{j'}) > 0$.

While processing statistical data, we do not know points v^{min} and v^{max}, and so we cannot connect these points by the straight line and calculate the difference between the actual curve and such a line. Nevertheless, a procedure can be proposed for evaluating the degree of convexity of the actual curves. This will be the value of their deviation from a straight line connecting the extreme points of observations statistically recorded [in this case, from the straight line connecting points x (1959) and x (1971)]. Let us explain this procedure.

The actual calculations in this section will be carried out not with statistics of values $v(t)$, but with statistics $x(t)$.

In the two-dimensional space of vectors $x = (x_{j'}, x_j)$ the following procedure is employed (for every particular j, j' being fixed). Let us denote observations (years of the period under consideration) by index $t = \tau, \tau + 1, \ldots, T$, where τ is the number of the observation when consumption reached a certain minimum (within the given series). The actual trajectories are thus considered for which, as a rule,

$$x_j^t \geqslant x_j^\tau \; (t > \tau)$$

holds for all j. Table 5.1 shows that

$$x_j(t) > x_j(\tau) \equiv x_j(1959); \; t = 1960, \ldots, 1971$$

appeared without any exception. That was a period with an overall consumer purchase growth exceeding $x(1959)$.

Then variables are to be calculated:

$$\bar{x}_j^t = x_j^\tau + (x_{j'}^t - x_{j'}^\tau) \cdot \frac{x_j^T - x_j^\tau}{x_{j'}^T - x_{j'}^\tau}, \tag{5.18}$$

where, x_j^τ, $x_{j'}^t$, $x_{j'}^\tau$, $x_{j'}^T$, x_j^T are actual consumption values for items j and j' statistically recorded; $\tau = 1959$, $T = 1971$; $t = 1960, \ldots, 1970$. It can easily be seen that $\bar{x}_j^t$ belongs to the straight line connecting points x^τ and x^T. Then the distance of the real trajectory from the straight line (5.18) for some point x^t, $t = (\tau + 1, \ldots, T - 1)$, can be found as the difference:

$$\bar{x}_j^t - x_j^t.$$

If the real trajectory is convex,

$$\bar{x}_j^t - x_j^t > 0$$

should be observed;
if it is concave,

$$\bar{x}_j^t - x_j^t < 0$$

should be observed.

Let us use the following percentage expressions of the distance between $\bar{x}_j^t$ and x_j^t:

$$F_j^t = \frac{\bar{x}_j^t - x_j^t}{x_j^t} \cdot 100 \; (t = \tau + 1, \ldots, T - 1). \tag{5.19}$$

The summed evaluation of the degree of curvature of the real trajectories will be determined with the aid of values

$$F_j = \sum_t F_j^t \; (t = \tau + 1, \ldots, T - 1). \tag{5.20}$$

Values F_j constitute the sum total of the relative deviations of the real trajectory from the straight line.

Calculated values $\bar{x}_j^t$ are given in Table 5.3. All were obtained under the condition that $j' = 0$; the actual data were used in the calculation for this item ("Food excluding alcoholic beverages") for the years of the period under consideration. Also, Table 5.3 indicates differences $\bar{x}_j^t - x_j^t$, where x_j^t were taken from Table 5.1.

Table 5.4 gives values F_j^t as calculated according to formula (5.19) and their sums: F_j. Table 5.5 indicates comparisons of consumer purchase growth rates Ix_j (1971/1959) with F_j.

Strict theoretical expectations may be stated if the statistics of values v are treated: Iv_j should be compared with values F_j as derived only from statistics v, rather than x. With certain reservations, these expectations were extended to the calculations with statistics x.

It is thus expected that, if $Ix_j < Ix_0$, then $F_j < 0$, and if $Ix_j > Ix_0$, then $F_j > 0$. As Table 5.5 shows, these expectations are supported, with one exception only: this test shows repeatedly that the curve for the item "Clothing and footwear" is concave, even though $Ix_2 > Ix_0$.

Also F_j is expected to be the smaller, the smaller is the corresponding Ix_j. To verify this expectation, values Ix_j and F_j were ranked in Table 5.5. The Spearman rank correlation coefficient[1] was calculated: $R = 0{,}79$. This is a rather high value. An additional characteristic is provided by estimating the a priori probability of arriving at value $S\ (d^2) = 12^1$: $Pr\ [S \leqslant S\ (d^2)] = 0.017$. In other words, it is highly unlikely that the estimated, rather low total sum of the squares of the rank deviations $S\ (d^2)$ and the corresponding high rank correlation coefficient R should appear by chance. It may be concluded that the purchase growth rates and the degrees of curvature of the trajectories correspond to each other with quite a high probability.

Also, the linear correlation coefficient of the series themselves of Ix_j and F_j was calculated and found to be rather high: $r = 0.77$. Note, however, that the correlation between Ix_j and F_j is not expected to be linear. That is why testing the rank correlation is of more general importance:

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1 The estimation is carried out here and below according to M. G. Kendall, see *Rank Correlation Methods*. Fourth edition, London, 1970.

Table 5.3

Calculated Values $\bar{x}_j^t$ and $(\bar{x}_j^t - x_j^t)$

	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
						$\bar{x}_j^t$					
1. Alcoholic beverages	10.51	10.69	10.95	11.19	11.45	11.68	11.90	12.19	12.40	12.68	12.89
2. Clothing and footwear	22.52	23.60	25.35	26.69	28.26	29.66	31.04	32.77	34.06	35.78	37.08
3. Housing	37.02	39.59	43.49	46.93	50.65	53.98	57.26	61.39	64.44	68.54	71.61
4. Hygiene and health	18.39	20.20	22.95	25.37	27.98	30.32	32.63	35.53	37.68	40.56	62.73
5. Transport and communication	16.79	18.29	20.57	22.58	24.75	26.70	28.62	31.03	32.81	35.20	37.00
6. Culture and leisure	16.13	17.44	19.42	21.16	23.04	24.73	26.40	28.49	30.04	32.12	33.67
7. Hotels, restaurants, cafés and other	19.08	19.94	21.25	22.41	23.66	24.77	25.88	27.26	28.29	29.67	30.70
						$\bar{x}_j^t - x_j^t$					
1. Alcoholic beverages	—0.01	0.05	—0.07	—0.41	—0.57	—0.53	—0.44	—0.42	—0.15	—0.08	0.09
2. Clothing and footwear	—0.11	—0.64	—1.28	—2.73	—2.25	—0.90	—0.46	0.56	0.50	—0.11	—0.19
3. Housing	0.57	0.77	1.11	0.27	1.32	2.35	3.42	3.99	2.72	2.03	1.87
4. Hygiene and health	0.62	0.58	1.42	1.83	2.18	2.50	2.33	2.12	3.30	2.75	1.16
5. Transport and communication	0.26	0.00	0.01	—0.08	0.09	0.55	0.37	1.16	1.52	1.15	1.19
6. Culture and leisure	—0.08	0.05	0.53	0.54	1.11	1.55	1.74	2.06	1.72	1.71	0.78
7. Hotels, restaurants, cafés and other	0.08	—0.10	—0.03	—0.13	0.16	0.07	—0.34	—0.04	0.50	0.28	0.29

Table 5.4

Values F_j, F_j^t and Their Ranks

	F_j and their ranks	F_j^t and their ranks										
		1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
1. Alcoholic beverages	−20.9 2	−0.1 3	0.5 5	−0.7 2	−3.6 2	−4.8 2	−4.3 1	−3.6 1	−3.3 1	−1.2 1	−0.6 1	0.7 2
2. Clothing and footwear	−26.9 1	−0.48 1	−2.6 1	−4.8 1	−9.3 1	−7.4 1	−3.2 2	−1.5 2	1.8 3	1.5 2	−0.3 2	−0.5 1
3. Housing	37.4 5	1.57 6	2.0 6	2.6 5	0.6 5	2.7 5	4.5 5	6.4 5	6.9 6	4.4 4	3.1 4	2.7 5
4. Hygiene and health	72.0 7	3.5 7	3.0 7	6.6 7	7.8 7	8.5 7	9.0 7	7.7 7	6.4 5	9.6 7	7.3 7	2.8 6
5. Transport and communication	20.5 4	1.56 5	0.0 3	0.1 4	−0.4 4	0.4 3	2.1 4	1.3 4	3.9 4	4.9 5	3.4 5	3.3 7
6. Culture and leisure	45.8 6	−0.47 2	0.3 4	2.8 6	2.6 6	5.1 6	6.7 6	7.0 6	7.8 7	6.1 6	5.6 6	2.4 4
7. Hotels, restaurants, cafés and other	2.4 3	0.4 4	−0.5 2	−0.1 3	−0.6 3	0.7 4	0.3 3	−1.3 3	−0.1 2	1.8 3	0.9 3	1.0 3

Note. For each item the first line shows values F_j, F_j^t, the second line – their ranks.

Table 5.5

Growth Rates of Purchases (Ix_j), Values F_j, H_j and Their Ranks

	Ix_j $\frac{1971}{1959}$	Their ranks	F_j	Their ranks	H_j	Their ranks	Ranks of Ix_j 1971/1959
0. Food, excluding alcoholic beverages	145.9	×	×	×	5.22	3	2
1. Alcoholic beverages	127.3	1	—20.9	2	6.15	1	1
2. Clothing and footwear	180.6	3	—26.9	1	5.71	2	4
3. Housing	219.2	4	37.4	5	4.76	6	5
4. Hygiene and health	274.2	7	72.0	7	4.52	8	8
5. Transport and communication	256.9	6	20.5	4	4.99	5	7
6. Culture and leisure	240.4	5	45.8	6	4.70	7	6
7. Hotels, restaurants, cafés and other	175.9	2	2.4	3	5.19	4	3
	$S(d^2)=12$ $R=0.79$ $Pr[S\leqslant S(d^2)]=0.017$ $r=0.77$				$S(d^2)=12$ $R=0.86$ $Pr[S\leqslant S(d^2)]=0.0036$		

the existence of a monotonous dependence is proved, and this may be both a straight line and a curve.

Such a dependence is an expression of a very subtle property of the law of structural shifts in consumption. The point is that the distance of the optimal curve (5.16) from the straight line connecting points v^{min} and v^{max} should exist, but it may not be large. Let us explain this assertion.

Let, in 1959, $v_0^{min}=57$ thous. mill. francs, $v_0^{max}=100$ thous. mill. francs; the ratio v_0^{min}/v_0^{max}, i.e., the degree of satiation of the need for food at the level of the historically established vector v^{min}, was, for the whole French population, 0.57, consequently, $q_0=0.43$. It will be shown below that, given this assumption, the actual processes of further consumption growth can be explained with a very high degree of accuracy. Suppose, for the item "Hygiene and

health", $v_4^{min} = 13.16$, $v_4^{max} = 62.81$ thous. mill. francs (this also results from the estimates below); then $q_4 = 0.79$. There are sufficient grounds for thinking that the actual differences between q_j and $q_{j'}$ exceed these limits only extremely rarely. In other words, given such q_j and $q_{j'}$, the highest degree of trajectory curvature should appear, but what value of the degree of curvature is expected theoretically?

Consider a point on the optimal trajectory for which $z_0 = 0.38$. This is the point where F_j^t achieves its maximum (from the diagram, given in the previous section, it is easily seen, that the optimal curve first leaves the straight line connecting points v^{min} and v^{max} and then draws back towards it, so that the maximum distance approximately corresponds to the middle of the curve). [1]

Let us find value of v_4^0, i.e., the optimal consumption level for the item "Hygiene and health", at the point where $z_{j'} = 0.38$. According to $A_{j'} = z_{j'}q_{j'}$ we obtain: $A_0 = 0.38 \cdot 0.43 = 0.1634$.

Then, according to (5.16):

$$v_4^0 = 13.16 + \frac{0.1634 \cdot 0.79}{0.79 - 0.38\,(0.79 - 0.43)} \cdot 62.81 = 25.57.$$

While, on the straight line connecting points v^{min} and v^{max}, given $z_{j'} = 0.38$, we have (see [5.16]):

$$\bar{v}_4 = 13.16 + 0.38\,(62.81 - 13.16) = 32.03.$$

The relative distance between v_4^0 and $\bar{v}_4$ is then:

$$F_4^0 = \frac{32.03 - 25.57}{25.57} \cdot 100 = 25\%.$$

This is the distance of the point on the optimal curve (5.16) from the corresponding one on the chord connecting points v^{min} and v^{max}. Yet we have so far calculated the distance not from this chord, but from that connecting the extreme points of observations statistically recorded. In this case, v_4 (1959) $= 16.60 > v_4^{min} = 13.16$; v_4 (1971) $=$

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[1] I. A. Itskovich derived the following formula for $z_{j'}$ meeting $max\ F_j^t$ (on the condition that $q_{j'} < q_j$):

$$z_{j'} = \frac{1}{1 + \sqrt{\dfrac{q_{j'}}{q_j - q_j^2}}}$$

In this case such $z_{j'} = 1/2.61 = 0.38$.

$= 45.51 < v_4^{max} = 62.81$; this means that the observed extreme consumption values for the item "Hygiene and health" are within the interval from v_4^{min} to v_4^{max}. The same is true of the item "Food". The chord connecting the actual extreme points v (1959) and v (1971) therefore lies much nearer to the optimal v_4^0 than that connecting v^{min} and v^{max}. Now we shall estimate the expected distance from such a chord.

Given $z_{j'} = 0.38$, $v_0^t = 57.0 + 0.38\,(100.0 - 57.0) = 73.34$.

Then, according to (5.18) and considering that, in this case, $x_j^t \approx v_j^t$, we obtain:

$$v_4^t = 16.60 + (73.34 - 62.05) \cdot \frac{45.51 - 16.60}{90.50 - 62.05} = 28.07.$$

The relative distance between v_4^0 and v_4^t is:

$$F_j^t = \frac{28.07 - 25.57}{25.57} = 9.8\%.$$

In Table 5.4 the reader will discover that the maximum distance of the real values v_4^t from the chord connecting points v (1959) and v (1971) was 9.6 per cent.

We have thus found that the greatest deviation of curve (5.16) for the item "Hygiene and health" from the straight line connecting points v (1959) and v (1971) should not exceed 10 per cent. Yet that is the maximum distance. At its other points, curve (5.16) is nearer to the above straight line. It follows that the average distance of curve (5.16) from the straight line connecting v (1959) and v (1971) should not be greater than 5 to 7 per cent.

Note, however, that 5 to 7 per cent is the estimate of the average distance for the optimal curve (5.16) for the case when it nearly attains its feasibly maximum curvature: remember that items 0 and 4 were taken, i.e., those with the maximum difference $q_j - q_{j'}$. As for the other items, the average distance of the optimal curve from the straight line connecting points v (1959) and v (1971) should be even smaller. Therefore, the law expressed by (5.16) means a definite curvature of optimal trajectories, but not clearly expressed one, such that the average distance between the optimal trajectories and some straight lines is measured literally in percentage points. In the case under consideration, the curvature values for all items should not exceed an average of 5 to 7 per cent.

Formula (5.16) shows, however, that this slight curvature should nevertheless be different for various j depending on values q_j (j' being fixed). That is, it may be expected that there are items for which the average distance of the actual curve from the straight line connecting v (1959) and v (1971) is 4-5, 3-4 per cent, etc. Consequently, given a generally weak curvature, the differences between items j with respect to their degree of curvature should be quite negligible. It seems that the consumer cannot, in his actual behaviour, be expected to effect such very small differences, i.e., that this quite subtle property of the law expressed by formula (5.16) would actually be realised. Nevertheless, *such differences are effected by the mass consumer*!

Precisely this fact is shown in Table 5.5. Indeed, values F_j are the sum totals of the relative deviations of the real curves from the straight lines connecting points v (1959) and v (1971). These sum totals result from 11 observations from 1960 to 1970. The average deviation of the real curves from the above straight lines can easily be calculated: it is $F_j/11$. The actual maximum average deviation (it is noteworthy that it appears in the case of the item "Hygiene and health") thus amounted to +6.5 per cent, while the smallest one—to —2.4 per cent. Seven items were covered by this range. The differences between them are no more than 1 to 2 per cent. These very subtle differences, as can be seen from Table 5.5, are rather closely correlated with the differences between Ix_j.[1]

We must return to the question already raised of the objective nature of the law governing mass consumer behaviour. Obviously, we are dealing with a really objective law, which determines the actions of consumer. A law should be very strong; it should manifest itself with considerable accuracy in mass consumer behaviour if the theoretically predicted curvature degrees originating from this law and differing from one another literally by percentage points and parts of a per cent appear clearly enough in real trajectories. A particular consumer hardly behaves strictly rationally, of course, but the law expresses the properties of *the optimum* in consumption, and *the totality* of consumers submit to this law, realising it in their behaviour.

Of further interest are the following observations. The degree of curvature was estimated not only for the trajectory

[1] Remember that values Iv_j and, with some reservations, Ix_j are considered to be indicators of degrees of non-satiation q_j.

as a whole, but also for each $t = (\tau + 1, \tau + 2, \ldots, T - 1)$. To what degree do these estimates correspond to the expected order?

Let us return to Table 5.4, which shows and classifies the estimates of relative distances from the straight line for each particular year. The ranks of these distances prove, on the whole, to be quite stable. As for the average rank, i.e., that of value F_j, in the overwhelming majority of cases they either do not differ from it or, if at all, do so by not more than 1. In only 7 cases out of 77 observations by year does the rank differ from the average by more than 1 (for example, for the item "Alcoholic beverages", in 1961 the rank of value F_j^t is 5, the average rank for this item being 2, etc.). This means that, not only for the period as a whole, but also in each year, the French consumer fulfilled the law of structural shifts in consumption with great accuracy. Only objective laws are observed this way; those produced by people are violated much more often and seriously.

The way of estimating the degree of trajectory curvature employed by us in these calculations is an heuristic one. Some other indicators might be offered of the distance between the chord connecting the extreme points of observations and the consumption growth trajectory within the interval between the extreme observations. All of them should be expected to result in approximately equal rank characteristics of the curvature's degree. Let us verify this by employing another possible (also heuristic) measurement procedure.

It follows from (5.18) that:

$$\overline{H}_j^t = \frac{\overline{x}_j^t - x_j^\tau}{x_j^T - x_j^\tau} = \frac{x_{j'}^t - x_{j'}^\tau}{x_{j'}^T - x_{j'}^\tau} = H_{j'}^t \text{ for all } j. \tag{5.21}$$

Thus, the chord connecting the extreme points of observations is characterised, at each point t, by equality of all values $\overline{H}_j^t$. If the real curve is convex, i.e., $\overline{x}_j^t - x_j^t > 0$, then

$$H_j^t = \frac{x_j^t - x_j^\tau}{x_j^T - x_j^\tau} < \overline{H}_j^t = H_{j'}^t,$$

consequently, $H_j^t < H_{j'}^t$. If it is concave, i.e., $\overline{x}_j^t - x_j^t < 0$, then $H_j^t > H_{j'}^t$.

Therefore, the ranks of H_j^t characterise the curvature of real consumption growth trajectories. Note that, while

verifying this property, all items may undergo the procedure of ranking, including j', for which the real curve obtained, if the same j' is used, is certainly a straight line—the bisectrix of the coordinate angle.

Values H_j^t were estimated for the same eleven years, and their totals were obtained, $H_j = \sum_t H_j^t$ (see Table 5.6). H_j are considered to be generalising curvature characteristics of trajectories. In Table 5.5 their ranks are compared with those of purchase growth rates. Given such a curvature evaluation, the rank correlation coefficient is even greater ($R = 0.86$), and the probability that this coefficient might be obtained by mere chance is even smaller than in the previous calculation.[1]

As Table 5.6 shows, the stability of the ranks obtained is also very high within the period. Only in 14 cases out of 88 do the deviations of ranks by year from the average exceed 1.

Additionally, the general conclusion is supported that in his real behaviour the French consumer with great accuracy realises the subtle properties of the law of consumption structure.

Third test: a retrospective forecast for 1962-1971. By examining Tables 5.4 and 5.6 we have already started dealing with the question of how accurately the properties of the structural law (5.16) manifest themselves not only over a long time interval, but also in individual years. The results seem promising. They allow us to expect that, on the basis of (5.16), we would succeed in obtaining sufficiently precise forecasts of mass consumer behaviour. At the same time, using the proposed utility function for forecasting is another test of it.[2]

Note that the properties of formula (5.16) facilitate forecasting. First, in order to forecast the consumption structure using formula (5.16), there is no need to forecast relative

[1] The reader must have noticed that ranks F_j and H_j do not coincide exactly. The different heuristic procedures employed for curvature estimates do not produce absolutely identical results. It would thus be reasonable to employ several techniques in order to achieve greater accuracy.

[2] Obviously, serious work on forecasting must be carried out by the experts and may by no means be reduced to employing some standard set of techniques. The author being not a specialist on France's economic affairs gives his computations only in order to draw the reader's attention to the prognostic potential of the utility function.

Table 5.6

Values H^t_j, H_j and Their Ranks

	H^t_j and their ranks											H_j and their ranks
	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	
0. Food, excluding alcoholic beverages	0.062	0.1248	0.219	0.304	0.394	0.475	0.55	0.65	0.75	0.83	0.90	5.22
	4	3	4	5	3	3	4	3	2	3	2	3
1. Alcoholic beverages	0.064	0.1060	0.25	0.45	0.60	0.66	0.71	0.80	0.78	0.86	0.873	6.15
	3	6	2	2	1	1	1	1	1	1	4	1
2. Clothing and footwear	0.068	0.16	0.29	0.46	0.52	0.53	0.581	0.62	0.70	0.84	0.91	5.71
	1	1	1	1	2	2	2	4	3	2	1	2
3. Housing	0.048	0.1058	0.192	0.297	0.36	0.42	0.4713	0.558	0.66	0.780	0.858	4.76
	7	7	7	6	6	6	7	7	6	6	7	6
4. Hygiene and health	0.04	0.10	0.17	0.24	0.32	0.39	0.474	0.58	0.62	0.73	0.864	4.52
	8	8	8	8	8	8	6	6	8	8	6	8
5. Transport and communication	0.051	0.1245	0.218	0.307	0.390	0.45	0.54	0.61	0.67	0.781	0.85	4.99
	6	4	5	4	4	5	5	5	5	5	8	5
6. Culture and leisure	0.066	0.1220	0.194	0.28	0.34	0.40	0.4712	0.556	0.65	0.75	0.866	4.70
	2	5	6	7	7	7	8	8	7	7	5	7
7. Hotels, restaurants, cafés, and other	0.056	0.13	0.222	0.313	0.38	0.469	0.579	0.66	0.69	0.81	0.88	5.19
	5	2	3	3	5	4	3	2	4	4	3	4

Note. For each item the first line shows values H^t_j, H_j, the second – their ranks.

prices (a difficult task in itself). Second, since the general formula of structural shifts is given, there is no need to use statistical data to determine the mathematical form of dependences according to which a forecast will be made for items; statistics are used only to determine the parameters (i.e., vectors v^{min} and v^{max}) of the given general dependence (5.16), and this greatly reduces the body of essential initial data.

All these simplifications are possible, of course, only if formula (5.16) itself is valid. Yet, in any case, the procedure for testing it by means of retrospective forecast is relatively simple.[1]

Strictly speaking, one should forecast values v_j^t and then proceed to x_j^t (for durables and semidurables, with the aid of a special formula; see below), but we shall start by applying formula (5.16) directly to statistics x_j^t. We assume that the reader will thus find it easier to test and perceive the proposed forecasting techniques with the aid of curve (5.16). The error is, in this case, expected to be small for the items including nondurables and services (i.e., such j for which $x_j^t \approx v_j^t$) and significant for those in which the share of semidurables and durables is great. As for these last items, below we shall have to estimate accumulated property, and its removal rate, and then repeat the forecast this time for values v_j^t.

The reader can familiarise himself with the first attempts at forecasting on the basis of formula (5.16) in the author's earlier publications (see the footnote to page 315). At present we are employing a general set of techniques developed earlier.

As can easily be seen, the parameters of formula (5.16) are vectors v^{min} and v^{max} (here x^{min} and x^{max}). The techniques for estimating them, as discussed below, can be called *the method of revealed needs* (normal needs and total ones).

Vector x^{min} is calculated first. Let us estimate

$$x_{j'}^{min} \equiv x_0^{min} = 57 \text{ thous. mill. francs}$$

[1] Note the sense of the concept *retrospective forecast* as used in this work. This is not a forecast for the period, the statistics of which were used for the calculation of formula (5.16) parameters. Forecasting is carried out strictly for a period to come and in this respect complies with the concept of an actual forecast; variables $x_j(t)$ and $v_j(t)$, $j \neq j'$, $t = 1962, \ldots, 1971$ are determined, the actual values of which were not used in determining the model's parameters. We call the forecast retrospective only because it has to do with a past period of time.

(everywhere in 1963 prices). The estimate relies on the theory according to which x^{min} is the consumption level fixed as the usual one; it is accepted that the usual nourishment level lags behind the one achieved by not more than 2-3 years. The consumption of foodstuffs (except of alcoholic beverages) in France increased annually by approximately 1.8 thous. mill. francs. In 1959 it amounted to 62.05 thous. mill. francs. By subtracting 3·1.8 thous. mill. francs and rounding, we obtain 57 thous. mill. francs. Let us also estimate

$$x_{j'}^{max} = x_0^{max} = 100 \text{ thous. mill. francs}$$

The estimate was made in order to observe that: $x_0^{max} > x_0^{1971}$ i.e., that x_0^{max} is substantially higher than any observed level of foodstuff consumption. It is obviously true that people's needs, including for food, are far from fully satisfied. This assertion is supported by the following: consumption growth does not stop although it ought to have ceased on reaching v^{max} (according to the sense of this vector). The same is evidenced by the differences in the consumption of different income groups.[1]

Now, to estimate x_j^{min} for any $j \neq 0$, we employ a standard procedure relying on the properties of formula(5.16).

By transforming (5.16) and assuming $x_j = v_j$, we obtain:

$$x_j - \frac{A_{j'} q_j}{q_j - z_{j'} q_j + z_{j'} q_{j'}} \cdot x_j^{max} - x_j^{min} = 0;$$

$$x_j - \frac{A_{j'} q_j}{q_j (1 - z_{j'}) + A_{j'}} \cdot x_j^{max} - x_j^{min} = 0.$$

By solving the system of two such nonlinear equations,[2] the values of two unknowns: x_j^{max} and x_j^{min} may be found. A substantial simplification of the calculation procedure is achieved if the equations are reduced to the linear type.

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[1] Total needs x^{max} should be distinguished from the effective demand x, which is much closer to x^{min} than to x^{max}. Note that a better technique for estimating x_0^{max} would be to employ the data on the consumption of foodstuffs in prosperous working population families. As sociological studies show, the body of working classes are striving to come as close as possible to this level. We lack data, however, on diet by group of families with different incomes.

[2] The equations are nonlinear because q_j are calculated using x_j^{max} and x_j^{min}.

Then $A_{j'}$ can be ignored in the denominator; its values for the preforecast points of time are inevitably small, since the values of $x_{j'}$ for these exceed $x_{j'}^{min}$ only negligibly. Note that $A_{j'}$ may not be ignored in the numerator, where it is not an item but a co-factor.

Given this simplification, we obtain:

$$x_j - \frac{A_{j'}}{1 - z_{j'}} \cdot x_j^{max} - x_j^{min} = 0.$$

This time the equation is linear: for x_j^{max} it has a coefficient (co-factor) depending only on variables $v_{j'}$, $v_{j'}^{min}$, and $v_{j'}^{max}$, which are given. If x_j are also given, only x_j^{max} and x_j^{min} will remain unknown. To find them, two simultaneous equations need to be solved:

$$\left.\begin{aligned} x_j(t_1) - \frac{A_{j'}(t_1)}{1 - z_{j'}(t_1)} \cdot x_j^{max} - x_j^{min} = 0; \\ x_j(t_2) - \frac{A_{j'}(t_2)}{1 - z_{j'}(t_2)} \cdot x_j^{max} - x_j^{min} = 0. \end{aligned}\right| \qquad (5.22)$$

In such a case, variables x_j, $x_{j'}$ are used for the points of time t_1 and t_2, when consumption growth takes place; to find $A_{j'}$, $z_{j'}$, values $x_{j'}^{min}$ and $x_{j'}^{max}$ should be given too. The requirements placed on determining points of time t_1 and t_2 are somewhat contradictory. On the one hand, these points should be sufficiently near each other in order that x_j^{min}, x_j^{max} (as well as $x_{j'}^{min}$, $x_{j'}^{max}$) might be assumed constant without perceptible error; otherwise the two equations would have had four unknowns. On the other hand, it is necessary to eliminate as completely as possible the effect of chance features of points of time t_1 and t_2, for which purpose the temporal distance between them should be increased.

The error made in linearising can have a fundamental effect on the estimation of x_j^{max} only: it is seen in (5.22) that the estimates of x_j^{min} inevitably prove close to the given x_j (to the smallest of them) and are mostly invariable with regard to the accuracy of the coefficient for x_j^{max}. It follows from this that, even if both x_j^{min} and x_j^{max} are found from equations (5.22), it is the estimate of x_j^{min} only that should be used.

Table 5.7

Retrospective Forecast of
(thous. mill.

	Initial data			x^{min}	Forc	
	1959	1960	1961		1962	1963
1. Alcoholic beverages	10.33	10.51	10.63	9.99	11.03 10.70 —2.96	11.60 10.78 —7.07
2. Clothing and footwear	21.45	22.63	24.24	18.27	26.53 25.83 —2.65	29.42 27.64 —6.06
3. Housing	34.47	36.45	38.82	29.51	42.38 41.53 —2.00	46.66 44.41 —4.82
4. Hygiene and health	16.60	17.77	19.62	13.16	21.53 21.22 —1.42	23.54 23.21 —1.40
5. Transport and communication	15.30	16.53	18.29	11.89	20.56 20.02 —2.62	22.66 22.03 —2.76
6. Culture and leisure	14.84	16.21	17.39	11.93	18.89 19.42 2.80	20.62 21.27 3.13
7. Hotels, restaurants, cafés and other	18.22	19.00	20.04	16.15	21.28 21.03 —1.19	22.54 22.16 —1.71
8. Sum total	193.30	202.90	214.60	169.03	230.50 227.35 —1.37	247.70 241.07 —2.68

Notes. 1. For each item the first line shows the actual values, the second – actual ones (per cent).
2. Additionally, the following data of the "Food excluding alcoholic

x_0^{min}	1959	1960	1961	1962	1963
57.00	62.05	63.81	65.60	68.30	70.68

Consumer Purchases
1963 francs)

cast								x^{max}
1964	1965	1966	1967	1968	1969	1970	1971	
12.02	12.20	12.34	12.61	12.55	12.76	12.80	13.16	
10.85	10.90	10.95	11.00	11.03	11.07	11.10	11.13	11.21
—9.73	—10.62	—11.26	—12.79	—12.09	—13.23	—13.30	—15.45	
30.51	30.65	31.50	32.21	33.56	35.89	37.37	38.74	
29.69	31.63	33.65	36.31	38.41	41.38	34.75	46.97	60.15
—2.67	3.20	6.82	12.74	14.45	15.30	17.38	21.24	
49.33	51.63	53.84	57.40	61.72	66.51	69.74	75.57	
47.68	50.75	53.95	58.17	61.49	66.20	69.94	75.03	95.80
—3.35	—1.71	0.20	1.34	—0.37	—0.47	0.29	—0.72	
25.80	27.82	30.30	33.41	34.38	37.81	41.57	45.51	
25.49	27.66	29.95	33.02	35.47	38.99	41.85	45.79	62.81
—1.20	—0.56	—1.15	—1.17	—3.16	3.13	0.66	0.62	
24.66	26.15	28.25	29.87	31.29	34.05	35.81	39.31	
24.35	26.56	28.90	32.04	34.55	38.18	41.13	45.22	63.06
—1.25	1.58	2.30	7.26	10.43	12.14	14.86	15.04	
21.93	23.18	24.66	26.43	28.32	30.41	32.89	35.68	
23.39	25.41	27.54	30.40	32.69	35.98	38.61	42.33	58.28
6.65	9.63	11.69	15.03	15.41	18.31	17.49	18.64	
23.50	24.70	26.22	27.30	27.79	29.39	30.41	32.03	
23.42	24.59	25.78	27.34	28.54	30.22	31.53	33.27	40.00
—0.36	—0.47	—1.66	0.15	2.71	2.82	3.67	3.87	
261.00	271.90	284.90	299.90	312.40	332.50	348.50	370.50	
256.53	270.97	285.88	305.41	320.63	342.01	358.85	381.51	471.30
—1.71	—0.34	0.34	1.84	2.64	2.86	2.97	2.97	

calculated values, the third – deviations of the calculated values from the

beverages" item were used:

1964	1965	1966	1967	1968	1969	1970	1971	x_0^{max}
73.25	75.55	77.83	80.68	82.80	85.64	87.77	90.50	100.00

In our estimations with system (5.22), it is taken that $t_1 = 1959$, $t_2 = 1961$. The resulting values of x_j^{min} are given in Table 5.7.

The estimate of x_j^{max} can be calculated separately, using equation (5.23) as obtained by transforming equation (5.16):

$$(x_j^{max})^2 - \left[x_j^t + \frac{1-z_{j'}^t}{A_{j'}^t}(x_j^t - x_j^{min})\right] x_j^{max} + \\ + \frac{1-z_{j'}^t}{A_{j'}^t}(x_j^t - x_j^{min})\, x_j^{min} = 0. \qquad (5.23)$$

The larger of the two solutions of this square equation should be taken as x_j^{max} (the other is systematically lower than x_j^{min}; but $x_j^{max} > x_j^{min}$ should be observed). 1960 was taken as t. The estimation results of x_j^{max} are given in Table 5.7.

Thus, to find the parameters of equations (5.16), the data for only three initial years were used: 1959-1961. Here lies the second advantage of formula (5.16).

In order to understand correctly the forecast with the aid of formula (5.16), it must always be remembered that the formula expresses the law of the consumption *structure*. It answers only the question of the amount of goods of item j that should be consumed when some consumption level of goods of kind j' is achieved. When precisely this will be achieved depends on the consumers' income growth rates. The authors of the models, when making consumption forecasts by item, assumed incomes to be given (exogenously; as expert estimates or calculations with a special model) by year. In our model, i.e., formula (5.16), the same part is played by expenditures on purchases of foodstuffs given exogenously. It can be said that this is an indicator reflecting the total income growth, i.e., a specific indirect representative of income growth. Statistical data confirm that, if incomes grow in general, the consumption of foodstuffs grows, though at a different rate (see Table 5.1). The calculation with formula (5.16) thus represents a consumption forecast for large aggregates depending indirectly on the total increase in consumer incomes. After variables x_j^t have been forecast, by totaling it may be discovered at which total amount of private consumer expenditures the corresponding vector of these purchases by item will be achieved.

Generally, the values of x_j $(j \neq j')$ can be forecast for any values of $x_{j'}$ between $x_{j'}^{min}$ and $x_{j'}^{max}$, but in making a retrospective forecast, it is reasonable to take as exogenous parameters only those values of $x_{j'}^t$ that were already recorded statistically. Then the x_j^t $(j \neq j')$ to be forecast can be compared directly with the actual x_j^t recorded by the statistics in the same year as the corresponding $x_{j'}^t$. In retrospective forecasts, th eexogenous parameters of the models are always used at their actual level.

Formula (5.16) is thus used only as that of the consumption structure: it does not tell when a given general consumption level will be achieved. It only indicates what its structure will be, given a particular general level.[1]

Consider the outcomes of the retrospective forecast for items $j = 3, 4, 7$. The share of durables and semidurables in them is relatively small. The forecast proved very accurate. For each item, 10 values of $x_j^t \approx v_j^t$ (for 1962-1971) were calculated. Thirty values in total were thus forecast. In no case does their error exceed (modulo) 5 per cent: 11 out of 30 errors are within 1 per cent, 11 errors from 1 to 2 per cent, 2 errors from 2 to 3 per cent, 5 errors from 3 to 4 per cent, and 1 error from 4 to 5 per cent. In 80 per cent of the cases the error is no greater than 3 per cent. This quality of the forecasts is very high, especially considering that they were made for a period of 10 years. The reader familiar with forecasting practices will draw precisely the same conclusion.[2]

The forecast for items with a relatively large share of semidurables and durables—"Clothing and footwear", "Transport and communication", "Culture and leisure"—was considerably less exact. This applies in particular to the period

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[1] Strictly speaking, changes in the consumption structure depend on the rate of total income growth, but this is a relatively weak dependence, which can at first be ignored in finding the *consumption* structure (vector v). It is much more substantial in determining the vector of current purchases (x), because it is the rate of removal of durables and semidurables from consumption and, therefore, the amount of purchases to replace those goods that depend on the income growth rate. See below.

[2] The estimation for the items under study is as accurate as to raise doubts concerning its correspondence to the concept of a forecast. Let us once more emphasise that we are dealing with a completely pure retrospective forecast, where no data were used for items $j \neq j'$, except those for 1959-1961.

following 1965. Note that, after 1965, a general reduction in consumer purchase growth rates took place; the annual rate from 1960 to 1965 was 105.9 per cent and from 1966 to 1971—105.3 per cent. A deceleration of the overall rise in incomes first, of course, affects purchases of durables and semidurables, which are specifically characterised by a high income elasticity. To improve the retrospective forecast further, we must proceed from the data on purchases x_j^t to estimating consumption volumes v_j^t and then return to the values of purchases through the indicators of the removal of goods from consumption and of the accumulated property increase (see next section).

As for the item "Alcoholic beverages", the retrospective forecast proved to be a substantial underestimate. Probably, in this case, the rate at which full need v_j^{max} increases is such that this value may not be assumed constant for a period of 10 years, v_j^{max} as obtained with formula (5.23) on the basis of 1959-1961 data may not therefore simply be used for a ten-year period. The techniques of forecasting must thus, in this and similar cases, be made more sophisticated: a procedure needs to be developed for forecasting changes in v_j^{max}. i.e., the law governing the growth of total needs must be formulated. We have not yet fulfilled this task. In the next section, an heuristic method to make the forecast more sophisticated will be set out.

The total amount of consumer purchases in 1971 was 370.5 thous. mill. francs' worth, including 90.5 thous. mill. francs' worth of foodstuffs. The items for which the forecast was calculated totalled 280 thous. mill; 153.1 thous. mill. francs, i.e., 55 per cent in this total, is the share of the three items for which the forecast turned out to be sufficiently exact. The errors for the other items in part cancel each other out.

In Table 5.7 the forecast for the total sum of consumer purchases is given. It was carried out not by summing the forecasts for particular items, but separately [i.e., the total was taken as $j = 8$, with the above techniques for x_j^{min} and x_j^{max} and then with formula (5.16) values x_8^t corresponding to x_0^t were estimated]. The error does not exceed 3 per cent by module. This fact is of importance as a test support that the consumption of foodstuffs may be considered as a sufficiently precise indicator of the overall consumer income level.

Further sophistication of forecasting techniques requires that the difference between variables v and x should be taken into account for items with a large share of durables and semidurables. Let us begin with the item "Clothing and footwear", using the example of which we shall set out general specific features of calculations for semidurables and durables.

A sophisticated retrospective forecast for the item "Clothing and footwear". More than 90 per cent of these goods are semidurables.[1] For example, in 1959 total expenditures on them amounted to 21.45 thous. mill. 1963 francs, expenditures on semidurables coming to 19.97; in 1971 the figures were 38.74 and 36.85 thous. mill. francs respectively. We shall not make any basic mistake by assuming (in order to simplify the estimates) that the item "Clothing and footwear" covers semidurables only.

We estimated the average lifetime of the goods making up this item as 4 to 5 years. Considering that, in 1959, the purchases of these goods amounted to 21.45 thous. mill. francs and in subsequent years systematically increased, it may be supposed that they also grew in previous years. These considerations were taken as the basis for estimating accumulated property of this item; by the end of 1958, accumulated property should have been approximately 90 thous. mill. francs [v_2 (1958) = 90.0]. Roughly, it was supposed that, for 4.5 years, property of 20 thous. mill. francs annually was accumulated on average.

Let us call the ratio of the amount of property removed in year t to its amount by the end of year $t - 1$ the rate of removal (r_j^t):

$$r_j^t = c_j^t / v_j^{t-1}, \tag{5.24}$$

where c_j^t is the amount of property removed.[2] Provided there is a regular gradual increase in purchases of semidurables and durables, the removal in year t would be smaller than purchases in year $t - 1$ and even smaller than the average amount of purchases for the time interval equal to the lifetime of the good. There is a well-known method of permanent stock-taking that allows the rate of removal to be estimated, but we lack data on purchases in 1954 to 1958, without which it is impossible to make use of this method.

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[1] See F. Pascaud, *op. cit.*, pp. 70, 138.

[2] Note that all the estimates of accumulated property are given for the end of some year t (the beginning of year $t + 1$).

On the basis of approximate estimates, we assumed the rate of removal of clothing and footwear to be equal to 0.18. This means that we assume the following: in 1954-1955 annual purchases of clothing and footwear amounted to an average of 16-16.5 thous. mill. francs in 1963 prices. The point is that, if the assumed lifetime is true, clothing and footwear purchased precisely in 1954-1955 were removed in 1959.[1]

We thus suppose that, prior to 1959, purchases of clothing and footwear tended to increase by 1 thous. mill. francs in 1963 prices on average per annum. This figure is hardly an exaggeration: after 1959, the purchases increased annually by 1-2 or even more thous. mill. francs (see Table 5.1). Our estimation $r_2^{1959} = 0.18$ does not, therefore, seem too low.

The statistical data on purchases of clothing and footwear for 1959-1971 are known. On their basis and on the assumption that $v_2^{1958} = 90.0$; $r_2^t = 0.18$ for all t it is easy to calculate estimates of the property actually accumulated by the end of each year under consideration. Let us use the obvious formula:

$$x_j^t = c_j^t + \Delta v_j^t, \tag{5.25}$$

where Δv_j^t is the increment in the property accumulated in year t ($\Delta v_j^t = v_j^t - v_j^{t-1}$). From this we obtain a recurrent procedure for estimating accumulated property:

$$c_j^t = r_j^t v_j^{t-1};$$
$$\Delta v_j^t = x_j^t - c_j^t;$$
$$v_j^t = v_j^{t-1} + \Delta v_j^t.$$

For example,

$$c_2^{1959} = 0.18 \cdot 90.0 = 16.2;$$
$$\Delta v_2^{1959} = 21.45 - 16.2 = 5.25;$$
$$v_2^{1959} = 90.0 + 5.25 = 95.25.$$

Then

$$c_2^{1960} = 0.18 \cdot 95.25 = 17.14, \text{ etc.}$$

The estimates of accumulated property derived in this way are given in Table 5.8. Below, the estimates of v_2^t are

[1] This reasoning is not quite exact because the item "Clothing and footwear" includes goods with a rather wide range of service life. Using averaged values we ignore, however, the internal structure of the item, which inevitably results in some error.

Table 5.8

Retrospective Forecast of the Accumulated Property and Purchases of «Clothing and Footwear»
(thous. mill. 1963 francs)

	Accumulated property			Annual increment of property			Replacement of removed items			Purchases			Share of replacement in purchases	
	Actual value	Calculated value	Deviation (per cent)	Actual value	Calculated value	Deviation (per cent)	Actual value	Calculated value	Deviation (per cent)	Actual value	Calculated value	Deviation (per cent)	Actual value	Calculated value
1	2	3	4	5	[illegible]	7	8	9	10	11	12	13	14	15
min	90.0													
1959	95.25			5.25			16.20			21.45			0.76	
1960	100.73			5.48			17.14			22.63			0.76	
1961	106.83			6.10			18.13			24.24			0.75	
1962	114.13			7.30			19.23			26.53			0.72	
1963	123.00			8.87			20.54			29.42			0.70	
1964	131.37	129.91	—1.1	8.37	6.91	—17.4	22.14	22.14	—0.0	30.51	29.05	—4.8	0.73	0.76
1965	138.37	136.31	—1.5	7.00	6.40	—8.5	23.65	23.38	—1.1	30.65	29.79	—2.8	0.77	0.79
1966	144.96	142.88	—1.4	6.59	6.56	—0.4	24.91	24.54	—1.5	31.50	31.10	—1.3	0.79	0.79
1967	151.07	151.39	0.2	6.11	8.51	39.3	26.09	25.72	—1.4	32.21	34.23	6.3	0.81	0.75
1968	157.43	157.96	0.3	6.36	6.57	3.4	27.19	27.25	0.2	33.56	33.82	0.8	0.81	0.81
1969	164.98	167.10	1.3	7.55	9.14	21.1	28.34	28.43	0.3	35.89	37.58	4.7	0.79	0.76
1970	172.55	174.23	1.0	7.57	7.13	—5.9	29.70	30.08	1.3	37.27	37.20	—0.2	0.80	0.81
1971	180.23	183.72	1.9	7.68	9.49	23.6	31.06	31.06	1.0	38.74	40.85	5.5	0.80	0.77
max		220.28												

regarded as actual because they are calculated using the actual data for purchases of clothing and footwear.

Now the data on actually accumulated property will be used for a retrospective forecast of v_2^t and x_2^t. The procedure is largely the same as that shown in the previous section. The main difference is that formula (5.16) is used directly to calculate v_j^t, while x_j^t is additionally determined with the aid of formula (5.25).

The parameters of formula (5.16) are variables v_0^{min}, v_0^{max}, v_0^t, v_2^{min}, v_2^{max}. The first four are already known; v_2^{max} can be derived from formula (5.23) by substituting v for x in it, indicators v_2^t having to be used for some t prior to the forecast period. In the previous section, 1960 was assumed to be such a t. It was found that, in order to estimate v_j^{max} for items with a large share of durables and semi-durables, the initial information must be used for a longer time period. Values v_2^{1963} were used in calculations with formula (5.23). Calculations for items 5, 6 and 1, shown in section, were made similarly.[1]

The estimate derived was $v_2^{max} = 220.28$ thous. mill. francs.[2] This is significantly higher than the estimate of accumulated property in any year of the forecast period (see the data in column 2). Given such a v_2^{max}, the forecast v_2^t (given in column 3 of the Table) proved very close to the actual ones. The forecast was, of course, in this case made for the period 1964-1971 (8 years) only. The highest error of the forecast is no more than 2 per cent (according to module). It is also significant that the error did not always have the same sign: for 1964 to 1966 the forecast is somewhat of an

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[1] In contrast to the previous section, therefore, year t for v_j^{max} estimation was chosen here in order to attain, in calculations using these values, the best approximation of the given consumer purchase statistics. Such a procedure would be correctly described as an *imitation* of the initial statistics, not a true retrospective forecast. Nevertheless, it is essential that the imitation using formulae (5.16) and (5.25) be sufficiently close to the real data.

[2] By inserting $v_0^{min} = 57.0$; $v_0^{max} = 100.0$; $v_0^{1963} = 70.68$; $v_2^{min} = 90.0$; $v_2^{1963} = 123.0$ in square equation (5.23) we obtain:

$$(v_2^{max})^2 - \left[123.0 + \frac{1-0.3181}{0.1368}(123.0-90.0)\right] v_2^{max} + \frac{1-0.3181}{0.1368}(123.0-90.0)\cdot 90.0 = 0.$$

The greater of the two values of v_2^{max}, as derived from this equation, is 220.28.

underestimate, while for 1967 to 1971 it is an overestimate compared with the estimates of actual property (see column 4 of the Table).

The forecast's quality should be assessed using not only the data on accumulated property, but primarily those on purchases: it is precisely on this point that we compare our estimates directly with the statistics. Obviously, the forecast was substantially better than that for the same item which was presented in Table 5.7. The greatest error is 6.3 per cent and the error is not regular in nature: the forecast values for four of the eight years were lower than the actual ones and for the four others—higher. Note that, if successive pairs of years (1964-1965, 1966-1967, 1968-1969, 1970-1971) are considered, the total error does not exceed (by module) 4 per cent. For the whole forecast period (8 years), the sum total of forecast x_2^t is 273.55 thous. mill. francs and of actual x_2^t—270.33. The error is 1.2 per cent.

Since the actual and forecast estimates of accumulated property are very close to each other, the differences between the two estimates of property removed are insignificant (see columns 8-10). The error in the estimate of the annual property increase remains significant (see columns 5-7). It is the increases, however, that are substantially affected by situational specifics in particular years. Such specifics are obviously not reflected by the general law expressed by formula (5.16), but they generate fluctuations around this law.[1]

The transition to the forecasting of accumulated property and purchases on this basis alone is, evidently, a productive method for improving the forecast for this item where the share of semidurables is very high. It goes without saying that the basic data on accumulated property by the beginning of the period and the estimates of the rate of removal would be better derived directly from the statistics, rather than with the aid of expert estimates. Yet on the whole it may be said that no method is likely to be found that would provide a much more accurate forecast.

[1] Let us note that the calculations as presented in Table 5.7 concern the sum total of annual purchases. Considering the error of the forecasts for the *increments* of these purchases by the year (these increments Δx_j^t coincide with Δv_j^t for $j = 3, 4, 7$), the error proves significant in spite of the high degree of accuracy of the forecasts of values v_j^t themselves for these j. The existence of chance fluctuations of values x_j^t is quite clear from the whole set of calculations.

Note, however, that we are able to derive this forecast only by processing the statistical data on purchases for the first 5 years, instead of for 3 years as above; the forecast period was correspondingly reduced from 10 to 8 years. Moreover, we chose this forecast only because we were convinced by the statistics of its sufficient accuracy. If such data were not available (i.e., in making an *ex-ante* forecast), many more difficulties would be encountered.

This means, however, only one thing: that the technique of obtaining parameters v_j^{max} needs further improvement. At the same time, if these parameters are well determined, formula (5.16) together with (5.25) provides a basis for rather exact forecasts. We obtain additional support in favour of this conclusion by examining the forecast for the other items. Let us stress straight away that, in all the following cases we used variables v_j^t for the same year, 1963, in calculating with formula (5.23). It follows that, below, we did not change the method for deriving basic data for the forecast and employed the standard procedure. Even more important is the fact that it had produced sufficiently exact results.

A sophisticated retrospective forecast for the items "Transport and communication", "Culture and leisure". In these cases, the fact that the items include not only durables, but also nondurables and services, may no longer be ignored. The work by F. Pascaud makes it possible accurately to divide purchases relating to these items into such two parts. The forecast was conducted for each part separately: for durables, the procedure described in the previous section was used; for nondurables and services—that presented in the section "Third test: retrospective forecast for 1962-1971". [In all cases the calculations using formula (5.23) were conducted on the basis of the data on magnitudes v_j^t for $t = 1963$.] Table 5.9 presents the results of these calculations.[1] The reader can see that the

[1] The Tables presenting the results of the forecast in detail are omitted to save space. Here we show only the estimates obtained of the following magnitudes (in thous. mill. 1963 francs):

	v^{min}	v^{max}
5. *Transport and Communication*		
5a. Durables	18.30	80.61
5b. Nondurables and services	9.61	38.53
6. *Culture and Leisure*		
6a. Durables	20.00	108.20
6b. Nondurables	6.14	16.05
6c. Tobacco	3.62	7.31
1. *Alcoholic Beverages*	9.99	13.83

Table 5.9

Retrospective Forecast of Consumer Purchases for Items 5, 6, and 1

	1964	1965	1966	1967	1968	1969	1970	1971
5. Transport and communication								
5a. Durables	6.32	6.50	6.94	8.24	7.88	9.56	9.19	10.94
5b Nondurables and services	17.07	19.16	19.72	21.53	22.97	25.02	26.68	28.94
Total	23.39	25.66	26.66	29.77	30.85	34.58	35.87	39.88
Actual Purchases	24.66	26.15	28.25	29.87	31.29	34.05	35.81	39.31
Deviation of Estimated Magnitudes from Actual, per cent	−5.15	−1.87	−5.63	−0.33	−1.41	1.56	0.17	1.45
6. Culture and leisure								
6a. Durables	7.35	7.94	8.99	11.06	10.80	13.63	13.56	16.92
6b. Nondurables	9.09	9.57	10.06	10.70	11.20	11.90	12.45	13.18
6c. Tobacco	4.88	5.07	5.26	5.51	5.69	5.94	6.14	6.39
Total	21.32	22.58	24.31	27.27	27.78	31.47	32.15	36.49
Actual Purchases	21.93	23.18	24.66	26.43	28.32	30.41	32.89	35.68
Deviation of Estimated Magnitudes from Actual, per cent	−2.[illegible]	−2.59	−1.42	3.18	−1.91	3.49	−2.25	2.27
1. Alcoholic beverages								
Estimated Purchases	11.85	12.06	12.26	12.50	12.67	12.89	13.04	13.23
Actual Purchases	12.02	12.20	12.34	12.61	12.55	12.76	12.80	13.16
Deviation of Estimated Magnitudes from Actual, per cent	−1.41	−1.12	−0.61	−0.64	0.96	1.01	1.90	0.56

forecasts relating to consumer purchases for items 5 and 6 proved substantially more accurate than those given in Table 5.7.

Apart from the fact that account was taken of accumulation processes, the improvement of the forecast was due to the basic data used being for the longer period (1959 to 1963, instead of 1959 to 1961). Let us show that this also led to a crucial improvement in the forecast estimates for the item "Alcoholic beverages", where quite considerable errors were indicated above.

A sophisticated retrospective forecast for the item "Alcoholic beverages". Table 5.9 presents a new version of the retrospective forecast for item 1. It was obtained provided that the calculation of v_1^{max} being carried out with the aid of formula (5.23), using $v_1^{1963} = 11.60$ (but not $v_1^{1960} = 10.51$ as earlier; see Table 5.7). As already noted, the forecast was an underestimate owing to the underestimation of magnitude v_1^{max}. Using the basic data for a longer time period corroborates this inference: this time $v_1^{max} = 13.83$ was obtained instead of the earlier 11.21. As a result, the forecast for the years 1964 to 1971 proved accurate, the error being irregular in sign and its maximum value equalling 1.9 per cent modulo.

The results presented in the last sections provide additional data for answering the question concerning the dependence of structural shifts in consumption on changes in relative prices. As can easily be seen, actual mass consumer behaviour in France for the period under consideration could be explained with great accuracy (with the aid of the retrospective forecast technique) without using any data on changes in relative prices.

General conclusions. The study using statistical data for consumer purchases in France yield rather typical results. They are, on the whole, close to those derived from the statistical data for more than 20 countries. The studies carried out show, in general, that in mass consumer behaviour we are dealing with a very strong and precisely operating law approaching, in these respects, the laws of natural sciences. An analogy with natural sciences allows conclusions to be drawn concerning the specific characteristics of formula (5.16), using the concept of the invariance of the laws in relation to certain factors.

First, this is invariance in relation to large aggregates of goods satisfying various needs (in relation to j), j' being

fixed. A law has the same presentation for all j and does not change if they are renumbered.

Second, this is invariance in relation to a large specified aggregate j which is a basis for calculating the structure of consumption. The law preserves its constant representation, given any j'. The specific consideration of foodstuffs as j' is merely for convenience of calculation and exposition of the results of the analysis, but essentially the same results can be obtained, given any j' of the large aggregates mentioned.

Third, this is invariance of structural shifts in consumption in relation to prices for those time periods during which vector v^{max} tends to change insignificantly; in this sense this is independence of the structure of consumption from relative prices.

Fourth, this is invariance of the presentation of the law in relation to changes in its parameters, i.e., vectors v^{max} and v^{min}, over space and time, and therefore in relation to the units of measurement employed for the aggregates. The presentation of the law proves to be the same for a certain nation at different stages of its development when $v^{min}(t)$ and $v^{max}(t)$ tend to change; it is also the same for different nations. At the same time, the law itself implies that parameters v^{min} and v^{max} are not constants and tend to change over space and time.

Since the objective consumption function (utility function of consumer goods) and the curve of the optimum increase in consumption generated by it were actually sufficiently corroborated, grounds are provided for drawing conclusions of common importance to the methodology of economic science. At least three such conclusions may be indicated.

The first applies to the issue of the *scientific productiveness of the objective approach* to cognition of social development phenomena, immediately appearing to be a result of subjective decisions taken by individuals. The subjectivist approach has so far failed to find the shape of the indifference surfaces that would permit specific features of the mass facts observed to be explained. The objective approach has made it possible to use the usual scientific method of presenting a hypothesis that is not immediately a generalised description of the facts, but conjecture on the researcher's part, but taken to consequences that can be subjected to actual verification. It is this method that led to the solution of the problem. Thus, the general Marxian idea asserting the exist-

ence of natural historical laws of the development of social processes was corroborated once more.

The second conclusion concerns the problem of *the form in which economic theory can be presented and developed.* In most advanced natural sciences, especially physics, theory assumes the form of a compact mathematical presentation, often a formula, from which specific features of the observed facts can, by empirically determining its parameters, be derived with an accuracy sufficient for all practical purposes; if the parameters are subject to variations, the consequences of practical actions or processes can be predicted on its basis. The possibility of employing the same form in economics is often questioned with reference to the complex nature of the subject. Yet a long-lasting tradition of precisely such a description of the most complex specific features of the economy is characteristic of Marxian economic theory. Let us cite, as an example, Marx's description of relations of exploitation in the formula of the rate of exploitation. In this particular case it seems demonstrated that the aggregate of empirical regularities of consumer behaviour, in both quantitative and qualitative form, can be derived from the utility function.

The third conclusion answers the question concerning *the possibility of constructing a scientific theory of the utility of products.* The development of subjectivism in the approach to the issue of utility made bourgeois theoreticians ultimately refute research on this issue in economics; in recent years, the opinion has become widespread that attempts to construct any theory of consumer behaviour should be given up in view of the chance nature and arbitrariness of this behaviour. These views are penetrated by agnosticism, which is, in general, an inevitable consequence of subjectivism. It is very instructive that, precisely in the field where the subjectivist methodology sees the greatest reasons to exist, in the field of consumer behaviour, in reality an objective law does operate, subordinating this behaviour to itself. Since such a law exists, a scientific theory is also possible in the full sense of this word, but it is possible only on the basis of the general methodology of Marxian political economy.

There seem to be good reasons for maintaining that the problem of measuring (and comparing) the utility of consumer goods, concerning which it has become common to assert in the literature that either it cannot be solved at

all, owing to the subjectivist estimates of utility, or at any rate not for the time being, *is, in principle, solved.*[1]

5.5. The Social Utility of Means of Production

The changes in the level and structure of social consumption are ultimately engendered by an increase in the productivity of labour based on the creation of more and more advanced means of production. For this reason, the change in the structure of their production should be analysed more closely.

Two aspects of the social utility of means of production. The use-value of means of production is dual in character. On the one hand, means of production are those for producing certain products, ultimately (or directly) consumer goods. On the other hand, means of production are those for economising on social labour and raising its productivity. Both aspects of the use-value of means of production are associated with their specialisation.[2]

Each means of production can serve to produce only a specific class of objects, no matter how wide it might seem (an exception being some universal means of production, for example, electric energy). Moreover, each means of production ultimately serves to produce some objects for non-productive purposes (personal consumption goods, military hardware and the like), i.e., products of Department II of social production (in Marx's classification).

This is rather complex economically. While the class of objects for the manufacture of which some means of production serves directly is always strictly determined by

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[1] Yet precisely for this reason other problems, in reality still outstanding, have arisen: of comparing utility for substantially disaggregated groups of goods with regard to their substitutability and mutual complementarity; of predicting changes in the system of requirements (i.e., forecasting shifts in vector N^{max}), and the like. In this work we cannot dwell in detail on the studies aimed at solving them.

[2] To complete the picture, it should be noted that there exist two other aspects of the use-value of means of production, determined by their technological characteristics. First, their properties affecting working conditions: in these characteristics, means of production as such constitute, in fact, consumer goods. Second, their characteristics determining the effect of technological processes on the natural environment. We shall not consider these aspects of the social use-value of means of production in any more detail.

the natural form of this means of production, the class for the manufacture of which it serves indirectly, through a number of intermediate stages, is largely physically indefinite. The further a given production process is from the final process of production of nonproductive goods, the greater this indefiniteness. For example, the equipment and raw materials of enterprises in the garment, knitware, or food industries can be used only for a very limited number of purposes and a relatively narrow class of consumer goods can be produced by them. Universal metalworking equipment, construction equipment, transport means, metals, cement, electrical power, etc. are quite a different matter. In fact, they are involved directly or indirectly in the production of all or almost all nonproductive goods. The destination of means of production would, however, become substantially narrower if not only their physical form were taken into account, but also the economic laws determining the structure of the output produced by society. For example, the metal produced is divided in economic terms into parts intended for the manufacture of quite specific goods, this being determined by the need for it and for metal goods in the various sectors of the national economy.

The dependence of the structure of the production of means of production on that of nonproductive goods varies in intensity at the different stages of the development of society. It is weaker during industrialisation, associated with considerably faster growth rates of Department I than of Department II. Its significance tends to rise when the growth rates of both departments of social production draw closer together. The sort of temporal lag separating the creation of a certain means of production from the effect obtained in the form of a certain nonproductive goods has decreased substantially.

The dependence of the structure of the production of means of production on that of the production of consumer goods is multistage in nature, since most kinds of means of production are used again to produce means of production. It is usually impossible to judge, for a certain specific means of production, whether it complies with the need to ensure the required pattern of consumer goods or not. Its utility is expressed directly only by the fact that it ensures the production of some other means of production, the latter of still others, etc., until it finally comes to means of production immediately employed in producing consumer goods.

The analysis of this first aspect of the utility of means of production provides, however, a certain objective basis for comparing their utility: the proportions of the production of means of production in general must correspond to those of the production of nonproductive goods.

From the other point of view, however, the utility of means of production is different even if the proportions of the production of consumer goods are adhered to completely; under all conditions, the dependence under consideration does not determine the physical structure of the production of means of production at all strictly, since a certain set of products of Department II can be obtained in very numerous ways, using different means of production. To determine further the physical structure of the output of Department I it is necessary to take account of the latter aspect of their use-value, i.e., the ability to economise on labour in the process in which they are employed. At this point we are concerned with technological progress as the basis for raising the productivity of social labour and, hence, that of any other social progress.

Here we can see particularly clearly the possibility and necessity of measuring and comparing the use-value of the most various means of production. The main, most important result of improving production techniques is savings of labour.[1]

The characteristic property of means of production that they affect the volume of labour input in producing output is an objective basis for comparing the social utility of the most various means of production from this point of view. The savings of social labour resulting from the

[1] Together with this, production techniques are usually subject to improvement in order to ensure the production of new kinds of product. Yet the very possibility of producing these arises in society only if the labour spent on producing the old kinds has been reduced, and consequently if resources are saved to satisfy the new needs. As far as many new kinds of means of production are concerned, savings of labour are not only the main result of introducing them into production, but also even the single *direct* result of this; there are very few means of production the immediate use-value of which is that, without them, a certain kind of output cannot be produced at all, i.e., they cannot be replaced where they are employed. This is particularly true with respect to consumer goods production. Here the number of cases where it would be technologically completely impossible to satisfy a given need if a certain kind of means of production were not available is extremely small.

employment of means of production are evidently determined by their technological properties, that is, by their use-value.

While technological progress is expressed in each industry by the introduction of the most diverse means of production, completely dissimilar in natural form, improvements of the means of production pursue one and the same aim and lead to one and the same outcome: a decrease in social labour inputs in producing aggregate social output. The saving of labour is, in each industry, a saving of social labour as a whole, a saving in the use of social labour power. Consequently, a common criterion for utility, i.e., the resulting saving of social labour, is inherently characteristic of the aspect of the use-value of means of production under consideration.

The existence of a common criterion for utility is a specific characteristic of means of production, being of extreme importance in clarifying the laws of national economic development and applying these laws in practice. As far as consumer goods are concerned, there is also a certain common criterion for utility, this being the total level of satisfaction of the set of personal needs (see the properties of function D). This is, however, rather a criterion common to all of them than one to be applied to particular consumer goods. Taken separately, in isolation from their specific interrelations and proportions, consumer goods cannot yield, in any sense of the word, a determined level of welfare, so this criterion, as a rule, is not applicable to particular consumer goods. It may only be applied to the various collections of consumer goods. A correct approach to comparing their utility thus consists in finding the optimum proportions between them so that the highest total level of consumption possible under the given circumstances might be achieved. Comparison of the utility of means of production is also attempted in order, ultimately, to find the proportions of their production, but this aspect of their utility (i.e., savings of labour) can be quantitatively determined in every particular case.[1]

[1] This does not mean that labour saving might be attributed to a particular kind of means of production. It is yielded by technical progress, this consisting in the replacement of some system of means of production by another one, associated with the different technology. For this reason, the magnitude of the utility of one particular means of production cannot be indicated quantitatively. Nevertheless, it is true that such a magnitude can be determined for a local set of means of production forming some technology and superseding some other technology. In contrast, there seems to be no (quantitative) concept

Improvements in the technological characteristics of means of production are always expressed by the aspects of their utility under consideration.

Two main aims are pursued in enlarging and renewing the range of instruments of labour manufactured: a cut in the expenditure of social labour in traditional industries and the creation of the needed technical facilities for new ones. Both aims are interrelated: the creation of new technology for traditional industries itself requires that new production industries emerge; on the other hand, in new industries the need arises to employ more effective kinds of technology.

Improvements in the production of quality specialised materials have the same main aims. First, they pursue the goal of reducing expenditure on producing traditional products. This can be exemplified by the substitution of synthetical materials for metals, wood, leather, and natural fibres in the production of a number of machines, construction materials, footwear, fabrics, and so on. Second, the aim may be to create the materials needed to produce some new products. Research and production, particularly at present, need more and more materials of increased durability and heat resistance, ultrapure materials and the like. Advances in and the prospects for such industries as electronics, aviation and aeronautics, the fuel and power, chemical engineering, and instrument-making, etc. are largely indebted to the creation of such materials. Various materials are also needed for enlarging the mix of consumer goods. The two aims of improvements in the production of specialised materials are closely interrelated: the creation of new materials often simultaneously reduces expenditures on production and provides for the production of new goods. Moreover, traditional materials are not necessarily bound to disappear entirely; on the contrary, it becomes possible to employ them on a wider scale in those productive activities to which their characteristics best correspond. Sort of "hybrids" of traditional and new materials are engendered, combining the valuable characteristics of both (for example, plastic-covered steel sheet).

In fact, the formalised presentation of utility of means of production was given above, in Chapter 2, during the

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itself of the individual utility of consumer goods, even taken in a full set.

presentation of the input-output model and the law of value, including such a consequence of it as different profitability of various firms and related technological progress.[1]

[1] Strictly speaking, the ratio of consumption to accumulation in the national income also belongs to the issue of utility theory. We omit here an exposition of the solution to this problem on the basis of Marx's economic theory, but such a solution does exist [see K. Valtukh, *Satisfying the Needs of Society, and National Economic Modeling*, Nauka Publishers, Novosibirsk, 1973, pp. 199-207 (in Russian)].

Appendix 1
to Chapter 4

Coefficients of Variation of Labour Intensity of Realised National Income for Nine Large Sectors of the US Economy
(per cent)

Year	Labour intensity without regard to labour reduction		Labour intensity with regard to labour reduction	
	without regard to sectoral shares in national income	weighted by sectoral shares in national income	without regard to sectoral shares in national income	weighted by sectoral shares in national income
A	1	2	3	4
1948	23.4	21.5	16.2	14.6
1949	19.8	16.3	15.6	12.8
1950	19.0	15.2	14.9	12.0
1951	21.6	17.4	15.4	12.0
1952	21.2	16.4	15.5	11.5
1953	20.0	14.7	14.2	9.7
1954	20.0	13.9	16.0	11.1
1955	18.6	12.7	16.1	11.0
1956	19.2	13.0	15.9	10.7
1957	18.8	12.0	15.8	10.7
1958	20.3	14.2	16.4	11.7
1959	19.6	12.9	17.1	11.3
1960	21.5	14.5	17.8	11.5
1961	22.1	14.9	17.6	11.6
1962	22.7	15.1	17.9	11.3
1963	23.5	15.7	17.8	11.0
1964	23.2	15.5	17.2	10.7
1965	24.7	16.9	16.9	10.0
1966	25.0	17.0	17.2	10.0
1967	25.4	16.8	16.9	10.2
1968	25.3	16.9	16.8	9.9
1969	26.3	17.2	16.7	9.7
1970	25.6	17.2	14.7	9.4
1971	25.5	17.5	15.1	8.7
1972	26.8	18.8	13.3	7.4
1973	30.1	21.9	12.4	7.1
1974	28.5	19.9	10.7	6.3
1975	29.8	21.2	13.3	7.4
1976	30.1	21.7	12.9	7.1
1977	30.7	22.5	13.2	7.3
1978	32.8	23.8	13.3	7.2
1979	33.4	24.2	11.6	6.4
1980	31.4	23.2	13.9	8.3
1981	33.3	24.5	14.7	8.6

Appendix 2

to Chapter 4

Coefficients of Variation of Labour Intensity of Realised National Income for Industries of the US Manufacturing (without Regard to Labour Reduction)

(per cent)

Year	Without regard to industries shares in national income	With regard to industries shares in national income	Year	Without regard to industries shares in national income	With regard to industries shares in national income
1948	24.5	25.6	1965	33.6	30.7
1949	27.1	27.3	1966	33.1	29.3
1950	30.7	31.1	1967	32.0	27.8
1951	28.7	29.2	1968	31.9	27.7
1952	28.4	27.6	1969	31.2	26.0
1953	30.2	29.0	1970	29.9	24.5
1954	30.3	29.2	1971	32.7	28.0
1955	32.4	32.0	1972	34.7	29.1
1956	31.1	29.7	1973	35.4	30.5
1957	32.2	29.7	1974	33.5	30.7
1958	31.4	27.6	1975	35.4	30.4
1959	33.7	30.0	1976	37.7	34.1
1960	32.4	29.1	1977	37.5	32.5
1961	33.1	28.6	1978	37.6	32.2
1962	33.0	29.3	1979	37.7	32.8
1963	33.2	29.8	1980	36.6	34.4
1964	33.1	29.7	1981	37.9	34.1

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Konstantin Valtukh is a scientific worker of the USSR Academy of Sciences Institute of Economics and Production Engineering (Siberian branch) and author of many books and articles in prestigious foreign journals as well as in Soviet ones. In his treatment of important theoretical problems he widely relies on economic and mathematical modelling.

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